

**Waste to energy conversion**  
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**Lecture - 32**  
**Tutorial on Anaerobic Digestion**

Good morning, now we will start discussion on a new module tutorial on anaerobic digestion.

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**Problem 1**

In a high rate biogas plant food waste is anaerobically digested to produce biogas. The slurry contains 8 % of solid food grains. The elemental composition of the food grains on dry basis is C: 58%, H:8%, O:26%, N:8% (mass basis). Around 80 % of the food grains are converted to biogas and all the converted hydrogen forms methane. If the flowrate of the slurry is 4500 litre per day, calculate the rate of biogas ( $\text{CO}_2 + \text{CH}_4$ ) production.

Solution	Basis
Flow rate of the slurry = 4500 L	
Dry solid food grains used = $4500 \times 0.08 = 360$ kg	
Assuming density of slurry = 1 kg/L	
	C present in 360 kg dry food grains = $360 \times 0.58 = 208.8$ kg
	H present in 360 kg dry food grains = $360 \times 0.08 = 28.8$ kg
	O present in 360 kg dry food grains = $360 \times 0.26 = 93.6$ kg
	N present in 360 kg dry food grains = $360 \times 0.08 = 28.8$ kg

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In previous module we have discussed on anaerobic digestion processes, and the production of biogas through this anaerobic digestion methods. In this module we will discuss some numerical problems and we will solve it. So, the statement of the first problems is in a high rate biogas plant food waste is anaerobically digested to produce biogas, the slurry contains 8 percent of solid food grains, the elemental composition of the food grains on dry basis is carbon 58 percent, hydrogen 8 percent, oxygen 26 percent, nitrogen 8 percent on mass basis. Around 80 percent of the food grains are converted to biogas and all the converted hydrogen forms methane. If the flow rate of the slurry is 4500 liter per day calculate the rate of biogas that is  $\text{CO}_2$  plus  $\text{CH}_4$  production. So, this is the problem statement now we have to solve it.

See if you see the problem statement we have given some information, and one basis is given here that is 4500 liter per day of slurry handling. So, we are handling 4500 liter per

day slurry, it is also given that 8 percent solid is present in the slurry. So, we can calculate how much solid is present in the slurry, then the percentage carbon hydrogen oxygen nitrogen is also given. So, we can get amount of elements carbon, hydrogen, oxygen, nitrogen present in the slurry then it is around 80 percent of the food grains are converted. So, at 80 percent of the food grain is converted. So, all these components will also be converted 80 percent we can assume, then we will see another condition is given that all hydrogen forms methane all converted hydrogen forms methane.

So, at first we calculate how much methane is produced on the basis of hydrogen conversion. Then from that calculation we get the carbon used for this methane production, and then we will get total carbon and how much carbon dioxide is remaining and that is that will be used for carbon dioxide production. So, this is have the methods which we will use to solve the problem.

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Basin — 4500 L per day.  
 Havy slurry = 4500 kg.  
 Solid =  $4500 \times 0.08 = 360 \text{ kg}$ .

80% Conversion

$C = 360 \times 0.58 = 208.8 \text{ kg}$	$\rightarrow 208.8 \times 0.8 = 167.04 \text{ kg}$
$H = 360 \times 0.08 = 28.8 \text{ kg}$	$\rightarrow 28.8 \times 0.8 = 23.04 \text{ kg}$
$O = 360 \times 0.26 = 93.6 \text{ kg}$	$\rightarrow 93.6 \times 0.8 = 74.88 \text{ kg}$
$N = 360 \times 0.08 = 28.8 \text{ kg}$	

Legend:  
 $C = 58\%$   
 $H = 8\%$   
 $O = 26\%$   
 $N = 8\%$

So, we have basis 4500 liter per day this is our basis, if we assume the density is one kg per liter then the mass is equal to 4500 kg 8 percent is slurry. So, slurry solid is equal to 4500 into 0.08. So, that is equal to we are getting 360 kg. So, that is equal to 360 kg then what is our carbon? Carbon is 58 percent, hydrogen is 8 percent and oxygen is 26 percent and then what is nitrogen that is also 8 percent. So, in the solid how much carbon is present 360 into 0.58, 360 into 0.58. So, that is equal to 208.8 kg. So, what is our hydrogen 360 into 0.08, 360 into 0.08, so that is equal to 28.8 kg.

Then what is our oxygen? Oxygen is 26 percent. So, that will be 360 into 0.26 8 kg and that is equal to we are getting 93.6 kg and what is our nitrogen 360 into 0.08. So, 28.8 kg, these are the elemental composition or amount of different components present in the slurry, then what we will do 80 percent is conversion 80 percent 80 percent conversion. So, carbon how much converted 280.8 into 0.8, 80 percent. So, that is equal to how much that is equal to 167.04 then how much hydrogen is coming 28.8 into 0.8, 28.8 into 0.8. So, that is equal to 23.04, this equal to 208 is equal to 208.8. So, this is this is.

So, now how much oxygen is converted that is equal to 93.6 into 0.8, 93.6 into 0.8 that is equal to coming 74.88 kg 74.88 kg. Now what we will do we have to find out how much methane is produced, how much hydrogen is converted we have got 23.04 kg.

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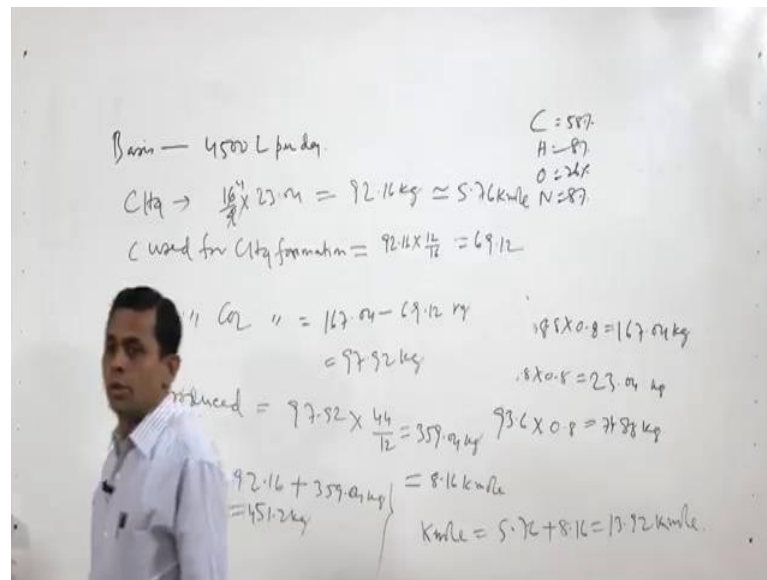
Basis — 4500 L per day.  
 $C \rightarrow \frac{16}{2} \times 23.04 = 92.16 \text{ kg} \approx 5.76 \text{ kmole}$   
 $C \text{ used for } CH_4 \text{ formation} = 92.16 \times \frac{16}{12}$   
 C =  $360 \times 0.58 = 208.8 \text{ kg}$   
 H =  $360 \times 0.08 = 28.8 \text{ kg}$   
 O =  $360 \times 0.26 = 93.6 \text{ kg}$   
 N =  $360 \times 0.08 = 28.8 \text{ kg}$   
 80% conversion  
 $208.8 \times 0.8 = 167.04 \text{ kg}$   
 $28.8 \times 0.8 = 23.04 \text{ kg}$   
 $93.6 \times 0.8 = 74.88 \text{ kg}$   
 C = 58%, H = 8%, O = 26%, N = 8%

Now, how much methane will be produced  $CH_4$  12 plus 4 that is equal to 16. So, 12 gram carbon will give 16 gram of methane that is why and 4 gram of hydrogen will give us 16 gram of methane. So, how much methane is produced 16 divided by 4 into how much this conversion is. So, how much it is 23.04, into 23.04. So, that is equal to 4 and that is equal to 92.16 kg; 92.16 kg. So, that is equal to 92.16 kg.

In terms of mole we can get this is divided by 16. So, that is equivalent to we are getting 5.76 kilo mole, 5.76 kilo mole. So, this the methane we are getting now how much carbon is used for the production of this amount of methane, we can get carbon used for

methane production that is equal to how much 92.16 into 12 by 16, 92.16 into 12 by 16 that is equal to 69.12. So, how much carbon we had? 167.04 conversion and 69.12 is for methane production, the rest carbon will be used for the production of carbon dioxide. So, carbon used for carbon dioxide production.

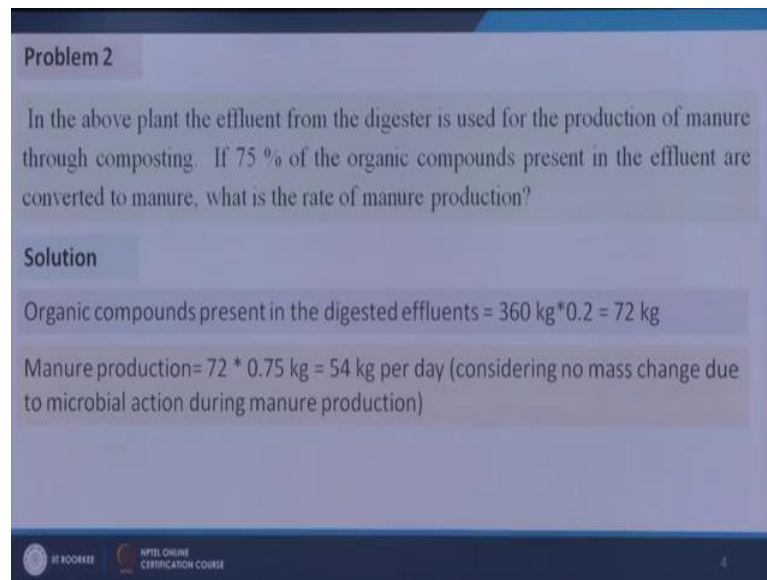
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So, carbon used for CO<sub>2</sub> production is equal to how much? We will be getting 167.04 minus 69.12 kg that is equal to 97.9 kg that is equal to 97.92 kg.

Then how much CO<sub>2</sub> is produced. So, CO<sub>2</sub> produced is equal to 97.92 kg into 44 divided by 12. 44 divided by 12 and that is equal to 359.04 kg. 359.04 kg in how many moles we can get is equal to divided by 44. So, it will be giving us is equal to 8.16 kilo mole. So, we have now what how much CO<sub>2</sub> and how much CH<sub>4</sub> is produced now. So, total CH<sub>4</sub> and CO<sub>2</sub> is equal to how much? We are getting 92.16 kg plus 359.04 kg our total is equal to 451.02 kg in terms of mass, but in terms of moles 5.76 kilo mole and here we are getting that is equal to 8.16 kilo mole. So, these two in terms of kilo mole, that is equal to 5.76 plus 8.16 we are getting total is equal to 13.92 kilo mole this kilo mole.

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**Problem 2**

In the above plant the effluent from the digester is used for the production of manure through composting. If 75 % of the organic compounds present in the effluent are converted to manure, what is the rate of manure production?

**Solution**

Organic compounds present in the digested effluents =  $360 \text{ kg} \times 0.2 = 72 \text{ kg}$

Manure production =  $72 \times 0.75 \text{ kg} = 54 \text{ kg per day}$  (considering no mass change due to microbial action during manure production)

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So, now the first problem is solved now we will see the problem number two. So, in this problem the statement is in the above plant the effluent from the digester is used for the production of manure through composting, if 75 percent of organic compounds present in the effluent are converted to manure what is the rate of manure production; that means, we have got 80 percent organics is converted to gas. So, remaining 20 percent is present in this in the slurry. So, we had 360 kg we had 360 kg of solids. So, out of this 360 kg 20 percent will be available in the slurry after biogas production. So, the solids present is equal to here organic compounds present in the digested effluents is 360 into 0.2 that is equal to 72 kg, and 75 percent of it is converted to manure. So, 75 percent means 72 into 0.75 that will be 54 kg per day. So, this will be the manure production.

Here we are assuming that there is no mass loss or gain due to microbial activities. So, this is the solution of this problem.

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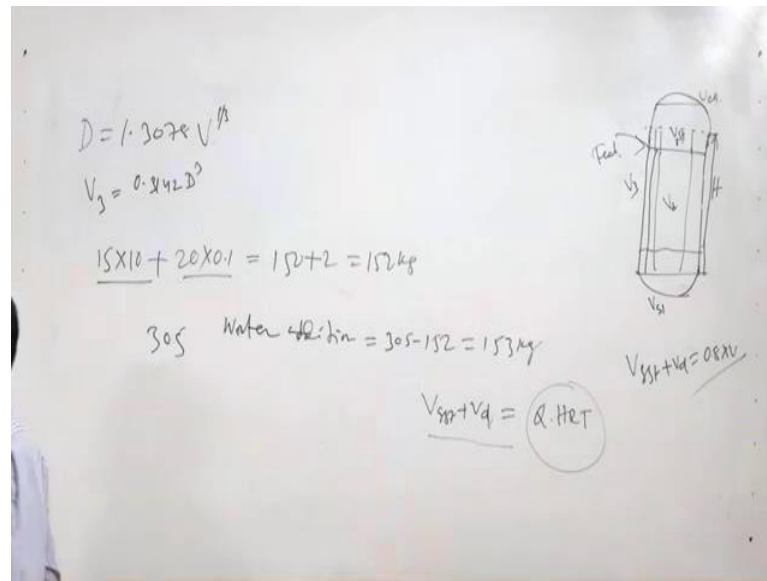
**Problem 3**

A family having 15 cows of bodyweight 200 kg each and 20 chicken of body weight 1.5 kg each, wants to setup a bio-gas production plant. What should be the volume of the anaerobic digestion unit suitable for this application and mention volume of various zones of the unit. Assume discharge per day of each cow is 10 kg and TS value is 16 % of the discharge. Similarly for chicken these values are 0.1 kg per day and 20 % respectively. HRT is 40 days. Daily gas production rate is 0.28 m<sup>3</sup>/kg TS/day or 0.4 m<sup>3</sup> /digester volume per day. If total volume of the digestion unit (V) is related with the diameter of the cylindrical portion as  $D = 1.3078 V^{1/3}$ , calculate the height of the cylindrical part of the unit assuming its volume as  $V_3 = 0.3142 D^3$ . Also calculate the volume of sludge zone, active digestion zone, gas collection zone and gas storage zone.

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Now, we will solve problem number three; the statement is a family having 15 cows of body weight 200 kg each and 20 chicken of body weight 1.5 kg each, wants to setup a biogas production plant what should be the volume of the anaerobic digestion unit suitable for this application, and mention volume of various zones of the unit. Assume discharge per day of each cow is 10 kg and total solid that is TS value is 16 percent of the discharge; similarly for chicken these values are 0.1 kg per day and 20 percent respectively. HRT hydraulic retention time is 40 days. So, daily gas production gas rate is 0.28 meter cube per kg TS per day or 0.4 meter per digester volume per day. If the total volume of the digestion unit that is V is related with the diameter of the cylindrical portion.

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Handwritten calculations on a whiteboard:

$$D = 1.3078 V^{1/3}$$

$$V_3 = 0.3142 D^3$$

$$15 \times 10 + 20 \times 0.1 = 150 + 2 = 152 \text{ kg}$$

$$305 \text{ Water} + 152 = 305 - 152 = 153 \text{ kg}$$

$$V_{sp} + V_d = Q \cdot H_{ET}$$

Diagram of a cylindrical digester unit with labels: Feed, V<sub>3</sub>, V<sub>d</sub>, V<sub>sp</sub>, V<sub>col</sub>, and H.

As D is equal to  $1.3078 V^{1/3}$  and calculate the height of the cylindrical part of the unit, assuming its volume as  $V_3$  is equal to  $0.3142 D^3$  to the power cube, also calculate the volume of sludge zone, active digestion zone, gas collection zone and gas storage zone. So, this is already discussed here various zones of the unit.

So, this is our statement, we have given the number of cows the number of chickens available in the family, and discharge per cow and per chicken it is given and the k values are also given that which will be used and it is also given that the TS present in the two discharges V and D relation is also given, see if we have say one digestion plant. So, it is given this is the sludge zone, sludge zone V sludge this is the gas collection, collection this is say storage and this is active zone V D, and we this is your cylindrical portion and this is our H and this is equal to  $V_3$  is equal to this cylindrical portion is equal to  $V_3$ .

Now,  $V_3$  is  $0.3142 D^3$  to the power to cube, and D is equal to  $1.3078 V^{1/3}$  to the power half. This V is equal to this V is equal to  $V_{col}$  collection plus  $V_{st}$  plus gas col gas storage  $g_{st}$  plus V D plus V sludge and this  $V_3$  is equal to this portion if this is our height this is our lower portion see this is our height. So, this is our  $V_3$  this portion is our  $V_3$  these are given here. So, we have to calculate the value of  $V_{st}$ , V D  $V_{st}$ , V col. Now what is our information given here the total sludge how much sludge we are getting we have 15 number of cows. So, 15 into 10 kg per day 10 kg per day, plus we



have 20 number of chicken. So, we have 20 number of chicken and it has 0.1 kg. So, we are having. So, this is your 0.1 kg. So, we are getting 150 20 into 0.1kg. So, that is equal to 150 plus 2, 152 kg. So, this is our discharge per day.

So, the chicken values are 0.1 kg per day. So, this is our similar the chicken discharge is 0.1 kg. So, we have 20 chicken. So, that is using 0.1 kg per day that is why 20 into 0.1 that is why 20 into 0.1 and then 15 into 20, 10 that is equal to 150 plus 2. So, one 52 kg the total discharge.

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**Solution**

Total discharge =  $15 \times 10 + 20 \times 0.1 \text{ kg} = 150 + 2 = 152 \text{ kg}$

TS in total discharge =  $150 \times 0.16 + 2 \times 0.2 = 24.4 \text{ kg}$

8 kg of TS is required to make 100 kg slurry (Since 8 % slurry is required for AD)

Hence 24.4 kg TS will be available in  $(100/8) \times 24.4 \text{ kg} = 305 \text{ kg slurry} \sim 305 \text{ L}$  (slurry density = 1 kg/L)

Thus, water to be added with 152 kg of discharge =  $305 - 152 \text{ kg} = 153 \text{ kg}$

Now

Gas storage volume ( $V_{gst}$ ) + active digestion volume ( $V_d$ ) = Slurry feeding rate (Q) \* HRT

$$V_{gst} + V_d = \text{Slurry feeding rate (Q)} \times \text{HRT}$$

$$V_{gst} + V_d = 305 \text{ (L/day)} \times 40 \text{ day} = 12200 \text{ L} = 12.2 \text{ m}^3$$

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Next will be TS in the total discharge, what will be the TS that is equal to 150 into 16 percent, 16 percent 0.16, that is for cow dung and this is for chicken that is 2 into 0.2 that is equal to 20 percent, that is 2 kg is coming from the chicken waste. So, and the solid content is 20 percent. So, 2 into 0.2 that is equal to 24.4, 150 into 0.16 plus 2 into point 0.2 that is equal to 24.4 kg. So, this is the coming. So, 0.16 means 16 percent of TS in carbon 0.2 means 20 percent is the chicken waste, now what is our condition the slurry will be having 8 percent TS. So, 8 percent TS if the slurry has, 8 kg of TS is required to make 100 kg slurry since the 8 percent slurry is required.

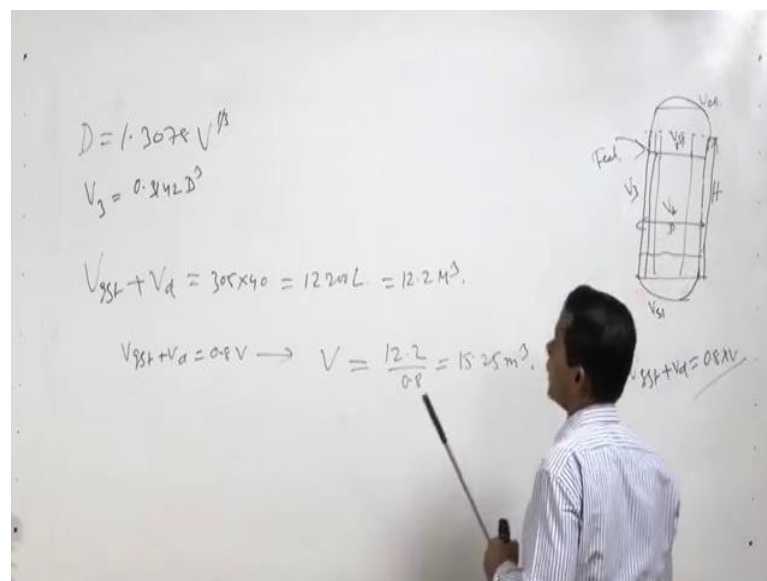
So, 24.4 kg is produced daily. So, what will be the volume of slurry to accommodate this 24.4 kg of TS? So, that is equal to 100 divided by 8 into 24.4 because to accommodate 8 kg, we need 100 kg total slurry weight if the density is equal to one, so 100 liter. So, 100 divided by 8 into 24.4 that is equal to 305 kg slurry that is 305 liter when we are



assuming that density is equal one kg per liter. Now how much material is produced by this the cow and chicken that is equal to 152 kg. So, how much water we need to add. So, water addition is equal to, water addition is equal 305 minus 152 minus 152. So, that is equal to 153, 153 kg water we have to add. So, 153 kg water we have to add.

Now, what will be the gas storage volume; as we have discussed in the previous module of anaerobic digestion that  $V_{gst}$  plus this  $V_{gst}$  gas storage plus  $V_D$  that is equal to 80 percent of total volume, this is our assumption that 80 percent of this and this is equal to slurry feeding rate into HRT. So, this is equal to also  $V_{gst}$  plus  $V_d$ ,  $V_D$  that is equal to  $q$  into HRT what is the retention time what is the feeding rate, what is the feeding and what is the retention time if we multiply this we will get  $V_{gst}$  plus  $V_d$  we have discussed in the anaerobic digestion module. So, here what is the slurry feeding rate slurry feeding rate is a word 305 liter per day into HRT. HRT is equal to how much that is equal to HRT 40 days. So, we have to multiply it. So, we multiply it 14 to 305. So, it is coming 12200 liter.

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So,  $V_{gst}$  plus  $V_D$  we are getting  $V_{gst}$  plus  $V_D$  we are getting how much 305 into 40, that is equal to 12200 liter or 12.2 meter cube we can get 12.2 meter cube, and another way of discussed again that  $V_{gst}$  plus that is equal to 0.8 into  $v$ . So, what will be  $V_{gst}$  plus  $V_D$  is equal to 0.8 into  $v$ .

So, what is V? V is equal to twelve12.2 divided by 0.8. So, that is equal to 15.25 meter cube. So, that is equal to 15.25 meter cube. So, now, we have got the total volume of these that is equal to 15.25 meter cube. Now we will have get the different zones now one information is given that D is equal to one what is D this is the D the diameter of the cylindrical part is equal to D, and that D is equal to 1.3078 into V to the power one third. So, what is the D? D is equal to you can getting here 1.3078 into 15.25 to the power one third, and that is equal to; that is equal to how much we are getting 3.24 meter. So, 3.24 meter. So, this equal to 3.24 meter that is equal to; and then what is V 3? We will put this D value here, 0.3142 into 3.24 cube.

Now, another important information which we have got here that what is V 3, V 3 we are getting from this equation.

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Handwritten mathematical derivation and a diagram of a cylindrical tank with a hemispherical top.

Diagram labels:  $V$  (Total Volume),  $V_{sh}$  (Hemispherical Volume),  $V_c$  (Cylindrical Volume),  $V_d$  (Small Volume at the bottom).

Equations:

$$V_{sh} = \frac{\pi D^2}{4} H$$

$$V_c = \frac{\pi D^2}{4} H$$

$$V_d = \frac{\pi D^2}{4} H$$

$$V = V_{sh} + V_c + V_d$$

$$V_{sh} = 0.05 \times 15.25 = 0.7625 \text{ m}^3$$

$$V_c = 0.15 \times 15.25 = 2.2875 \text{ m}^3$$

$$V = 0.7625 + 2.2875 = 3.05 \text{ m}^3$$

Another is this is V 3 that is cylindrical part. So, pi D square by 4 into H, that is also the V 3. So, here we will put that is equal to pi D square by 4 into H that is equal to 0.3142 into 3.24 per cube, that is equal to D we can write it here D also, D cube now D square D cube cancel. So, h is equal to 0.3142 D into 4 divided by pi and.

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Again

$$V_{gst} + V_d = 0.8 * V \Rightarrow V = 12.2 / 0.8 \text{ m}^3 = 15.25 \text{ m}^3$$

$$\text{Given } D = 1.3078 V^{1/3} = 1.3078 * (15.25)^{1/3} = 3.24 \text{ m}$$

Further

$$V_3 = 0.3142 D^3 = \pi \left( \frac{D^3}{4} \right) * H \Rightarrow H = (0.314 * 4 * D) / \pi = (0.314 * 4 * 3.24) / \pi = 1.29 \text{ m}$$

$$V_{col} = 0.05 V = 0.05 * 15.25 \text{ m}^3 = 0.7625 \text{ m}^3$$

$$\text{Assuming } V_{sl} = 15 \% \text{ of } V = 15.25 * 0.15 = 2.288 \text{ m}^3$$

$$V_{gst} = 0.5 * (V_{gst} + V_d + V_{sl}) * K = 0.5 * (V - V_{col}) * K = 0.5 * (15.25 - 0.7625) * 0.4 = 2.898 \text{ m}^3$$

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So, H is equal to 1.29 meter. So, by this calculation we are getting H is equal to 1.29 meter. Now we have got the value of H, now we need to calculate the V collection, what is the V collection that is 5 percent of V, what is sludge collection sludge volume that is equal to 15 percent of V.

So, now we have got the V value. So, what will be V? V collection is equal to 0.05 into 15.25 and that is equal to 0.7625 meter cube, then what is our V sludge? That is equal to 0.15 into 15.25 and that is equal to 2.288 meter cube. So, that is equal to 2.288 meter cube. So, we have got this one we have got this one we have to calculate these and this now in the previous module we had seen that  $V_{gst}$ ,  $V_{gst}$  come here. So,  $V_{gst}$  is equal to 0.5 into  $V_{gst}$  plus  $V_d$  plus  $V_{sl}$  into K what is this K? K is nothing, but 0.4 that is the meter cube per meter gas produced meter cube of gas produced per cube of volume of reactor per day. So, that is equal to 0.5 into  $V_{gst}$  plus  $V_d$  plus  $V_{sl}$  into 0.4.

Now, what is  $V_{gst}$  plus  $V_d$  plus  $V_{sl}$  plus  $V_{col}$  that is equal to total volume. Total volume is equal to  $V_{col}$  plus  $V_{sl}$  plus  $V_{gst}$  plus  $V_{col}$ , V collection. So, this three is equal to we can get equal to V minus  $V_{col}$ , V collection and this V is equal to 15.25 and V collection is equal to 0.7625, these two it is becoming this one, 15.25 into 0.7625. So, this into 0.5 into 0.4 that is giving 2.898 meter cube, that is  $V_{gst}$  we are getting now.

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Alternatively,

$$V_{gst} = 50\% \text{ of daily gas yield} = 0.5 * TS * \text{gas production rate (m}^3 \text{ per kg TS/day)}$$
$$= 0.5 * 24.4 * 0.28 \text{ m}^3/\text{kg TS} = 3.416 \text{ m}^3$$

Thus conservative value of  $V_{gst} = 3.416 \text{ m}^3$

Now

$$V_d = (V - V_{gst} - V_{col} - V_{sl}) = 15.25 - 3.416 - 0.763 - 2.288 = 8.738 \text{ m}^3$$

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Now, alternatively we also know that  $V_{gst}$  can be calculated on other thumb rule that is 50 percent of daily gas yield that is 0.5 into TS or total solid into gas production rate that is meter cube per kg TS per day. So, these value is equal to 0.28 as given in the statement. So, 0.25 TS is equal to 24.4 as per the statement given we have calculated and then this is equal to 3.416 meter cube. So,  $V_{gst}$  one formula is giving us 3.416 meter cube.

Another formula is giving us 2.898 meter cube. So, we have to be conservative. So, we will be taking the larger value. So, 3.416 meter cube. So,  $V_{gst}$  is equal to 3.416 meter cube.

So, then what is  $V_d$ ?  $V_d$  is equal to  $V$  minus  $V_{gst}$  minus  $V_{sl}$  minus  $V_{col}$ . So, everything we have got that is 15.25 we have got this one this one this one and this one. So, I have seen here, 15.25 minus 3.416 minus 0.763 minus 2.288. So, we are getting 8.738 meter cube. So, now, we have got the values of all the zones  $V_{sl}$ ,  $V_d$ ,  $V_{gst}$  and  $V_{col}$  as well as the total volume. So, the problem is solved. So, up to this in this module so.

Thank you very much.