#### INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

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# NPTEL ONLINE CERTIFICATION COURSE

#### **Unit Operations of Particulate Matter**

# Lec – 13 Liquid Fluidisation

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Welcome to the 3<sup>rd</sup> lecture of week 3 and in this lecture we will discuss liquid fluidisation.;

(Refer Slide Time: 00:26)



So if you see this image in the first image the particles are away from each other it means particles are already at fluidized condition so this condition occurs when liquid velocity is equal to the minimum fluidizing velocity why I am telling this that liquid velocity because here we are discussing liquid fluidization, so fluid we are considering as liquid, so this is the condition when minimum fluidization achieved.

Now if you see second image here particle are separated far from each other they are more way then these are in figure 1 and this is achieved when liquid velocity will be much higher than that of minimum fluidizing velocity, so liquid velocity is much higher than minimum fluidizing velocity, so you see here even if velocity are much greater than minimum fluidizing velocity particle are separated almost uniform in the bed particle are spread almost to uniform in the column.

So this is the effect of liquid fluidization through liquid bubbles are not formed in the bed and therefore in this case particulate fluidization occur not aggregative fluidization.

(Refer Slide Time: 01:57)



So when we are considering liquid fluidized system these are generally characterized by regular expansion of bed that takes place as the velocity increase from minimum fluidization velocity to terminal falling velocity of the particles so you see here when we are discussing liquid fluidization the expansion of bed is regular it is not like that that some in some section of the bed more particles are available and some are very in some places no particle is available the particles are spread the particles are spread uniformly in the bed and that is due to liquid fluidization.

And this happens when the velocity of the fluid is greater than minimum fluidizing velocity and lesser than terminal falling velocity now what happens beyond terminal falling velocity because beyond terminal falling velocity the particle will be carried away by the liquid because when we are considering terminal falling velocity of the particle it means particles falls with a uniform velocity.

Now the fluid velocity is greater than terminal falling velocity the fluid take the particle with it and it can be carried away so that is the condition when we have that condition we should obtain when we have to transport the solid from liquid, no win this case as bed is uniformly as bed is regularly expanded now in this case when bed is regularly expanded it means particle which are available that are spread uniformly.

So void age we can calculate inside the bed because almost uniform void age will be appear uniform space for void age will be appear every where so that can be related with the velocity so Richardson and Zaki 195 showed that for fluidization of uniform particles  $u_c/u_i = e^n$  so  $u_c$  is basically\ the operating velocity at fluidization occur  $u_i$  is the velocity at infinite condition and how we will calculate  $u_i$  that we will discuss and this should be equal to void age power n.

So n is the index over here uc is empty tube fluidization velocity or the actual velocity of fluid and how we can calculate ui expression is this log ui  $-\log u0 - d/dt$  so you see here u0 we are considering with free velocity of the particle so here indirectly we are relating actual velocity particles and terminal falling velocity of particles with void age.



Now values of index n which we have seen in the last equation ranges from 2.4 to 4.8 for fluidization at a given value of Galileo number and that can be calculated from this equation 4.8 - n / n- 2.4 and which is related with goalie number by this expression where d is basically particle diameter and dt is the diameter of the column or tube so considering this expression we can calculate value of n.

Now further Khan and Richardson 1989 they have proposed a relation between  $u_i$  and  $u_0 u_i$  we have seen for  $u_i$  we have already seen 1 expression in the last slide this is another and more resent expression this is  $u_i/u_0 = 1$ - 1.15 d/ dt<sup>0.6</sup> so u0 is basically terminal falling velocity of the particles and from here we can calculate ui so once we calculate the value of  $u_i$  and velocity inside the fluid inside the fluidized bed is known so we can calculate void age but before that we have to calculate index and by equation which we have just discussed.

(Refer Slide Time: 06:50)

Another approa	ch for co	mputin	index
$n = \left[a_o + b_o\left(\frac{d}{d_r}\right)\right] \left(1\right]$	Re <sub>γ</sub> )‴	Re <sub>T</sub> =	$\frac{d_t u_0 \rho}{\mu}$
Reynolds no. Re <sub>T</sub>	a <sub>o</sub>	bo	m
<0.2	4.65	20	0
	4.40	18	-0.03
0.2 < Re <sub>T</sub> < 1.0	4.40		
0.2 < Re <sub>T</sub> < 1.0 1.0 < Re <sub>T</sub> < 200	4.40	18	-0.01
0.2 < Re <sub>T</sub> < 1.0 1.0 < Re <sub>T</sub> < 200 200 < Re <sub>T</sub> < 500	4.40 4.40 4.40	18 0	-0.01 0.1

Now another approach for computing index n is this where  $n = a_0 + b_0 (d/dt) (Re_t)^m$  now what is this Reynolds number  $t^n$  is it is basically  $d_t u_0 \rho/\mu$  where  $\rho$  and d are the property of fluid,  $u_0$  is terminal falling velocity, so you see Reynolds number t is nothing nut the Reynolds number at terminal falling velocity, now when we are considering d/dt over here d is for particle and dt is for column or tube.

So this can be consider when we have to consider the effect of valve, how we consider the effect of valve because this expression is almost negligible when we are considering dt value should be very high, what is the meaning of this that if column diameter is very high in comparison to particle diameter it means the effect of valve will not be then the falling off then the movement of particle will not be affected by effect of valve then the movement of particle will not be affected by the valve of column.

But this will be considered when diameter of tube is not very large in comparison to diameter of the particle so through this equation we can calculate n now here we have again few factors like  $a_0 b_0$  and m and that we can calculate for different Reynolds number range we can calculate these

coefficients value we can see the value of these coefficients for different Reynolds number from this stable.

Therefore this correlation has been proposed for bed of spherical particle however with allowable error same may be used for non spherical particles while replacing d with the volumetric diameter of particle so you see using this expression we can calculate value of n for a spherical particle as well as non spherical particle but for a non spherical particle instead of diameter we have to consider volumetric diameter of the particle.

And you understand how we define volumetric diameter is the diameter of a sphere having same volume has that of the particle and here we will discuss a few examples to illustrate how the governing equation which we have developed which we have derived how these equation will be useful for fluidized for fluidization process how we compute different parameter how we compute minimum fluidizing velocity, mass flow rate etc. So to illustrate that let us start with example 1.

(Refer Slide Time: 09:55)



And in this example a bed consists of uniform spherical particles of diameter 2.5mm so particles are spherical in shape it is having 2.5mm diameter and density 5000 kg/m<sup>3</sup> a liquid having viscosity  $2mNs/m^2$  and density 1050 kg/m<sup>3</sup> is used to fluidized these particles we have to calculate minimum fluidizing velocity, so how we will calculate minimum fluidizing velocity if you remember second lecture of week 3 there we have discussed derivation of different expression.

Which can be used to calculate minimum fluidizing velocity analytically there we have discussed expression with this spherical particle there we have discussed expression of minimum fluidizing velocity of for a spherical particle and for non spherical particle as well, so for a spherical particle we have the expression for Reynolds number as well as Galileo number therefore first of all we will proceed to calculate Galileo number and that is  $Ga = d^3 \rho(\rho_s - \rho) g/\mu^2$ .

Putting all values over here diameter we know density of fluid and particle we know viscosity of fluid we know so considering all values putting all values over here we can calculate Galileo number as  $1.59 \times 10^5$  so Reynolds number relates Galileo number for spherical particle in this equation where Reynolds number mf is the Reynolds number at minimum fluidizing condition so for  $e_{mf} 0.4$  because we do not know the voltage.

So we have assumed voltage as 0.4 so Reynolds number  $m_f$  should be obtain by this expression while keeping all values while keeping the value of Galileo number here so the value of Reynolds number  $m_f$  should be 54.724 once we know Reynolds number at minimum fluidizing condition we can calculate minimum fluidizing velocity. So that should be  $\mu/d \rho$  Re<sub>mf</sub> putting all parameter over here we can obtain minimum fluidizing velocity as 0.0417 m/s and that is 41.7 mm/s, so in this way we can calculate the minimum fluidizing velocity.



Liquid having density 850 kg/m<sup>3</sup> and viscosity 5 mNs/m<sup>2</sup>, is passed vertically upwards through a bed of catalyst, which contains approximately spherical particles of diameter 0.15 mm and density 3000 kg/m<sup>3</sup>. Bed is packed such as particles of diameter 'd' are closely packed in a cube of side '2d'. Compute the mass flow rate per unit area of bed at which (a) fluidisation, and (b) transport of particles occur.

#### Solution

(a) Mass flow rate per unit area of bed at which fluidisation occur depends on minimum fluidisation velocity  $G'_{mf} = \rho u = (0.0055e^3/(1-e))(d^2(\rho_s - \rho)g)/\mu \qquad e = e_{mf}$ 

Now here I am having second example and in this example liquid having density 850 kg/m<sup>3</sup> and viscosity 5 mNs/m<sup>2</sup> is passed vertically upward through a bed of catalyst which contains approximately spherical particles of diameter 0.15 mm, so you see diameter of particle is very small and density 3000kg/m<sup>3</sup> so here again the bed of spherical particle with very small diameter so bed is packed such as particles of diameter d are closely packed in cubes of side 2D.

So this is the important line like here we have 2 particles in single side of cube, so size of cube should be 2D compute the mass flow rate as per unit area of bed at which fluidization and transport of particles occur, so we have to calculate mass flow rate per unit cross sectional area when fluidization will occur and when particle will be transported, so when we are considering fluidization condition what happens, that fluidization when fluidization will occur when minimum fluidizing velocity condition may be achieved.

(Refer Slide Time: 14:25)



So first of all we have to calculate minimum fluidizing velocity and then that should be multiplied with density so that we can calculate mass fluid per unit cross sectional area so lets start the calculation first of all we have to calculate minimum fluidizing velocity and here we have spherical particles is and diameter of particles are very less that is 0. 15mm.

So it is very fined particles so equation will be applicable over here so here we have we cn assume equation so  $u_{mf}=0.0055(\ e^3_{mf}/1-e_{mf})\ d^2(\rho_s-\rho_b)g/\mu$ . So here we should knew value of emf along with other parameters are already known to  $e_{mf}$  we should calculate. Once we know the minimum fluidizing velocity we can calculate mass flow rate per minute area of the bed because here we have the multiplied by with density.

So when we multiplied by the density the velocity we can get mass flow rate per minute in cross sectional area but before that we should calculate voltage so how we can calculate voltage in this problem diameter of the particle is deal and two particles in line on one side of the tube. So what is the shape is basically cubic you have to consider of side 2d through this wy we cn calculate this packing is done.

So when we consider the cubes side is 2d so when for example if considering this face of the cube so here this is one side so two particles will be dual. Similarly here we have hve two particles, here we have two particles so if we are considering cube this side of the cube of the sid 2d it means four particles will lie over here. Similarly four particles will lie over here because this side is should also have length has 2d so 2two particles will lie over here so in this cube when side 2d is considerate means total 8 particles will be used to prepare this cube.

(Refer Slide Time: 16:54)

As no value of the voidage is available, e will be estir closely packed spheres of diameter d in a cube of sid	nated by considering eight e 2d.
Volume of spheres = $8(\pi/6)d^3$	
Volume of the enclosure = (2d) <sup>3</sup> = 8d <sup>3</sup>	d = 0.15 mm
Voidage, e = [8d <sup>3</sup> - 8( $\pi$ /6)d <sup>3</sup> ]/8d <sup>3</sup> = 0.476, say, 0.48. $u_{mf} = 0.0055 \left(\frac{c_{mf}^3}{1 - c_{mf}}\right) \frac{d^2(\rho_s - \rho)g}{\mu}$	
u = 0.0055(0.48) <sup>3</sup> ×(15 <sup>-4</sup> ) <sup>2</sup> ×(3000-850) × 9.81)/((1 - 0.4 = 1.11 × 10 <sup>-4</sup>	48) × 5 × 10⁻³
$G'_{mf}$ = 850 × 1.11 × 10 <sup>-4</sup> = 0.0944 kg/m <sup>2</sup> s	

So volume of this pairs is equal to  $(8\pi/6)d^3$  so 8 should be because e have seen 8 particles will lie in cube and this is the volume of single sphere that is  $\pi/6 d^3$  so this will become calculate total volume of sphere. Now volume of the enclosure is because side is 2d so 2d<sup>3</sup> that is 8 d<sup>3</sup> should be the total volume of the enclosure.

Now how we can calculate void age so void age can be defined as the total volume of liquid divide by the volume of the enclosure so volume of liquid means volume of enclosure minus volume occupied by the sphere. So that should be  $[8d^3-8(\pi/6)d^3]/8d^3$  so value comes sa 0.476 e can take this 0.48.

Once you know the value of void age we can calculate minimum fluidizing velocity by this be expression so while putting all values in this expression we can calculate minimum fluidizing velocity as  $1.11*10^{-4}$  and mass flow rate when this velocity will be multiplied with density of solid we can calculate mass flow rate at which fluidization will occur and that should be equal to  $0.0944 \text{ kg/m}^2\text{s}$ .

(Refer slide Time: 18:30)

b) Tra he te As par hus, S	nsport of the particles will occur when the fluid velocity is equal to rminal falling velocity of the particle. ticle size is very small, laminar flow of particle may be assumed and stoke's law is applicable.
Using = ((15 = 0.00	Stokes' law : u <sub>0</sub> = d²g(ρ <sub>s</sub> – ρ)/18μ <sup>4</sup> )² × 9.81 × (3000-850))/(18 × 0.005) 52 m/s
Re = (	15 <sup>-4</sup> × 0.0052 × 850)/0.005 = 0.134
As Rey	nolds number is less than 0.2, Stoke's law will be applicable.
The re	quired mass flow = (0.0052 × 850) = 4.42 kg/m²s

So this is the condition t which fluidization will occur and let us start the second part were we have to transport the particles now how the particles will be transported that we have already discussed that when velocity of the fluid would be equal to or the greater than the terminal falling velocity on the particle.

Then only it can be transported with fluid so here we have to calculate terminal falling velocity now to calculate laminar falling velocity first of all we should understand the which zone particle is moving. So as we have discussed earlier the diameter of the particle is very small so we can assume the flow over there. Once laminar flow over there we should applied a strokes law over here in a strokes law the terminal cycling velocity will be in these squares putting all values over here we can calculate that terminal cycling velocity as 0.0052 m/s. so now once we know this we can calculate the Reynolds number and that Reynolds number as 0.34 so you should understand that as this Reynolds number is less than .2, stroke's law will be applicable easily over here.

So once here we have the terminal falling velocity or terminal cycling velocity we can calculate require mass flow rate while multiplying this with density of solid so that comes as 4.42kg/m<sup>2</sup>s. So here we should understand how we have calculated mass flow rate for fluidization condition and for transportation condition.

(Refer Slide time: 20:14)

particles as :	e' is related to fluid velocity 'u <sub>c</sub> ' for particulate fluidisation of uniform $\frac{u_c}{u_0} = e^n \qquad \text{where u}_0 \text{ is the free falling velocity.}$
For particles of value of 2.6. It the relation be colution	f glass with free falling velocities of 40 and 20 mm/s the index n has a f a mixture of equal volumes of the two particles is fluidised, what is etween the voidage and fluid velocity?
Considering Voidage of la	unit volume of each particle as 1 m <sup>3</sup> arge particles = e <sub>1</sub> , volume of liquid = e <sub>1</sub> /(1 - e <sub>1</sub> ).

So here we have discussing third example for liquid fluidization and in this example bed void age e is related to fluid velocity you see for particles fluidization of uniform particles has you see  $u_c/u_o=e^n$  so here terminal cycling velocity  $\mu$  is consider for particles of glass for particles of glass will free falling velocity of 40mm per second and 20 mm/s the index n has a value of 2.6.

So you see what is given that here we have two particles one is falling with 40mm/s cycling velocity and another one is falling with 40mm/s velocity and another is falling with 20mm/s velocity and value is given as 2.6. If a mixture of equal volumes of two particle is fluidized what is the relation between void age and fluid velocity, so here you see what we have to calculate is the velocity as well as void age relationship, velocity means the you see as a function of e that we have to find.

So let us start the solution here we have considered unit volume of each particle as  $1 \text{ m}^3$  because if you consider the problem it is given as the mixture of equal volumes of two particles has to fluidized, so in that case volume of two particle should be same and that we have assumed as  $1\text{ m}^3$ . Now void age of large particle is  $e_1$  and volume of liquid is  $e_1/(1-e)$ , so how we define  $e_1$ ,  $e_1$ should be equal to volume of liquid divided by volume of liquid plus volume of solid.

So when we solve this equation and calculate volume of liquid it will come out as  $e1/(1-e_1)$  into volume of particle and that we have taken as 1 m<sup>3</sup>. Similarly void age of small particle that is e2 volume if liquid is e2/1-e2 so volume of liquid we have defined.



(Refer Slide Time: 22:46)

Now total volume of the solid that is because equal volume so  $2m^3$  is there total volume of liquid we have derived in the last slide and both expression should be added, so total volume of the system is volume of solid that is 2 plus volume of liquid, so this is total volume of system. Now void age is volume of liquid divide by volume of the system so putting all expressions over here and resolving it we can have void age as  $e_1+e_2-2e_1e_2/2-e_1-e_2$ .

(Refer Slide Time: 23:22)



This is the void age and in the problem falling velocity of particle 1 is 40 and second is 20, so free falling velocity ratio is 2:1 then e1 can be defined as  $e_1=(u/u_{01})^{1/2.6}$  so this is the expression of e we have given and for first particle terminal settling velocity is 0.01 and similarly for second particle  $e_2$  can be  $u/u_{01}/2$  because it is  $\frac{1}{2}$  than the 1 a power 1/2.6. So you see we can have e2 in terms of e1 and that should be  $e_2$  should be equal to  $e_1 21/2.6$  and e1 should be  $(u/40)^{1/2.6}$ .

Now substituting e1 and e2 in this equation we can find this equation and resolving it we can get equation of this form.

(Refer Slide Time: 24:37)



Further resolving the expression of e as a function of u we can get this expression and then we can further simplifying this and once we collect value of u we can obtain equation as  $u^{0.7692}$ -3.649(1+e) $u^{0.3846}$ +13.08e=0 so this equation we have obtained now this is quadratic equation how we can say because this power is just double then this so this is the quadratic equation once we solve this quadratic equation we can get  $u^{0.3846}$ =1.824(1+e) and this expression.

So here you see  $\pm$  both compounds are available because this is quadratic equation, so what is the correct expression of u and e to know this we have to draw the relation of u and e for + component as well as for – component.



So when we are drawing graph with positive component then here we have u and voidage so this flat curve and then slide increment so here we have flat curve and then slide increment is observed. Further if we draw the minus component here we have u in nm/s and here we have void age, so here void age and so here velocity is continuously increased with the voidage. So when we consider first graph here flat curve is obtained however we know that when the velocity increase void age increases.

So that should be in continuous relationship so here when we are comparing positive component and negative component, negative component gives more realistic relationship and that should be the final relation we have asked to draw between u and e. (Refer Slide Time: 26:56)



And here we have summary of the lecture, in this lecture liquid solid system that is liquid fluidisation is discussed relationship between fluid velocity and void age is described and finally we have discussed few examples on liquid fluidisation.

(Refer Slide Time: 27:13)



And here we have the references through you can read liquid fluidisation in detail and that is all for now, thank you.

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