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Unit operations of Particulate Matter

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Welcome to the 2nd lecture of week 3 which is on fluidization, now if you remember lecture 1 of week 3 there we have defined fluidization we have discussed minimum fluidization condition how it achieves that we have discussed now in this lecture we will see how to compute minimum fluidization velocity for fluidization process analytically as well as experimentally.

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So you see if flow conditions within the bed are streamline the relation between fluid velocity you see pressure drop ΔP and voidage is given for a fixed bed of spherical particle of diameter d by Carman- kozeny equation so you see here we are considering a streamline motion a

streamline movement of fluid it means we are considering laminar flow and for that purpose Carman-Kozeny equation is defined for fixed bed, so first we will see this equation of our fixed bed and then we will transpose we will translate to the fluidized bed.

So for fixed the Carman-Kozeny equation is this $uc = 0.00055 e^3 1 - e$ whole square pressure drop that is $\Delta P d^2 / \mu l$, so pressure drop across the bed μ is the fluid viscosity l is the height of bed d is the particle diameter and it is specifically proposed for spherical particle, so this is for fixed bed now for fluidize bed the buoyant weight of the particle is counter balanced by the frictional drag.

Now what happens in fluidize bed if you consider if you remember the second if you remember the first lecture of week 3 the last slide of that lecture contains the derivation of pressure drop of fluidized bed. So that pressure drop relation we have obtained like this that is pressure drop equal to $11 - e \rho s - \rho$ into g if you remember this expression we have derived this donned forced balance.

Now when we put this pressure drop over here we can have the velocity for fluidized condition, so in that case u_c should be equal to 0.0055 $e^3 1 - e d^2 \rho s - \rho g / \mu$. So while putting pressure drop for fluidized condition we can obtain this velocity u_c for fluidized bed, now as we have to define the fluidizing as we have to define the minimum fluidizing velocity some other factor we have to consider.

Minimum fluidising velocity

As the upward velocity of flow of fluid through a packed bed of uniform spheres is increased, the point of incipient fluidisation is reached. The corresponding value of the minimum fluidising velocity (u_{mf}) is then obtained using e_{mf}

$$u_{mf} = 0.0055 \left(\frac{e_{mf}^3}{1 - e_{mf}}\right) \frac{d^2(\rho_s - \rho)g}{\mu}$$

The value of e_{mf} will be a function of the shape, size distribution and surface properties of the particles. Substituting a typical value of 0.4 for e_{mf} :

$$(u_{mf})_{e_{mf}=0.4} = 0.00059 \left(\frac{d^2(\rho_3 - \rho)g}{\mu} \right)$$

Because as the upward velocity of flow of fluid through a packed bed of uniform spheres is increased the point of incipient fluidization is reached incipient fluidization you understand that is minimum fluidization and that we have already discussed in the last lecture, so the corresponding value of minimum fluidizing velocity that is u_{mf} is then obtained using e_{mt} if you remember the last slide where we have define the void age of the bed in terms of e now velocity corresponding to that is u_{c} once we have to define the velocity for minimum fluidization condition that u_{mf} .

So void age which is obtained at minimum fluidization condition that also we have to consider and that will be represented by emf, so the same equation but with different terminology umf we are considering and emf we are considering, value of emf will be a function of shape size distribution and surface properties on the particle substituting a value of emf as 0.4 we can get minimum fluidization velocity is 0.00059 and this expression is put as it is from the previous equation.

So you see here though we have define we have to calculate void age but if void age is equivalent to 0.4 the minimum fluidization velocity we can calculate directly from this

expression, so all this expression are valid tell the laminar flow occur or tell the validity of Carman-Kozeny equation will be found so when the flow regime at the point of minimum fluidization or incipient fluidization.

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Is outside the range over which Carman-Kozeny equation is applicable like if flow is not a streamline it is necessary to use one of the more jungle equations for pressure gradient in the bed such as Ergun equation, so when we see the Erguns equation it is denoting the pressure drop across the bed and the whole equation is like this where d is diameter of the sphere with the same volume and same area ratio as the particle, so diameter should be equal to the diameter of a sphere having same volume.

And same surface area as the particle, substituting $e=e_{mf}$ at minimum fluidization condition and velocity u_{mf} we will put over here, so and pressure drop condition pressure drop expression we will consider for pressure drop we have define for fluidizing condition all these expression we will put over here so this is the expression for Erguns equation when minimum fluidization condition will be achieved.

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Now we will multiplied both side of Ergun's equation at minimum fluidization condition by this factor that is $\rho d^3/\mu^2 1$ – emf then we multiplied this we are getting a equation like this, now in this equation here if you consider this particular expression this is resembling to or this is equal to Galileo number, Galileo number is defined through these parameters and similarly if we are considering this factor this expression what is this, that is nothing but the Reynolds number at minimum fluidization condition.

So because here we have minimum fluidization velocity so accordingly Reynolds number would be at minimum fluidization condition, so what we are doing here is we are converting whole Erguns equation at minimum fluidization condition in dimension less form, so final equation should be Ga that is Galileo number equal to $150 \ 1 - e_{mf} / emf^2$ that is already dimension less and here we have Reynolds number and this parameter this factor we have kept as it is and further this factor will be equal to Reynolds number square at minimum fluidization condition so you see it is in terms of dimension less groups. (Refer Slide Time: 08:15)



For typical value of e_{mf} is equal to 0.4 Galileo number should be like this 1406Re at minimum fluidization condition plus 27.3 Re² at minimum fluidization condition, so this final equation we can obtained at $e_{mf} = 0.4$ once we solve this we can get Reynolds number 24.7 and whole expression is showing like this similarly for void age 0.45 Reynolds number we can find by this we can obtained by this equation.

Now once we have the Reynolds at minimum fluidization condition that we have already seen how we have how we can get that for a stream line flow and for non stream line flow also, once we have this Reynolds number value we can calculate the minimum fluidisation velocity through this, so by this way we can calculate minimum fluidisation velocity analytically. But this is specifically used for spherical particles. (Refer Slide Time: 09:25)



Now how we can calculate minimum fluidisation velocity experimentally, the minimum fluidising velocity u_{mf} may be determined experimentally by measuring the pressure drop across a bed for both increasing and decreasing velocity and plotting the results. So what happens we can carry out fluidisation experiment and we have to note down the data of pressure drop as well as velocity, and that pressure drop and velocity data we can draw in the graph as shown over here.

So first of all we draw this graph for increasing velocity and then for decreasing velocity we can observe this nature of the graph, so the two best straight lines are then drawn through a experimental point and the velocity at which their point of intersection is taken as the minimum fluidisation, minimum fluidising velocity, so how we can calculate minimum, how we can calculate minimum fluidising velocity is this is one straight like this is another straight line if we join these two line this should be the intercept so that intercept is resembling to minimum fluidising velocity.

So in this way we can calculate minimum fluidisation velocity experimentally, so till now we have seen the minimum fluidisation velocity expressions for spherical particle only, but in

general particles are, particle carry different shapes so how we can consider different shapes of the particle to compute minimum fluidisation velocity or minimum fluidising velocity.

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For this WEN and YU examined the relationship between voidage at minimum fluidising velocity that is e_{mf} and particle shape ϕ_s , ϕ_s particle shape when we are considering, we are speaking about sphericity of the particle which is defined as $\phi_s=d/d_p$ so this expression is given by WEN and YU. From this condition we can calculate whatever d we have used earlier all that d will replaced with ϕ_s into d_p , where d is $6V_p/A_p$ and dp is $6V_p/\pi$.

So here you see this d_p is basically diameter of the particle having equal volume as that of the particle which is nothing but V_p . In practice the particle size d can be determined only by measuring both volumes like V_p and areas A_p of the particles, so once we know V_p as well as area Ap then only we can define diameter d as we can see from this equation, since this operation involves somewhat tedious experimental technique because we have to observe two parameter for the characterizing, we have to observe two parameter to characterize the particle.

It is more convenient to measure particle volume only and then work in terms of d_p and the shape factor. So you see dp is only based on particle volume so once we calculate the particle volume we can carry out, we can define the particle diameter it is not necessary to define, necessary to calculate or to obtain volume as well as area of the particle.

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The minimum fluidising velocity is a function of both e_{mf} and ϕ_s and Yu showed that these two quantities are in practice inter-related so voidage as well as shape of the particle will be interrelated by how we can say that because voidage can be obtained based on shape of the particle itself like if particles are sphere the uniform voidage we are, we can obtain throughout but if particle shape are quite irregular we can have small voidage somewhere and somewhere we have large voidage, so e_{mf} will be directly associated with the shape of the particle.

So these authors like we, I speaking about WEN and YU have published experimental data of e_{mf} and ϕ_s fro wide range of well characterized particles and it has been shown that the relation between these two quantities is essentially independent of particle size over wide range. So you see the relation between e_{mf} and ϕ_s they have proposed two different relations and these relations are independent of the particle size.

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The experimental information carried out by the authors are shown over here they have plotted e_{mf} verses ϕ_s and they have shown different points experimental point over here and when they draw the line of these equations for e_{mf} and ϕ_s the solid line shows first expression that is $1 - e_{mf}/\phi_s^2 e_{mf}^3 = 11$ and dotted lines shows second expression between shape factor as well as e_{mf} , so you once you see the plot of these two line you can see that these line satisfy the experimental data nicely.

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 $\begin{aligned} & \text{Minimum fluidization for irregular shape particles} \\ & \text{Substituting } (\phi_t * d_\rho) \text{ for } d \\ & (1 - e_{wf})(\rho_s - \rho)g = 150 \left(\frac{(1 - e_{wf})^2}{e_{wf}^3}\right) \left(\frac{\mu u_{wf}}{d^2}\right) + 1.75 \left(\frac{(1 - e_{wf})}{e_{wf}^3}\right) \left(\frac{\rho u_{wf}^2}{d}\right) \\ & \text{Multiplying both sides by } \frac{\rho d_0^3}{\mu^2 (1 - e_{wf})} \\ & (1 - e_{wf})(\rho_s - \rho)g = 150 \left(\frac{(1 - e_{wf})^2}{e_{wf}^3}\right) \left(\frac{\mu u_{wf}}{\phi_s^2 d_\rho^2}\right) + 1.75 \left(\frac{1 - e_{wf}}{e_{wf}^3}\right) \frac{\rho u_{wf}^2}{\phi_s d_\rho} \\ & \frac{(\rho_s - \rho)\rho g d_p^3}{\mu^2} = 150 \left(\frac{1 - e_{mf}}{e_{wf}^3}\right) \frac{1}{\phi_t^2} \left(\frac{\rho d_\rho u_{wf}}{\mu}\right) + 1.75 \left(\frac{1}{e_{wf}^3}\phi_s\right) \left(\frac{\rho^2 d_\rho^2 u_{wf}^2}{\mu^2}\right) \\ & \left(\frac{1 - e_{wf}}{e_{wf}^3}\right) \frac{1}{\phi_t^2} = 11 \quad \left(\frac{1}{e_{wf}^3}\frac{1}{\phi_s}\right) = 14 \\ & Ga_p = (150 \times 11) Re'_{wfp}' + (1.75 \times 14) Re'_{wfp}'^2 \end{aligned}$

Now what happens we have to obtained minimum fluidising velocity for irregular particle, so it was the Ergun's equations if you remember where pressure drop we have replaced e we have replaced with e_{mf} and u we have replaced with μ_{mf} , here d should be replaced with $\phi_s xd_p$ to consider shape of the particle, and along with replacing d with ϕ_s and d_p along with this we have to multiply both side with this factor, so here we have replaced d with $\phi_s^2.d_p^2$.

And similarly d should be replaced as $\phi_s x d_p$ so here you see d² is replaced with $\phi_s^2 d_p^2$ and in this expression d is replaced with $\phi_s d_p$ multiplying both side by this we can obtain this expression and then we can replace $(1-e_{mf}/e_{mf}^3)1/\phi_s^2=11$ as just we have seen these relationship was proposed by WEN and YU.

And similarly, this expression we can equate to 14 so after putting these value and here you understand this is Galileo number this is Reynolds number at minimum fluidising condition so putting all these factors we can obtain Galileo number for the particle because this is not of regular shape, we can have 150x11 this can be replaced by 11 and here this is replaced by Reynolds number for particle at minimum fluidising condition.

And similarly 1.75x14, 14 we can obtain from here and this is Reynolds number of particle square.

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Minimum fluidization for irregular shape particles $Ga_{p} = (150 \times 11)Re'_{mfp} + (1.75 \times 14)Re'^{2}_{mfp}$ $Ga_{p} \text{ and } Re_{mfp} \text{ are the Galileo number and the particle Reynolds number at the point of incipient fluidisation, in both cases with the linear dimension of the particles expressed as d_{p}.$ $Re'^{2}_{mfp} + 67.3Re'_{mfp} - 0.0408Ga_{p} = 0$ $Re'_{mfp} = 33.65[\sqrt{(1 + 6.18 \times 10^{-5}Ga_{p}) - 1}]$ $u_{mf} = \left(\frac{\mu}{d_{p}\rho}\right)Re'_{mfp}$

So this is the relation of Galileo number and Reynolds number of particle and once we solve this to get the Reynolds number, Reynolds number for different particle shape we can obtained by this equation that is 33.65 and in bracket we have this expression, so once we have obtained Reynolds number of particle of different shape we can calculate minimum fluidsing velocity for different shape particles.

Now further we will relate minimum fluidisation velocity as well as terminal settling velocity of the particle, you understand what is terminal settling velocity when particle is moving without having any net force acting on this so the minimum fluidization, so at terminal velocity particle will move with the fluid.

Minimum fluidization velocity in terms of terminal settling velocity

The minimum fluidising velocity, $u_{m\nu}$ may be expressed in terms of the freefalling velocity u_0 of the particles in the fluid. The Ergun equation relates the Galileo number Ga to the Reynolds number Re_{mf} in terms of the voidage e_{mf} at the incipient fluidisation point. $(1 - e_{mf}) = (1.75)$

$$Ga = 150 \left(\frac{1 - e_{mf}}{e_{mf}^3}\right) Re'_{mf} + \left(\frac{1.75}{e_{mf}^3}\right) Re'_{mf}$$

The particle Reynolds number $Re'_0(u_0d\rho/\mu)$ for a sphere at its terminal falling velocity u_{0} also is a function of Galileo number.

Thus, it is possible to express Re'_{mt} in terms of Re'₀ and u_{mf} in terms of u₀. For a spherical particle the Reynolds number Re'₀ is expressed in terms of the Galileo number Ga as follows: $Re'_0 = (2.33Ga^{0.018} - 1.53Ga^{-0.016})^{13.3}$

So minimum fluidization velocity u_{mf} may be expressed in terms of free falling velocity, so here first we should understand what is the difference between minimum fluidization velocity and terminal velocity? To a terminal falling velocity, minimum fluidization velocity is obtained when fluid will start moving with the particle, and terminal falling velocity is that velocity where the particle will be carried by the fluid. So you understand minimum fluidization velocity particle will start moving it will not be fluidized completely but at terminal falling velocity the particle will be carried out by the liquid, so it will be beyond fluidization condition.

So minimum fluidization velocity u_{mf} will be exposed in terms of free falling velocity, the Erguns equation relates the Galileo number and the Reynolds number in terms of void age for minimum fluidization condition is this. that just you have seen specifically for spherical particle and similarly when we have to co relate the particles Reynolds number act terminal falling velocity with Galileo number. We can obtain this particular relation that is Reynolds number, at terminal falling velocity = 2.33Ga $^{0.018} - 1.53$ Ga $^{-0.016 \ 13.3}$, so the derivation of this you can find in volume 2.



So here we have the graph, this is drawn between the ratio Reynolds number at a terminal falling velocity and Reynolds number at minimum fluidization velocity, that we have plotted, o you see this graph it shows the ratio of the Reynolds number at a terminal settling velocity or terminal falling velocity and Reynolds number minimum fluidization velocity against the Galileo number, where e_{mf} is considered a parameter. So you see how does this we have obtained a given Reynolds number we can calculate Reynolds number at velocity at this expression and Reynolds number for fluidization where e_{mf} have to put.

So for seeing Galileo number at minimum fluidization condition and terminal falling condition we obtain the ratio of this we can calculate and we can allot this ratio over this graph, so you see different authors have already carried out the experiment then this curves satisfied the experimental data. Therefore ratio of Reynolds number at terminal falling velocity minimum fluidization will be related with the Galileo number using these curves. So here you can see when the Galileo number is very small at that point the ratio of Reynolds number falling velocity minimum fluidization velocity is higher in comparison to Galileo number. Minimum fluidization velocity in terms of terminal settling velocity At $e_{mt} = 0.4$ $Ga = 18Re'_0$ (Ga < 3.6) $Ga = 18Re_0' + 2.7Re_0'^{1.667} \quad (3.6 < \mathrm{Ga} < 10^5) \qquad \qquad Re_{mf}'^2 + 51.4Re_{mf}' - 0.0366Ga = 0$ $(Ga > 10^5)$ $Ga = \frac{1}{4}Re_0^{'2}$ For low values of Remi (<0.003) and of Ga(<3.6), the first term may be neglected and: $\frac{Re'_0}{Re'_{mf}} = \frac{u_0}{u_{mf}} = 78$ $Re'_{mf} = 0.000712Ga$ Thus, $Re'_0 = 0.0556Ga$ Again, for high values of Rend (> 200) and Ga(>105), $Re'_{mf} = 0.191Ga^{1/2}$ $\frac{Re'_0}{Re'_{mf}} = \frac{u_0}{u_{mf}} = 9.1$ Thus, $Re'_0 = 1.732 Ga^{1/2}$

So here we have shown relation to different5 Galileo number and the Reynolds number at terminal falling velocity, for different zone of the Galileo number at $e_{mf} = 0.4$ that is we can obtain. Now for low value of Reynolds number that is <0.003 and Ga< 3.6, so once we have this as Reynolds number very small this term can neglect. We can calculate the minimum fluidization condition, using these two terms, so Reynolds number minimum fluidization condition obtained as 0.000712 Ga and similarly Reynolds number at terminal falling condition, we can obtained by this because here the Galileo number is , 3.6, so this condition we can obtain, this expression we can obtain.

The ratio of these two will give value 78 which is the ratio of terminal settling velocity and minimum fluidization velocity because the rest of the parameter becomes equal. Again for higher Reynolds number that is Reynolds number at minimum fluidizing condition >200 and then Galileo number >10⁵, so these are the expression for Reynolds number at two different condition and the value is 9.1, so you see at < Galileo number, higher ratio is obtained and higher Galileo number lesser ratio is obtained.

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Therefore if you see this value it shows that u_0 / U_{mf} is much larger for low values of Galleo number, generally obtained with small particles, than the high values, so for the smaller particle the ratio of u_0 / U_{mf} is larger than the large particle. For particulate fluidization with liquid the theoretical range of fluidizing velocities is from a minimum of U_{mf} to a maximum of u_0 . So you see for the particulate fluidization the velocity of fluid should be between u_0 / U_{mf} . That is lower and upper range of fluid velocity in particulate fluidization.

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So here we have to summarize this lecture and this summary consist of summary of lecture 1 and lecture 2 of week 3 and it goes as, fluidization is defined along with it types, fluidization characteristics are discussed. Minimum fluidization velocity is defined and effect of shape on it is discussed, and finally relation between minimum fluidization velocity and terminal setting velocity is discussed.

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Here we have the references basic book which I have referred you can go through these books for further detailing and that is all for now thank you.

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