INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
NPTEL
NPTEL ONLINE CERTIFICATION COURSE
Mechanical Operations

Lecture – 05
Characterization of collection of particles-2

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Welcome to the fifth lecture of the week one, that is characterization of collection of particle, part 2. If you remember the lecture 4 that is characterization of collection of particle, part 1.

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Where we have discussed the particle size distribution based on screen analysis, Now once I am having the particle size distribution data we have represented this in a tabular form. Now in this particular lecture we will discuss how to represent the particle size distribution data in graphical form. So that can be represented on X axis and Y axis plot that is 2D plot.

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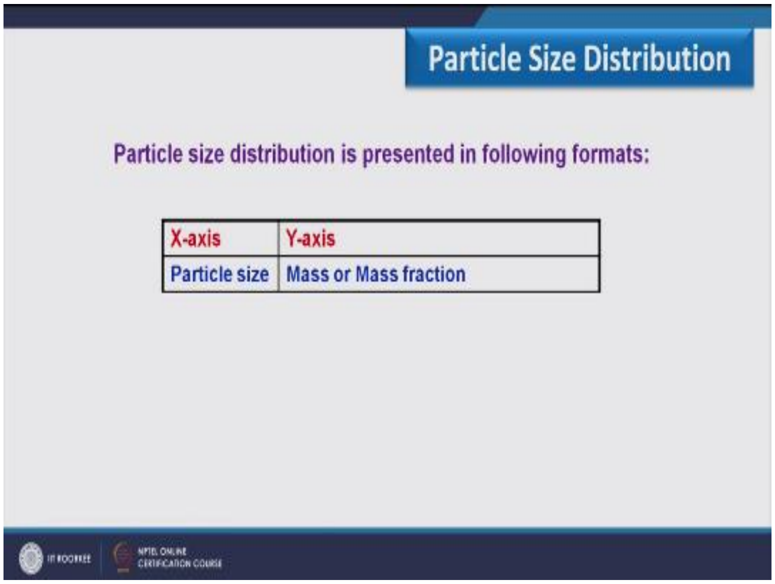

On X axis we will consider particle size and on Y axis we will plot mass or mass fraction. How many other ways we can represent this is if I consider particle size on X axis.

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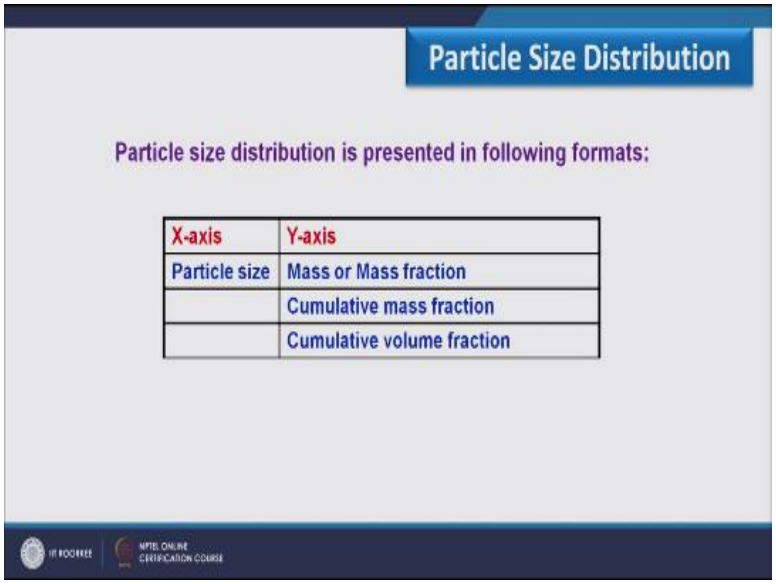

And cumulative mass fraction on Y axis. Further if I consider particle size on X axis and cumulative volume fraction on Y axis, so by these three means we can represent particle size distribution data graphically. Therefore, the distribution can be reported either in terms of frequency that is differential form, or by cumulative that is integral form.

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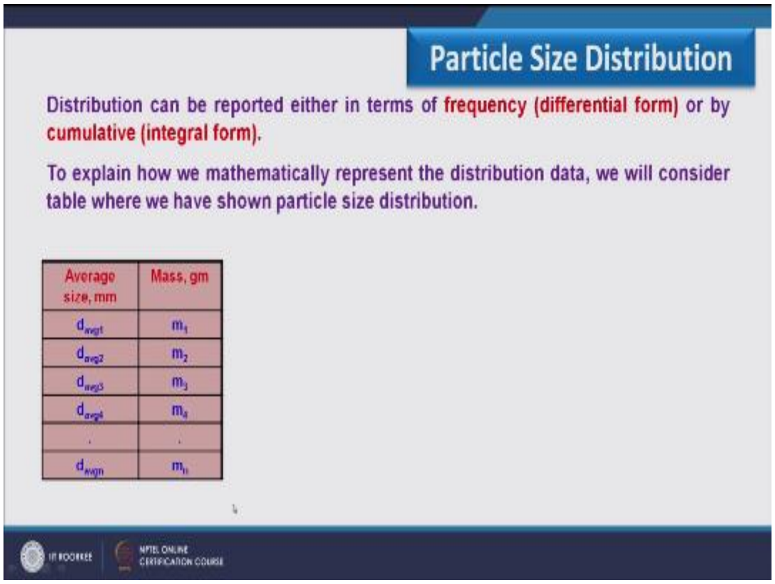

To explain how we mathematically represent the distribution data we will consider the table where we have shown particle size distribution data. If you remember this is the table where we have shown average size that is d_{avg} which is computed by considering the arithmetic mean of sizes of screens used in that fraction, and m_1 is the mass available on that particular screen. So here we have the data from a screen analysis and we can represent this in.

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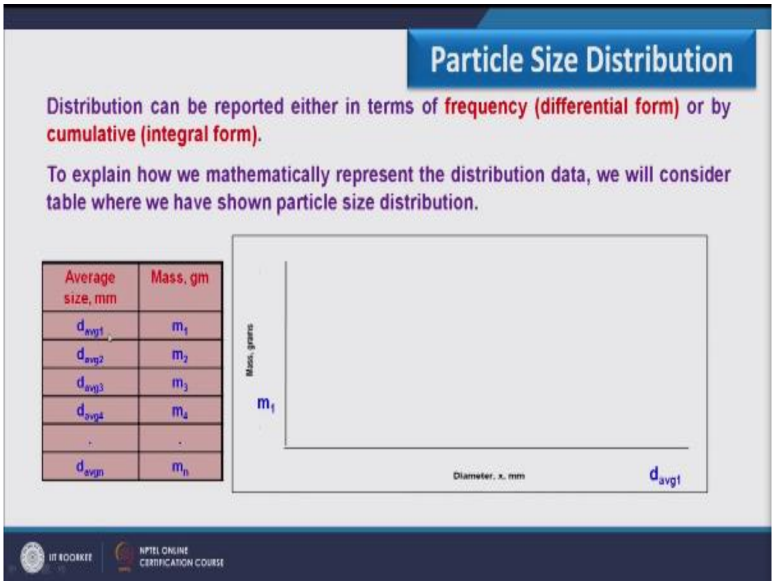

Graphical form considering that is particle size that is diameter d_{avg} and the mass available correspond to that d_{avg} size. So here I am having the value of d_{avg1} and m_1 we can put on Y axis, now why I have written d_{avg} for this because if you remember the data which we have shown in this table the topmost has the highest opening. So it would have the largest purchase size it will have the largest value, and as we keep on moving downward the size of particle will keep on decreasing. So obviously d_{avg} will fall farthest in this graph.

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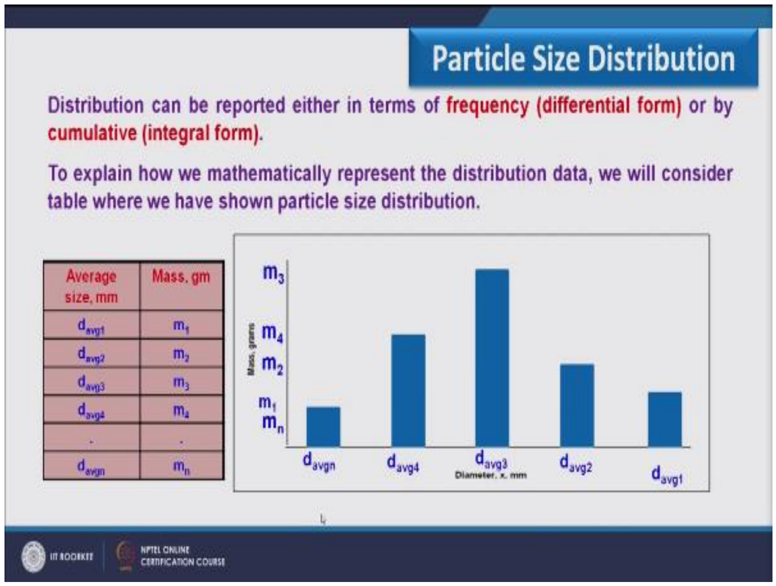

So considering value of m_1 and d_{avg} we can show the data, we can plot the bar chart for this, so here if you see this graph here I am having the value of d_{avg} which we have drawn farthest and considering this m_1 and d_{avg} we can plot the bar. And similarly we can plot the bar for other fractions also, and then we can write the values of diameter as well as fractions. So here if you see this graph here on Y axis we have represented this in terms of mass. If I want to represent this is terms of mass fraction then obviously its nature will not be changed, only the value of Y axis will change.

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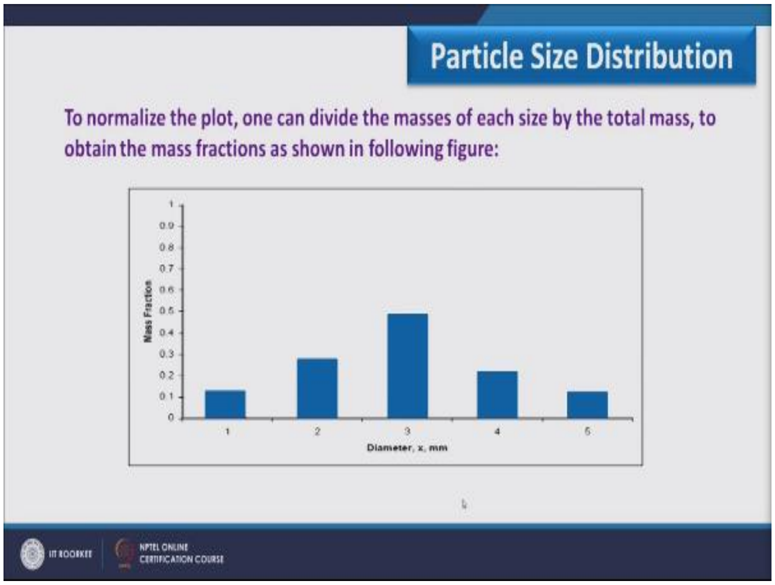

Which I have shown in this graph, which we called as normalized graph. So here instead of mass we can consider mass fraction, rest of the things will remain same. So this is basically the differential form or we call it frequency chart for particle size distribution.

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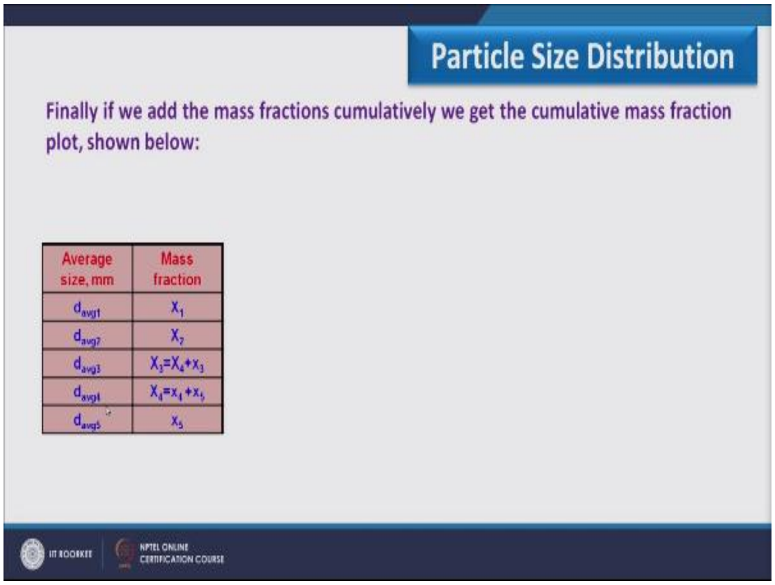

If I draw the cumulative graph we have to add all mass fraction to get the cumulative mass fraction plot as we have shown in this table. In this table if you consider the first column that is all d_{avg} value we have written and here I am having the mass fraction. So if I consider the lowest value that is the minimum size of the particle and if it has the fraction X_5, so correspond to d_{avg5} the value is $X_{5,}$ if I consider d_{avg4} here I am having the value X_4+X_5 because we are making the cumulative from bottom to top.

Now why we are doing this from bottom to top, because if I consider this d_{avg4} it means if I consider this opening.

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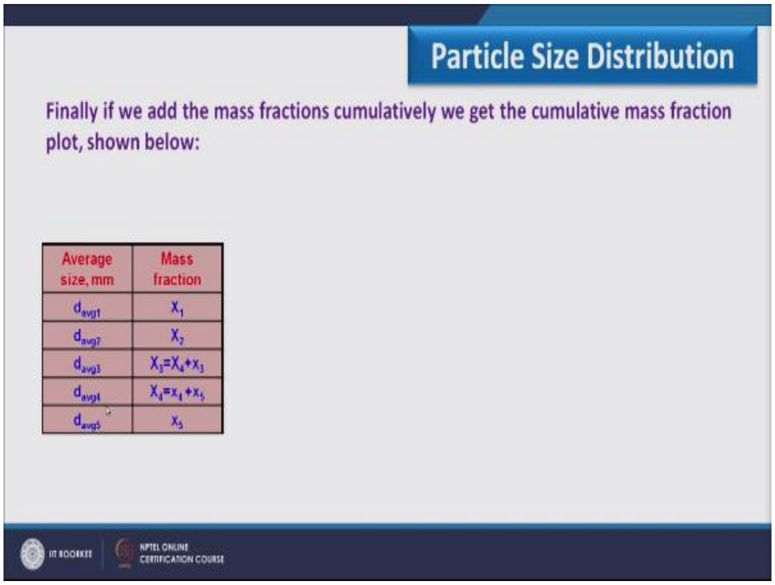

The mass correspond to this opening that is X_4 and mass below to this all will be passed from this, therefore we can consider correspond to d_{avg4} the total mass which is the summation of X_4 and X_5 and we have represented this with X_4.

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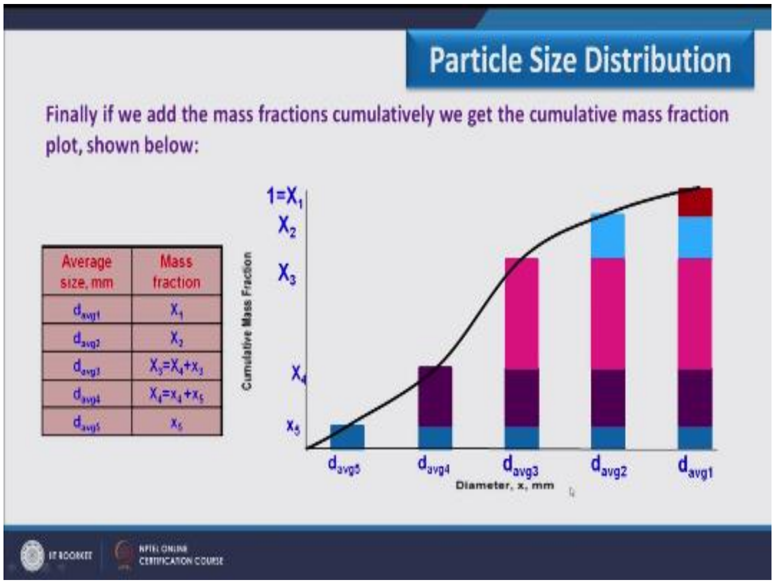

So if I want to draw the cumulative plot on X axis we have represented the diameter and on Y axis we can represent cumulative mass fraction. For d_{avg5} correspond to X_5 we can have the bar chart like this. If I want to draw for d_{avg4} we have to join X_4 as well as X_5 together. So correspond to d_{avg4} we have X_4, X_4 value which has X_5 as we have shown correspond to d_{avg5} and X_4 would be added over here, so correspond to d_{avg} total mass fraction would be X_4, and similarly if I consider for d_{avg3} the cumulative mass is X_3, X_4 and X_5, summation of all these will be called as cumulative mass fraction correspond to d_{avg3}.

And in similar line we can write for other fractions also here you see, so once I keep on cumulating from bottom to top when I reach to the top correspond to X_1 the value should always be equal to 1. So this we call the cumulative graph, if I do this and if I join the center of these bars together we can have the cumulative plot like this. So this is basically the cumulative plot, we will use this curve for the computation purpose in subsequent lectures.

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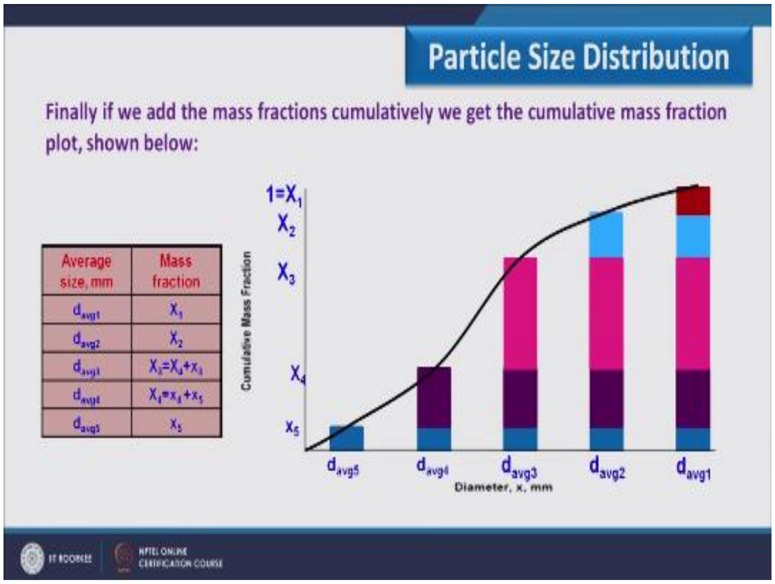

But here you should understand how to plot the cumulative mass fraction curve. Now here in this slide I have shown how the particle size and shape is measured though we have already discussed this but whatever we have discussed that is the manual effort. If we want to compute this through the instrument how we will do this.

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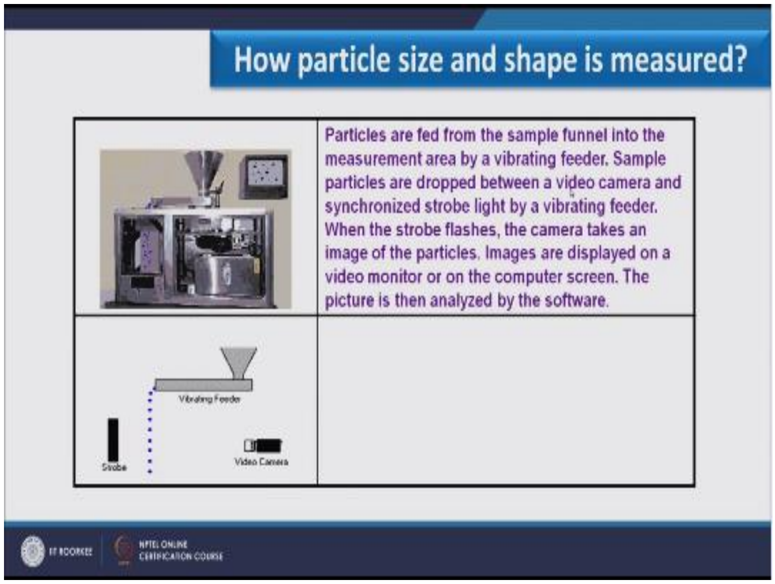

So this is basic instrument for this and we call it particle size analyzer, here particles are fed from a sample funnel into the measurement area by vibrating feeder. Now if you see this here we have the funnel and this is the vibrating feeder. Once the feed is coming to the funnel it drops in this feeder, it vibrates so due to that vibration material is continuously fall between the analyzer. So here you see this is the funnel and this is the vibrating feeder, material is continuously fall in the analyzer, now here if you see this particular section this is called strobe and here we have the video camera.

This continuously takes the picture of this as particles falling in between these two. So sample particles are dropped between a video camera and the synchronized strobe light.

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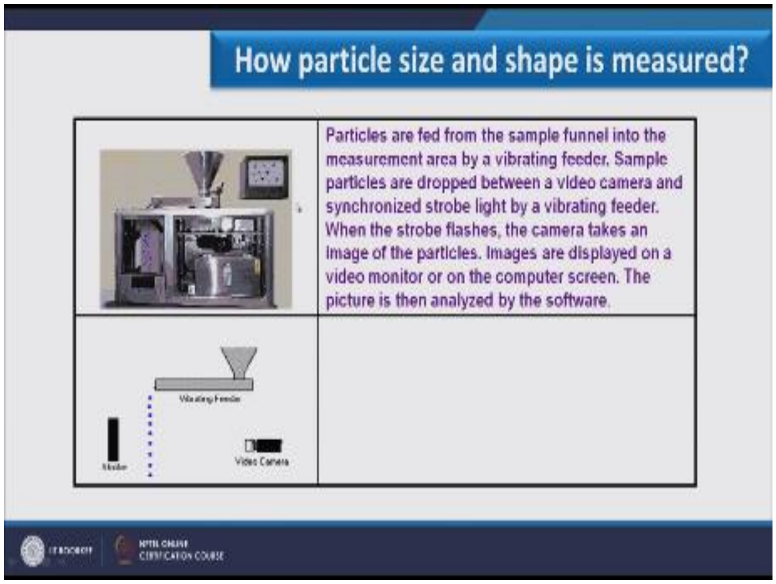

By a vibrating feeder, when the strobe flashes the camera takes the image of the particle and this image are displayed on the associated computer and the picture is then analyzed by the software. So if you see this particular image which is reflected on a computer screen this is basically the pictures taken by this video camera. So after a short interval continuously these images are coming and we can analyze this.

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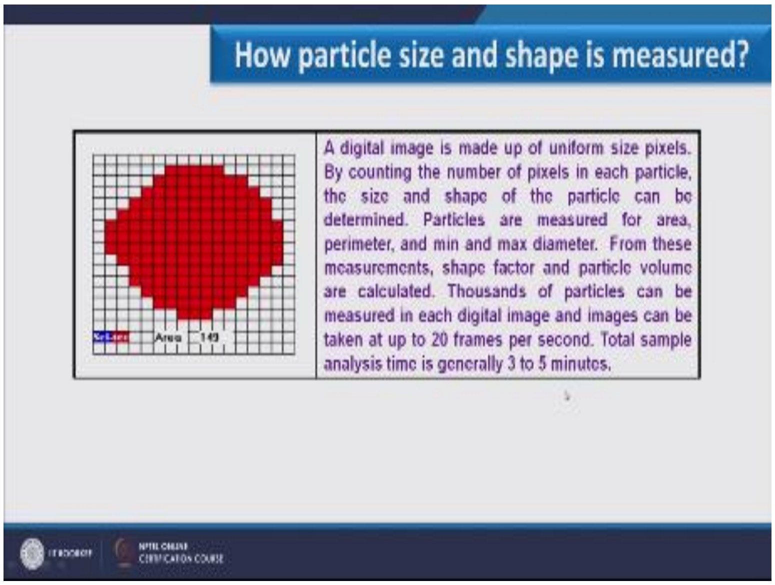

How we can analyze this it is shown in this slide, so if you see this particular image where a single particle is shown which we call a digital image. So a digital image is made up of uniform size pixels. If you see these square or rectangular shape cells these are called the pixel which are of uniform size. By counting the number of pixels in each particle the size and shape of particle can be determined. If I consider the diameter on this axis you can simply count this is 1, 2, 3, 4, 5 and similarly up to 15 we can calculate in this, we can count in this. So if you consider the largest diameter it is, it will have the value equal to 15.

So using this image particles are measured for area, periphery and minimum and maximum diameter. From these measurements shape, factor, and particle volumes are computed. Thousands of particles can be measured in each digital image and images can be taken at up to 20 frames per second. So total sample analysis time is generally 3 to 5 minutes, so this is the method how we measure the particle size as well as shape using the analytical instruments.

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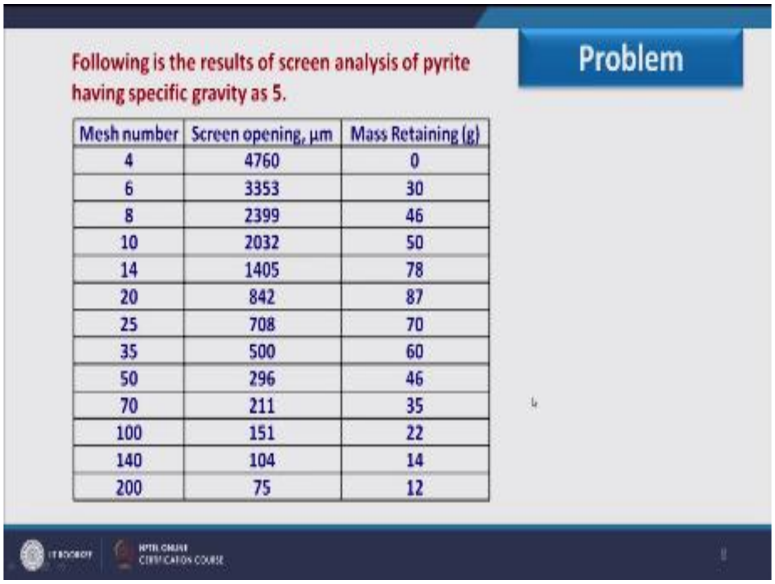

Mesh number	Screen opening, µm	Mass Retaining (g)
4	4760	0
6	3353	30
8	2399	46
10	2032	50
14	1405	78
20	842	87
25	708	70
35	500	60
50	296	46
70	211	35
100	151	22
140	104	14
200	75	12

Now in this particular lecture I will demonstrate worked example, here I have the problem, following is the result of a screen analysis of pyrite having the specific gravity as 5. So if you see this table here we have represented the data which we have got from the screen analysis which includes mesh number from 4 to 200 and screen opening from 4760 to 75, so here if you see this mesh number here we have used the American standard then the Indian standard screen and mass retained on each screen is also shown over here.

So what we have to compute for this problem, the first is the specific surface area.

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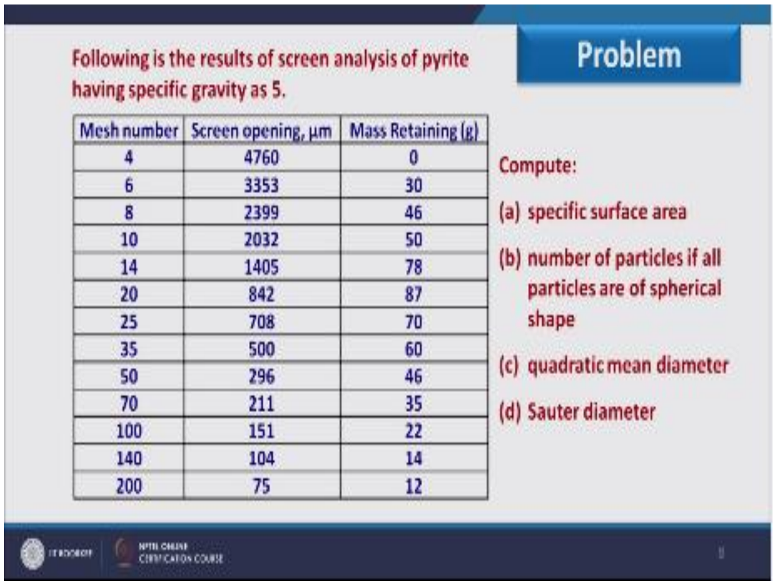

Secondly we will compute number of particles if all particles are of spherical shape, so here we have to consider the uniform shape of particle that is a spherical. Third we have to calculate quadratic mean diameter and finally we will compute sauter diameter for the mixture. Now to calculate the specific surface area on number of particle or quadratic mean dia, first of all we have to calculate the.

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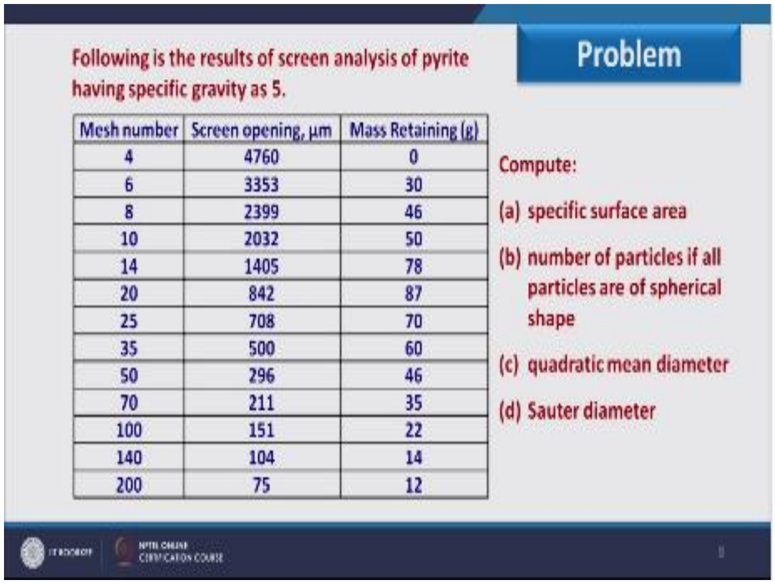

Average diameter of the particle in a particular fraction, for calculation of average diameter we should know from which screen material is passed and to which screen it is retained so if I consider the first one

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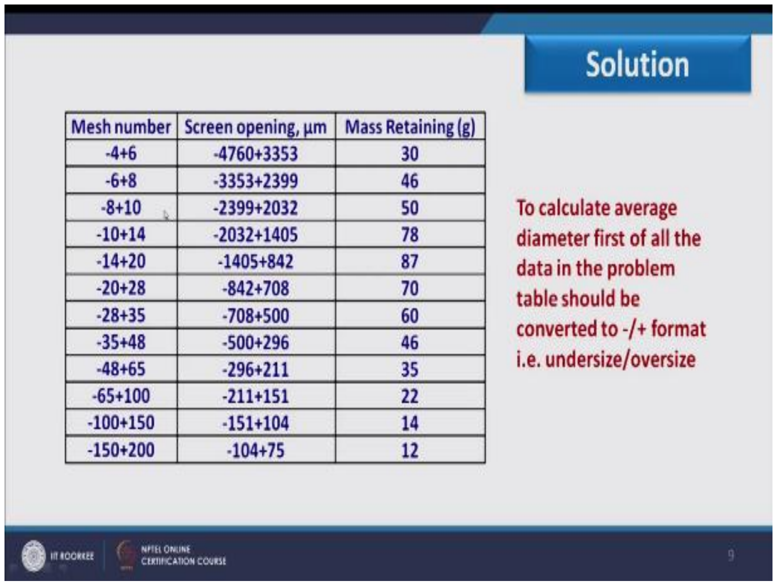

Mesh number	Screen opening, µm	Mass Retaining (g)
-4+6	-4760+3353	30
-6+8	-3353+2399	46
-8+10	-2399+2032	50
-10+14	-2032+1405	78
-14+20	-1405+842	87
-20+28	-842+708	70
-28+35	-708+500	60
-35+48	-500+296	46
-48+65	-296+211	35
-65+100	-211+151	22
-100+150	-151+104	14
-150+200	-104+75	12

Here I am having - 4 +6 it means material which is passed from 4^{th} screen and retained on 6^{th} screen is basically 3o gram. If you remember the previous slide.

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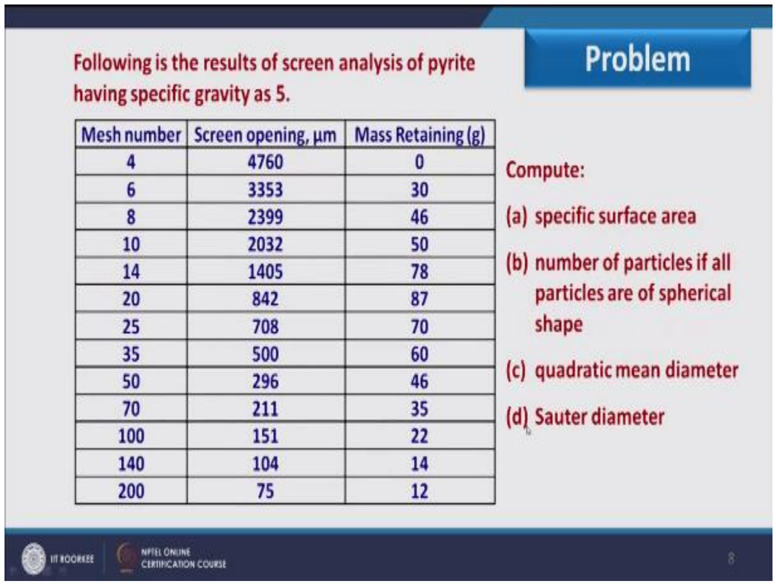

Here what we have the value correspond to 4 we have 0 mass retained, it means nothing is retained on 4th mesh screen so whatever is passed from 4th screen is retained on 6th screen is equal to 30 gram. So same we have represented over here that is -4+6, if I consider screen opening that is -4760 + 3353 micro meter, this we have shown and corresponding to this we have the mass fraction equal to 30 gram and similarly we can show for rest of the fraction in -+ format to compute the average diameter of particles in each fraction.

And I hope you are understanding what is – what is +, - shows whatever

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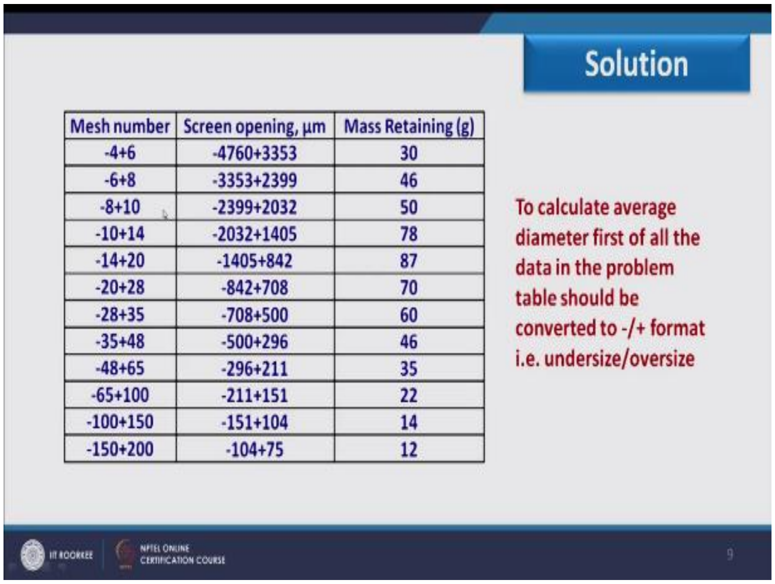

Mesh number	Screen opening, µm	Mass Retaining (g)
-4+6	-4760+3353	30
-6+8	-3353+2399	46
-8+10	-2399+2032	50
-10+14	-2032+1405	78
-14+20	-1405+842	87
-20+28	-842+708	70
-28+35	-708+500	60
-35+48	-500+296	46
-48+65	-296+211	35
-65+100	-211+151	22
-100+150	-151+104	14
-150+200	-104+75	12

Is the undersize to 4 and + shows whatever is the oversize. Here if you see this -4 and +6 in other word we can say -4 is basically showing the undersize to 4, it means whatever is passed through 4 and +6 is basically showing the material which is larger than correspond to 6 mesh number opening. So -4 is basically undersize to 4 +6 is basically over size to 6 so -+ we also call it undersize as well as oversize format.

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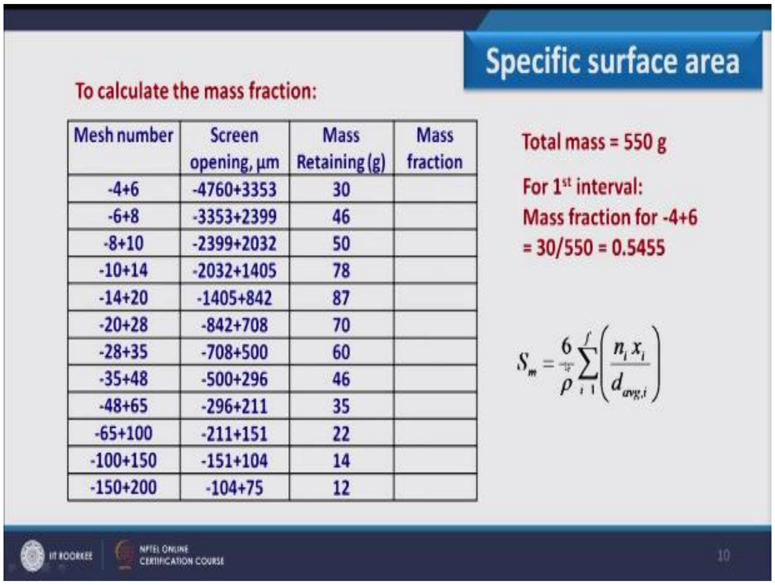

Now we will compute a specific surface area. To calculate a specific surface area this is the expression of specific surface area where the expression goes as $6/\rho\ \Sigma_1^f\ (n_i\ x_i/d_{avg.i})$, so all these parameters we will consider we will compute one by one and then we will calculate the specific surface area. First of all we will concentrate on mass fraction, so total mass if you count the mass retained on each screen if you count all these value it will be equal to 550 gram, for first interval that is -4 +6 it has mass retain as 30.

So mass fraction for first interval is 30/550 = 0.5455 so we can write the value of mass fraction of first interval here. In the similar line I can calculate mass fraction for other fractions also, for example -20 +28 it has 70, 70/ 550 so the value comes as it is shown over here, so if you join all this if you add all these mass fraction it would be equal to one, so here I have calculated mass fraction correspond to all fractions and in this slide.

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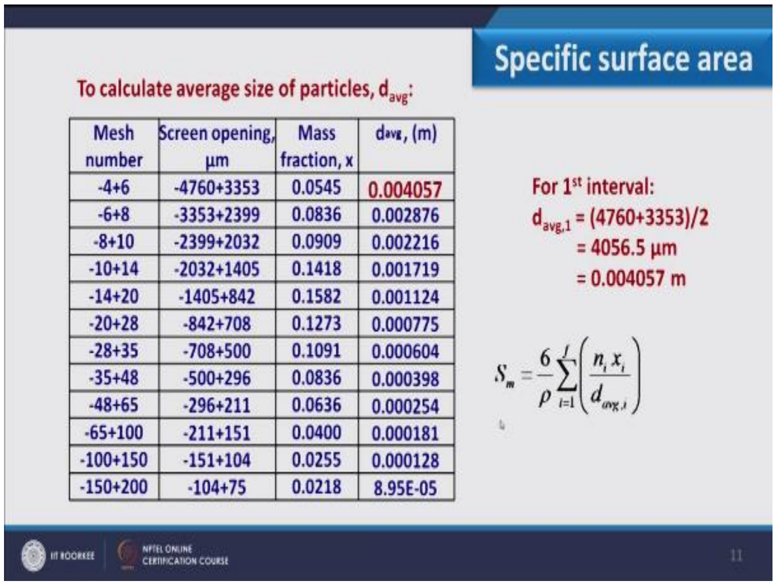

Mesh number	Screen opening, μm	Mass fraction, x	d_{avg}, (m)
-4+6	-4760+3353	0.0545	0.004057
-6+8	-3353+2399	0.0836	0.002876
-8+10	-2399+2032	0.0909	0.002216
-10+14	-2032+1405	0.1418	0.001719
-14+20	-1405+842	0.1582	0.001124
-20+28	-842+708	0.1273	0.000775
-28+35	-708+500	0.1091	0.000604
-35+48	-500+296	0.0836	0.000398
-48+65	-296+211	0.0636	0.000254
-65+100	-211+151	0.0400	0.000181
-100+150	-151+104	0.0255	0.000128
-150+200	-104+75	0.0218	8.95E-05

$$S_m = \frac{6}{\rho}\sum_{i=1}^{I}\left(\frac{n_i x_i}{d_{avg,i}}\right)$$

We have to calculate d average for first interval, how we can compute the d average, the arithmetic mean of screen opening of 4 and 6 mesh number screens. So it would be 4760+ 3353/2 so there is 4056.5 micro meter, in terms of meter we can represent this as 0.004057 and this value we can write over here. So here if you see this expression for specific surface x i and d average i for all fraction we have computed and we have written in this table. Further we have to calculate the value of ni that is specific surface ratio. Now if you remember this specific.

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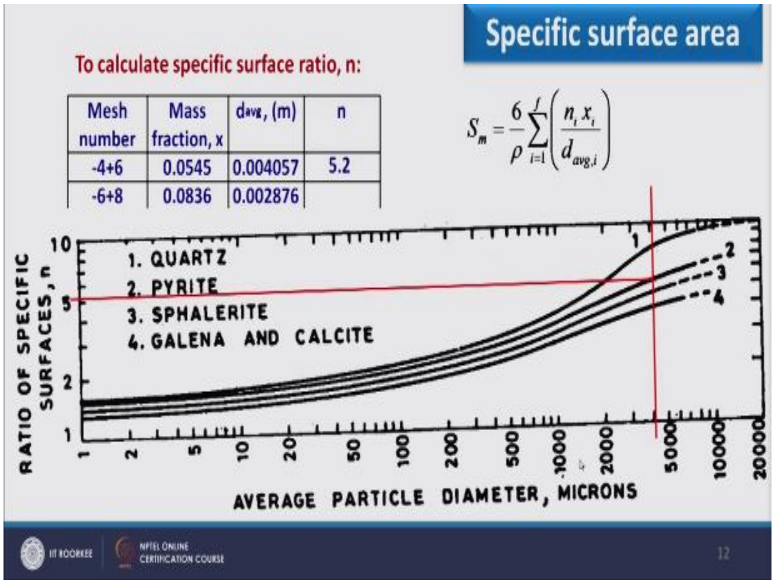

Surface ratio for different material we can see this value through a standard graph and here I am having that graph which is plotted between ratio of a specific surface, that is n versus average particle size diameter that is in microns. So if you remember the material we have the material as pyrite which is shown by two number plot in this graph. So first value is 4057 micro meter the average particle diameter, so correspond to 4057 we can place a line and correspond to this value as well as 2 number plot we can calculate the, we can see the value n from this graph which comes as 5.2.

So you can see that using this graph we can calculate, we can find the value of n that is specific surface ratio very easily and similarly we can write them.

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Specific surface area

To calculate specific surface ratio, n:

Mesh number	Mass fraction, x	d_{avg}, (m)	n
-4+6	0.0545	0.004057	5.2
-6+8	0.0836	0.002876	4.7
-8+10	0.0909	0.002216	4.2
-10+14	0.1418	0.001719	3.8
-14+20	0.1582	0.001124	3.4
-20+28	0.1273	0.000775	3
-28+35	0.1091	0.000604	2.8
-35+48	0.0836	0.000398	2.6
-48+65	0.0636	0.000254	2.3
-65+100	0.0400	0.000181	2.2
-100+150	0.0255	0.000128	2.1
-150+200	0.0218	8.95E-05	2

$$S_m = \frac{6}{\rho}\sum_{i=1}^{J}\left(\frac{n_i x_i}{d_{avg,i}}\right)$$

N value for other fractions also. Now I am having all values to calculate specific surface ratio, we already know the value ρ that is the density because we have given specific gravity of the material, therefore if you see this expression.

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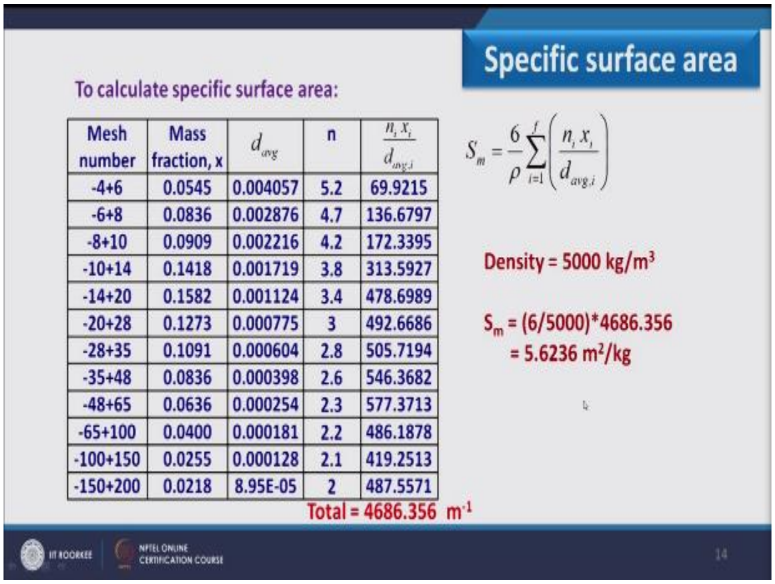

Mesh number	Mass fraction, x	d_{avg}	n	$\frac{n_i x_i}{d_{avg,i}}$
-4+6	0.0545	0.004057	5.2	69.9215
-6+8	0.0836	0.002876	4.7	136.6797
-8+10	0.0909	0.002216	4.2	172.3395
-10+14	0.1418	0.001719	3.8	313.5927
-14+20	0.1582	0.001124	3.4	478.6989
-20+28	0.1273	0.000775	3	492.6686
-28+35	0.1091	0.000604	2.8	505.7194
-35+48	0.0836	0.000398	2.6	546.3682
-48+65	0.0636	0.000254	2.3	577.3713
-65+100	0.0400	0.000181	2.2	486.1878
-100+150	0.0255	0.000128	2.1	419.2513
-150+200	0.0218	8.95E-05	2	487.5571

Total = 4686.356 m^{-1}

We have make the Σ of n_i, x_i over d average i for all fraction where i vary from 1 to f if I am having all fractions as f we can simply compute this because we have already computed n, x and d average for individual fractions. So for first interval we can calculate n_i, x_i, and d average i as 5.2 x 0.0545/0.004057 and value comes at 69.9215, this we can write in the cell. In the similar line we can calculate the value in rest of the fractions and total summation of this would, we can simply add all these value of n_i x_i / d_i which comes as 4686.356 per meter and density of the material is 5000. So I am having all values, we can simply calculate a specific surface area of mixture that is 6/5000 which is the density of the material into total n_i x_i / d_i for all fractions. So 5.6236 m^2/kg is the specific surface area of the mixture.

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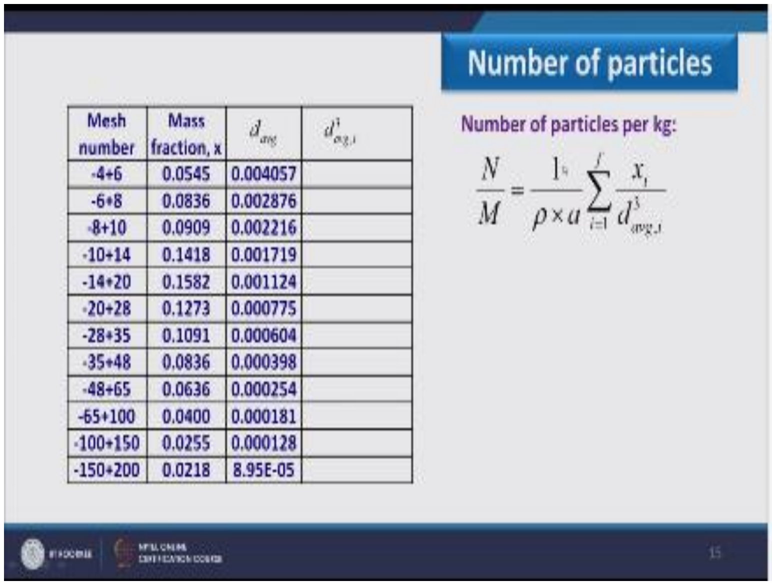

Further I have to calculate number of particles in a mixture, so number of particles per kg we can write like this N/M.

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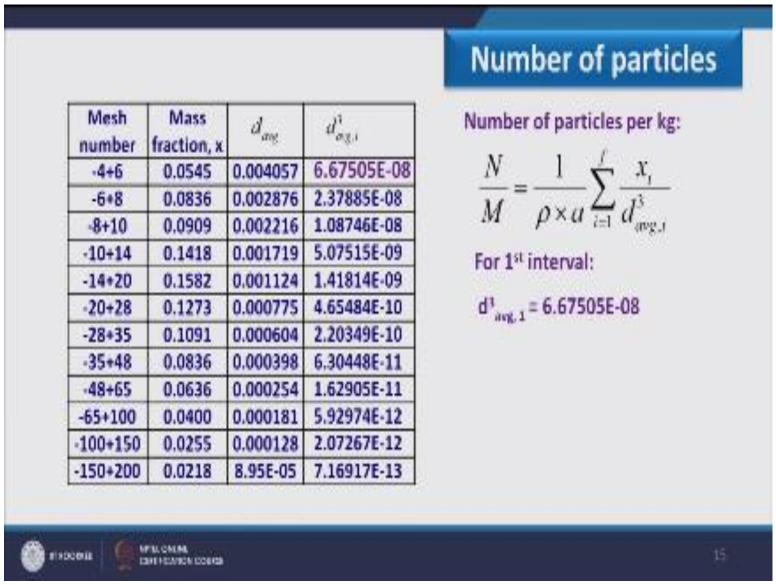

An expression is 1/ (ρ x a) Σ x_i /$d^3_{avg\ i}$$^{3.}$ So first of all we know d average for each fraction we have to compute $d^3_{avg\ i}$^{3} for each fraction, the first interval $d^3_{avg\ 1}$^{3} value is coming as it is shown over here which we can write at the proper place in the table and similarly $d^3_{avg\ i}$^{3} for rest of the fraction can be computed. We have already computed mass fraction for a specific surface area computation, so that we can use over here.

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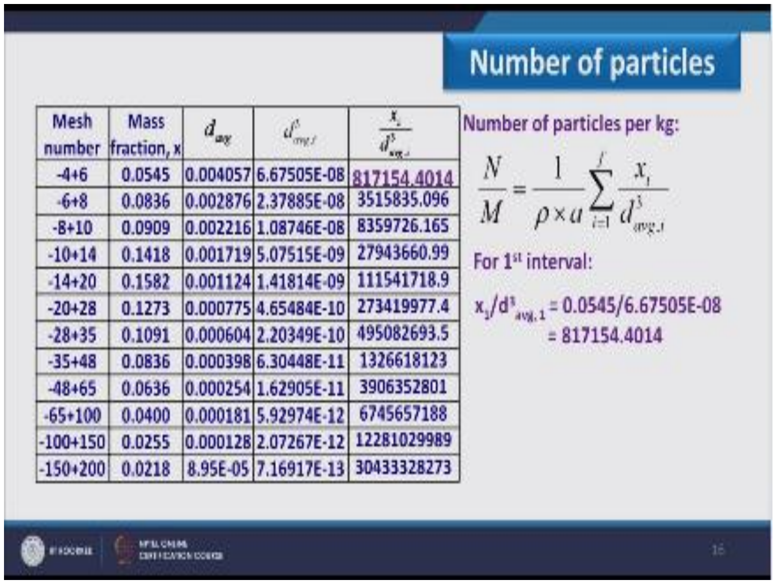

So now we have to calculate x_i and $d_{avg\,i}^3$ for each interval, for first interval we can write x1/ $d_{avg\,1}^3$ and whatever value we are having is 817154.4014, this value we can write over here and similarly we can write value in rest of the intervals. As we have summation over here.

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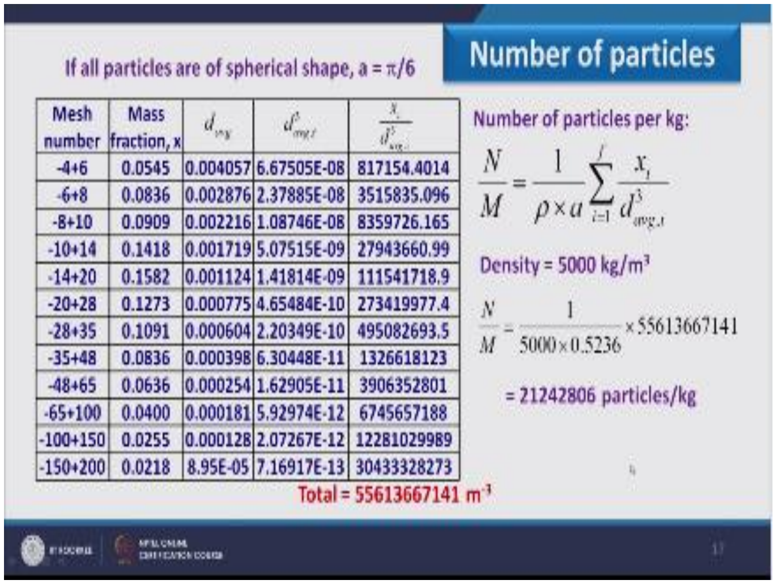

So this we have to add all the value of x_i and $d_{avg\ i}^3$ we have to add, the value comes like this, such a huge value and density I am having 5000kg/m$^{3.}$ I am having all particles of spherical shape the value of A should be Π/6. So it comes as 0.5236 so putting all the value over here we can have 21242806 particles/kg. So using particle size distribution data we can calculate specific surface area and number of particles in the mixture, in continuation to this we can calculate quadratic mean diameter which we also call surface mean diameter.

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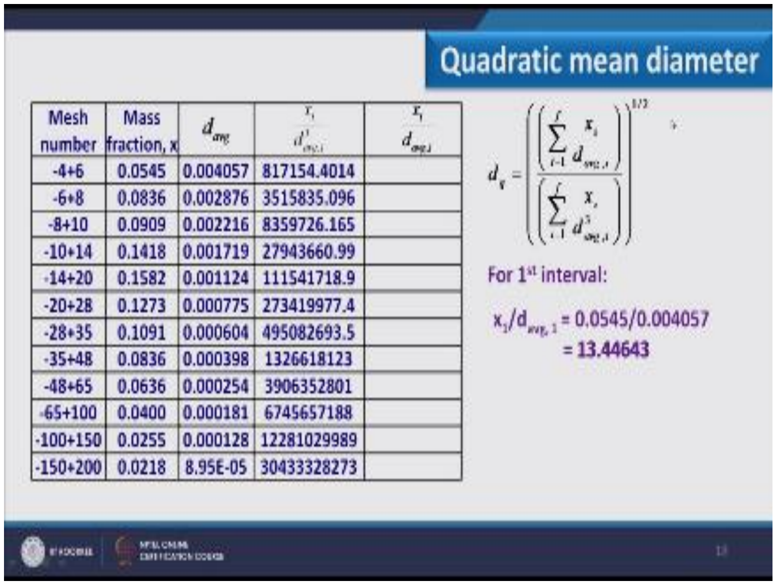
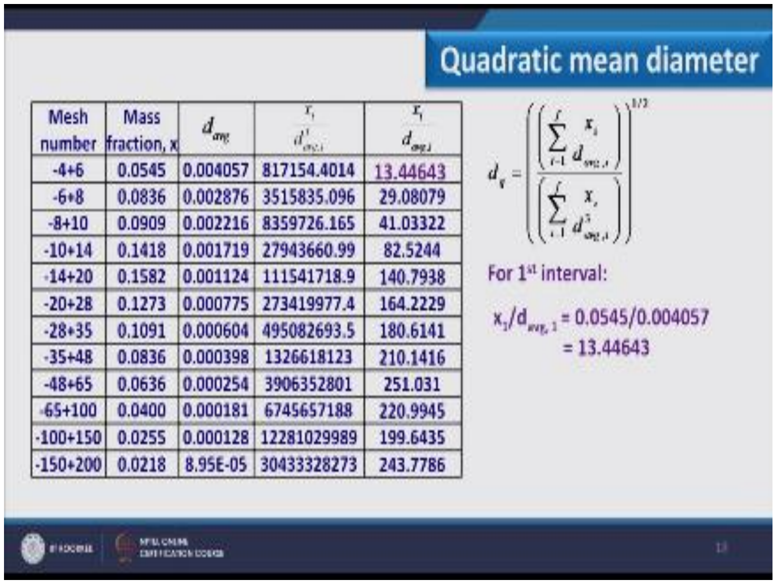


Mesh number	Mass fraction, x	d_{avg}	$\frac{x_i}{d^3_{avg,i}}$	$\frac{x_i}{d_{avg,i}}$
-4+6	0.0545	0.004057	817154.4014	
-6+8	0.0836	0.002876	3515835.096	
-8+10	0.0909	0.002216	8359726.165	
-10+14	0.1418	0.001719	27943660.99	
-14+20	0.1582	0.001124	111541718.9	
-20+28	0.1273	0.000775	273419977.4	
-28+35	0.1091	0.000604	495082693.5	
-35+48	0.0836	0.000398	1326618123	
-48+65	0.0636	0.000254	3906352801	
-65+100	0.0400	0.000181	6745657188	
-100+150	0.0255	0.000128	12281029989	
-150+200	0.0218	8.95E-05	30433328273	

So that is d^3 and the expression goes as [xi/ d^3 $_{avg\ I}$ Σ i=1 to f]/ [xi d^3 $_{avg\ I}$ Σ 1=1 to f] and the whole is power ½. So this is the expression, in the previous slide for computation of number of particle we have already calculated xi and d^3 $_{avg\ I.}$

(Refer Slide Time: 21:51)



Which we have shown in this column, now we have to compute xi, d $_{avg\ I}$. For first interval x1 d $_{avg\ I}$ we have computed using mass fraction as well as d average value and the value comes as 13.44643 and we can write the value over here. Similarly we can write the value in rest of the columns also. Now once I am having all values of xi $d^3_{avg\ I}$ as well as xi d $_{avg\ I}$ for all fraction we can simply sum them up to have the value of summation of these particular expression. So xi/ $d^3_{avg\ I}$ give the value of 55613667141 and xi/ d $_{avg\ I}$.

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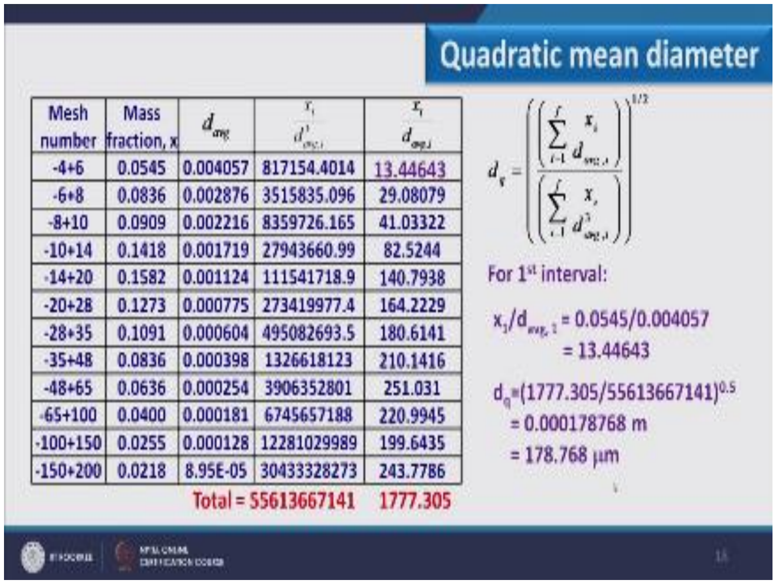

It gives the value as 1777.305 so using these two value we can calculate quadratic mean diameter and it value comes as, and its value comes as 178.768 micrometer. In this way we can calculate the mean particle size of a mixture, please remember this.

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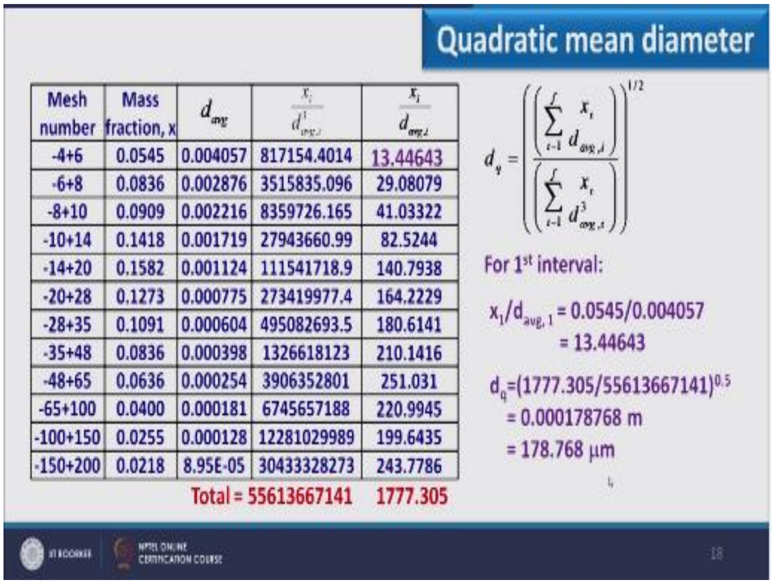

Is the mean diameter or mean particle size of the mixture not a particular fraction? Further I can calculate Sauter diameter, we are having the expression.

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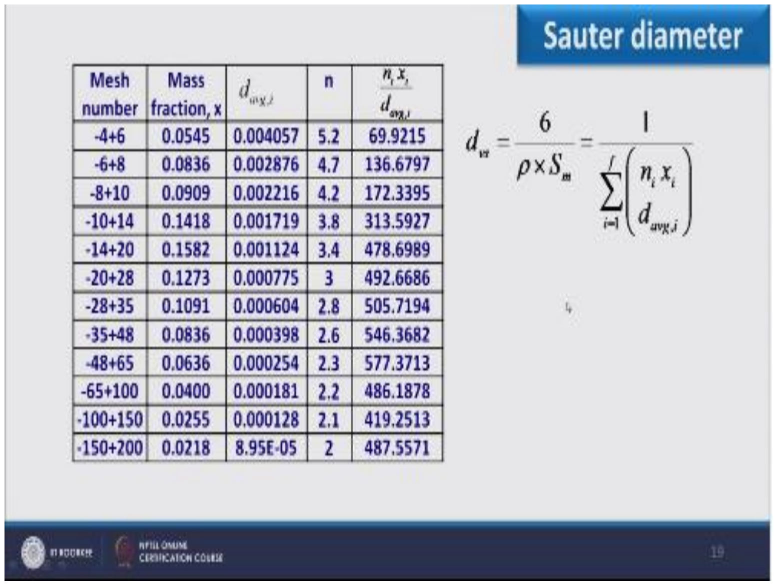
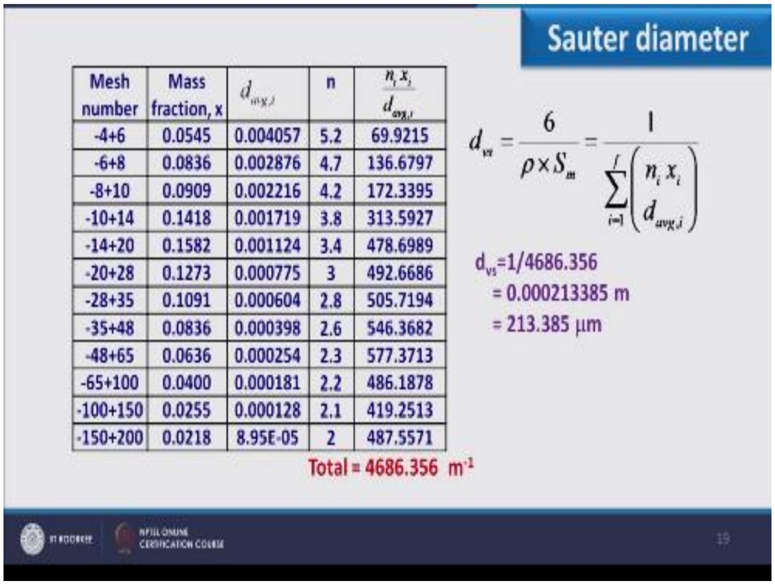


$d_{vs} = 6/\rho S_m$ where S_m is specific surface area which we have just computed. Specific surface area means it is surface area per unit mass and if you remember Sauter diameter is the volume surface ratio, so here we have to multiply S_m with the ρ to calculate the volume surface ratio and if I put the expression for Sm over here the final expression of Sauter diameter is 1 over $\sum n_i x_i$ over $d_{avgi.}$ So here in this slide we can write the value of $n_i x_i / d_{avgi}$ which we have computed once we were computing the specific surface area of the mixture, all these value we can find we can have.

(Refer Slide Time: 24:21)



The ∑ of this we can have 4686.356 per meter so considering this value we can calculate Sauter diameter as 1 divided by this complete value and Sauter diameter comes as 213. 385 micro meter. So in this way we can calculate the specific surface area, number of particles, quadratic mean dia also we call at surface mean and Sauter diameter we can calculate once I am having the particle size distribution data. So that is about the, that is all for this worked example.

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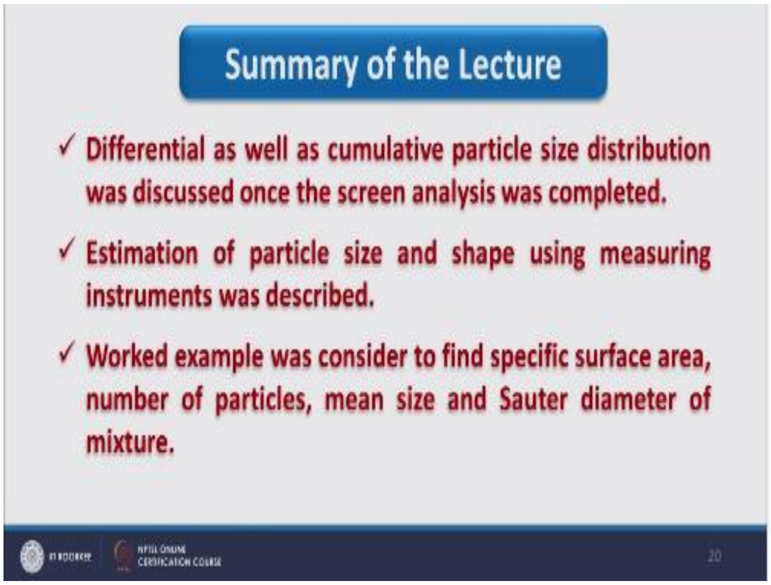

So summary of this lecture goes as differential as well as cumulative particle size distribution was discussed once we are having screen analysis data. Secondly in this lecture we have discussed the estimation of particle size and shape using measuring instrument and finally we have solved one example and we have computed the specific surface area and number of particle mean size and Sauter diameter of the mixture, and that is all for this lecture 5, thank you.

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