INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
NPTEL
NPTEL ONLINE CERTIFICATION COURSE
Mechanical Operations

Lecture-18
Particle dynamic-Examples

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Welcome to the third lecture of week 4 which is on particle dynamics and here we will discuss some of the work example on particle dynamics. So the topic of this lecture is particle dynamics, examples. Let us consider the first example on particle dynamics.

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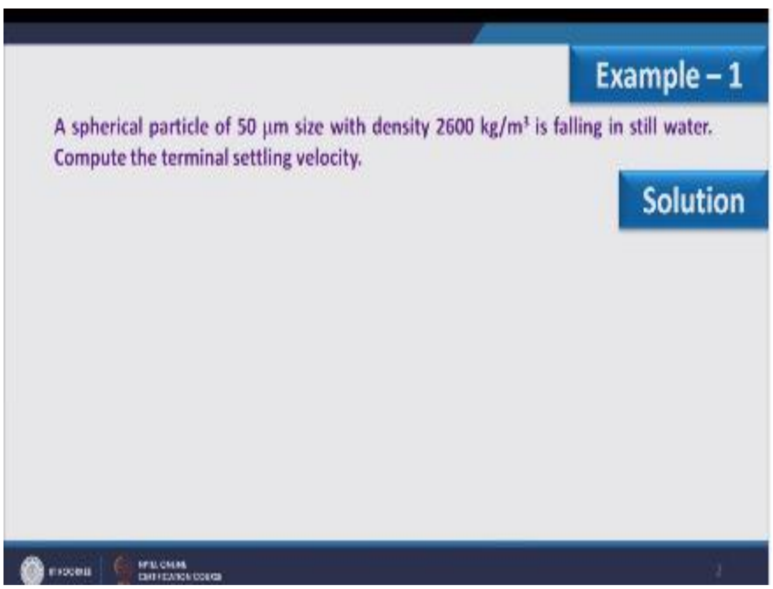

It says a spherical particle of 50 micrometer size with density 2600kg/m^3 is falling in still water, what we have to compute is terminal settling velocity of particle. So you see to solve this, to calculate terminal settling velocity we should know what is the region in which particle is falling, whether it is laminar region or turbulent region. So when you see the language of this question it will speak about the region on its own, how?

Here you see the particle diameter is 50 micrometer, so that is very less. So when we consider very small particle it is usually falling in laminar zone, if you remember the Stokes' law application there we have discussed the small fine particle and micro organism, all these will follow the Stokes' law, it means they fall in laminar zone.

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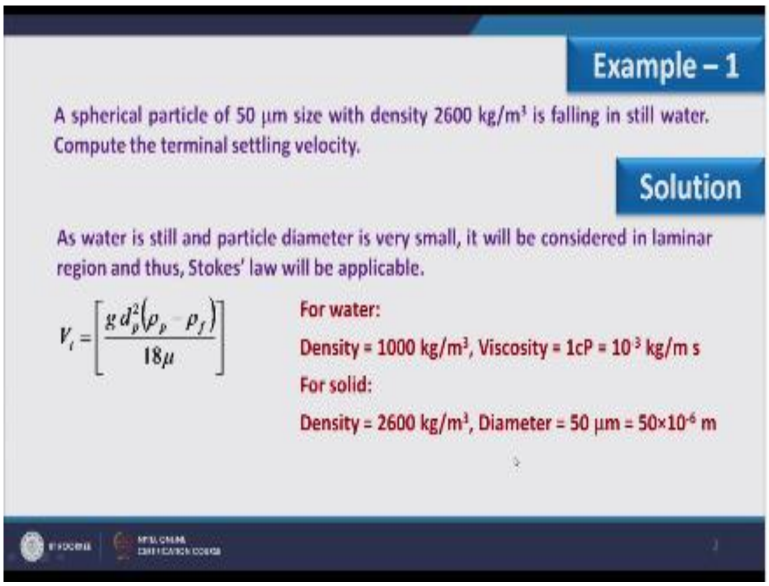

So as water is still and particle diameter is very small it will be considered in laminar region and therefore Stokes' law will be applicable. This is the expression of Stokes' law, that is $gd^2{}_p(\rho_p - \rho_f)/18\ \mu$, this is the Stokes' law and fluid which we have is the water, so for water density we have taken as $1000 kg/m^3$ and viscosity we have taken as 1cP which is 10^{-3}kg/MALE_2>>>' and the parameters used for solid density is 2600 that is given in the problem and diameter is 50 micrometer which is 50×10^{-6}m.

So all parameters are known to us so we can calculate the terminal settling velocity directly by following the Stokes' law, so considering these value.

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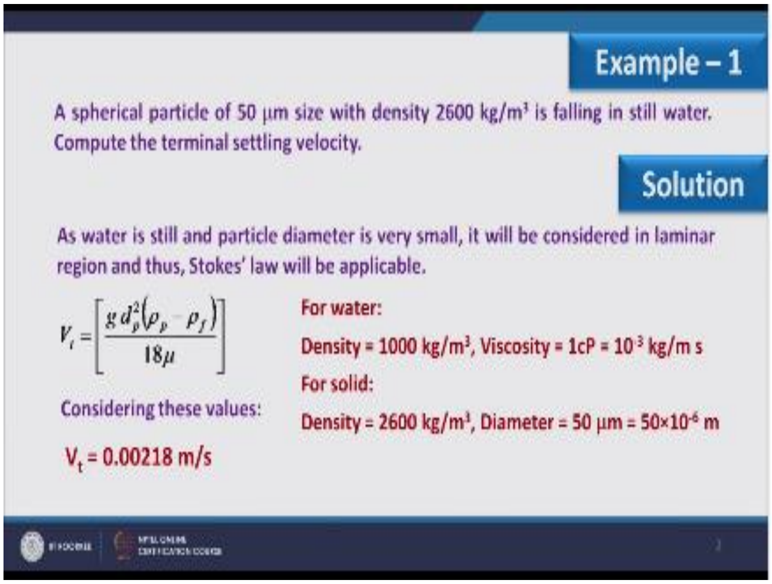

V_t value we can obtain which is having a value 0.00218 m/s, so if you see this problem how we can find the region of flow.

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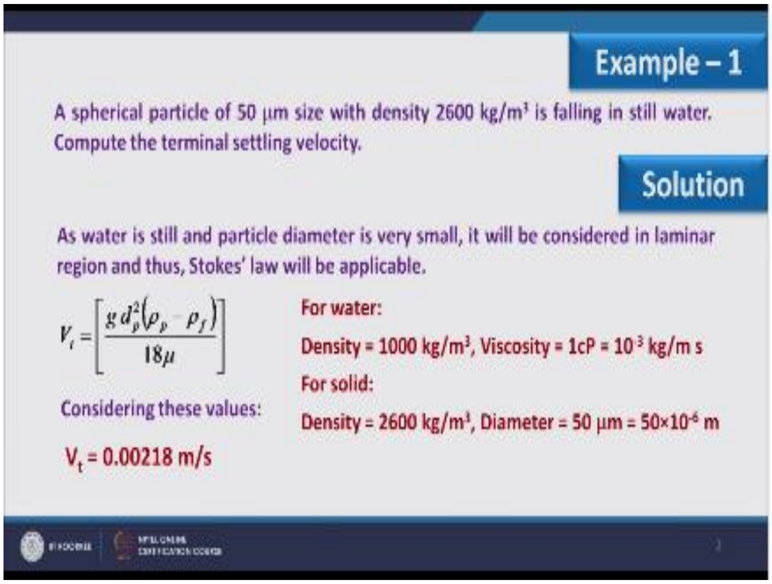

That is purely based on particle diameter and therefore we have used Stokes' law here.

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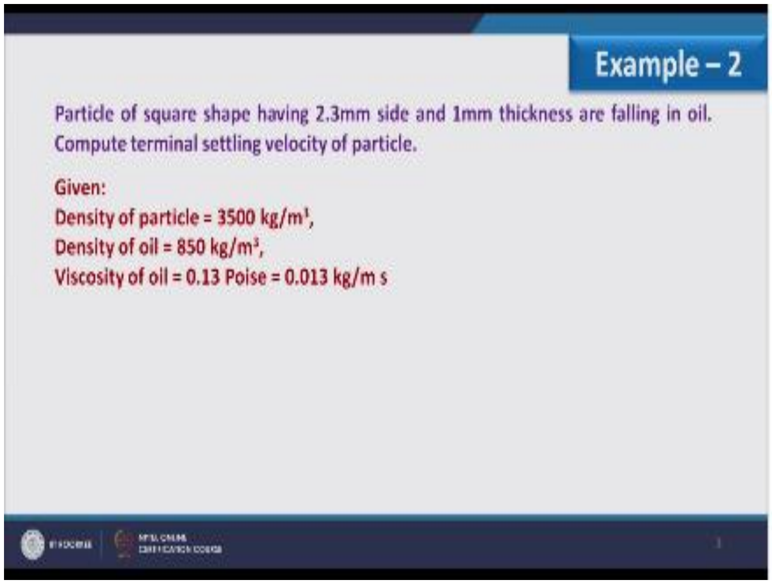

In example two what happens, the particle of square shape having 2.3mm side and 1mm thickness are falling in oil, we have to calculate the terminal settling velocity of particle, so here you see the particle is not a spherical, non-spherical particle is considered over here and the liquid is not water but it is oil.

So what is the purpose to take this example is when the particle is non uniform which, so when particle is not non-uniform, non spherical how the terminal settling velocity will be commuted and this can be applicable to the real situation also because in real situation most of the particles are non-spherical. So this example will be helpful in that condition.

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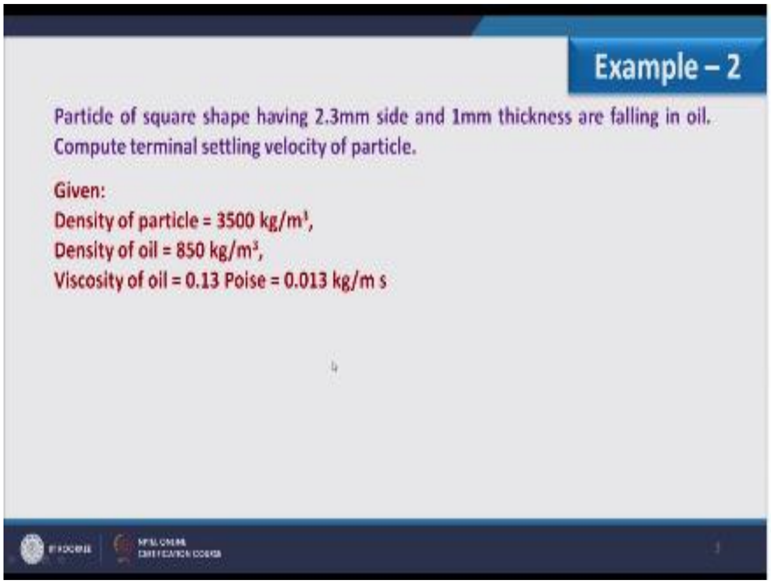

And the given parameters are density of the particle that is 3500kg/m^3, density of oil 850kg/m^3 and viscosity of oil is 0.013 kg/m^2. So let us start the solution of it.

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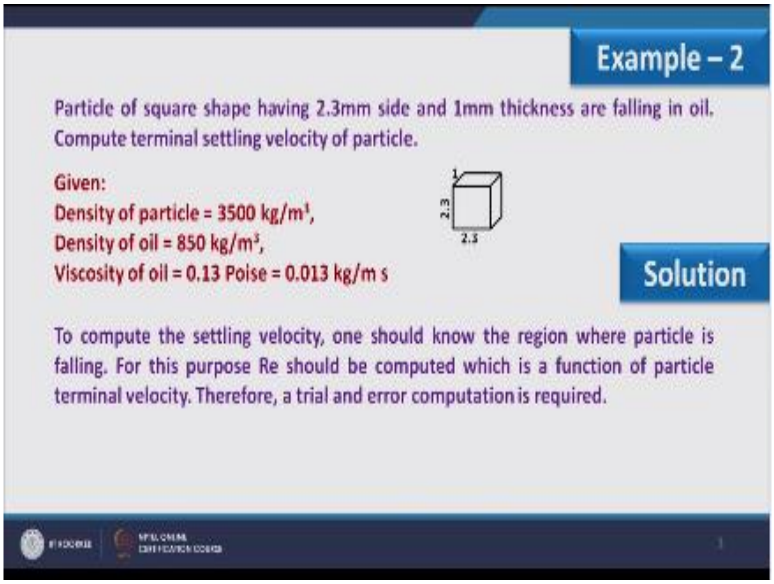

Now as far as particle is concerned that is, it is having side of 2.3 so that is the square particle 2.31 side another side is also 2.3 however thickness of the particle is 1mm. So this particle dynamics we have to calculate we have to find the terminal settling velocity of this particle, and as far as solution is concerned to compute the settling velocity one should know the region where particle is falling. If you remember the last example there we can aware about the region by considering the particle size that was very small but here particle size is significant.

So how we can find the region in which particle is falling, until unless I will not aware about the region in which particle is falling I cannot calculate, I cannot know the large number and therefore I cannot calculate f_D and further terminal settling velocity expression cannot be used.

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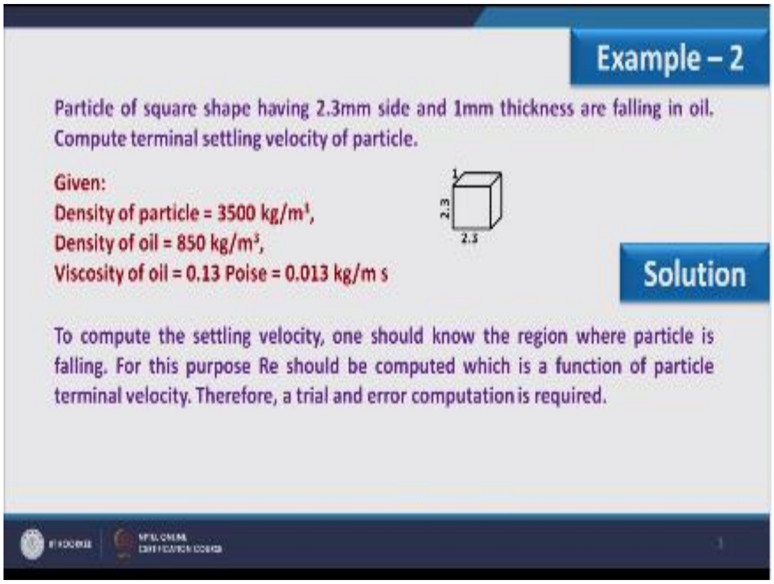

So to know the region we should know the large number and for that we know that terminal settling velocity so you see this is all trial and error based computation.

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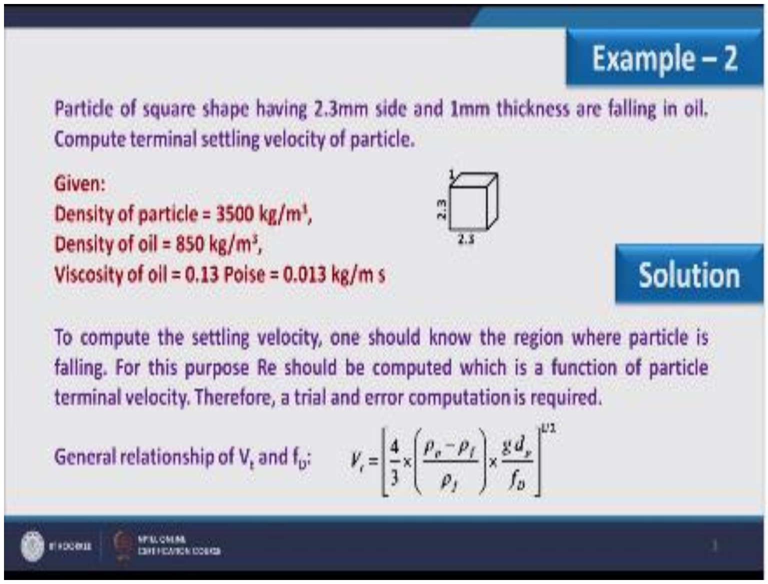

And to carry out this generalized expression of terminal settling velocity and f_D we have consider which is shown over here.

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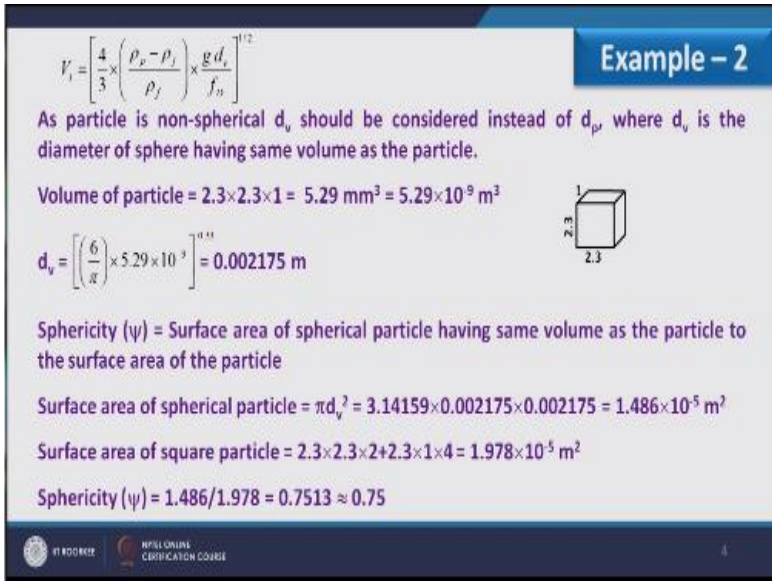

So what happens, this is the expression of V_t as a function of $f_{D,}$ now as particle is non spherical that is square in shape so instead of dp we have consider d_V that is the volume diameter of particle which is defined as the sphere diameter, d_V is basically the sphere diameter which is having same volume as the particle so to calculate d_V first of all we should know what is the volume of particle.

So if you consider this particular particle which is the object of the present example, here we have 2.3 and this side is also 2.3, so 2.3 x 2.3 x 1 would be the volume of particle which comes out as 5.29mm^3 or 5.29 x 10^{-9} m$^{3.}$ Once I am having the volume of particle I can calculate the volumetric diameter and that would be equal to volume of particle into 6/ π and the complete expression has the power 0.33. So solving this we can calculate d_V as 0.002175m, so if you remember the effect of shape particle shape on terminal settling velocity along with d_P we also know the sphericity of the particle.

So that we can use the given chart of f_D and Reynolds number, so to calculate sphericity first of all we will define it like surface area of a spherical particle having same volume as the particle to surface area of particle, so surface of spherical particle, we already know the volumetric diameter of particle so the surface area would $\pi\ d_V^2$ so π value is this and diameter we have already calculated.

So surface area of spherical particle comes out as 1.486 x $10^{-5} m^2$. Further we have to calculate surface area of square particle, so you see here how I can calculate surface area of this particle, if you see this is one phase and similar phase present opposite to this and here we have this another phase which is have total four phases of this size, so surface area of this phase we can calculate as 2.3 x 2.3 x 2 because 2 phases are available of this size. Further 2.3 x 1 x 4 because four phases are available of this size where the area is 2.3 x 1, so 2.3 x 2.3 x 2 + 2.3 x 1 x 4.

So total surface area of square particle is 1.978 x $10^{-5} m^2$, therefore we can calculate sphericity as surface area of spherical particle divided by surface area of square particles so it comes out as 0.75. Basically it is coming as 0.7513 and we have approximated it to 0.75, so now we know the volumetric diameter of particle as well as the shpericity and this is the expression of terminal settling velocity as a function of f_D.

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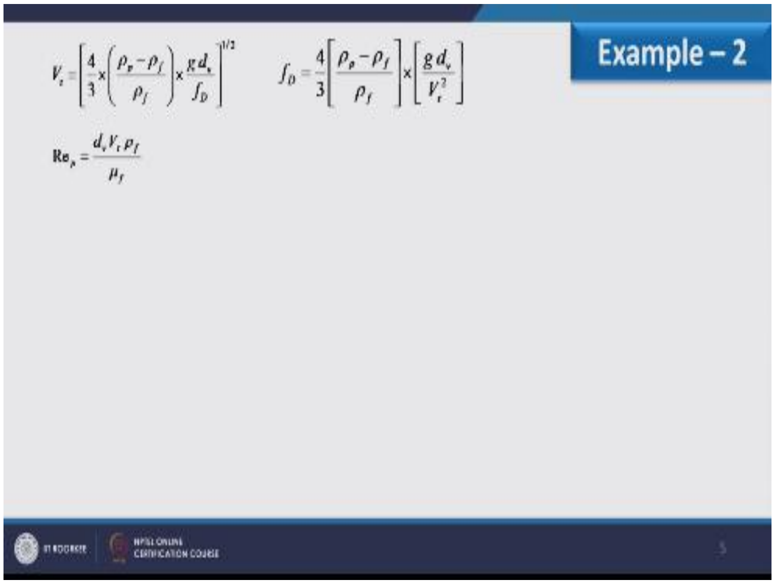

So further we have rearranged the expression and write the whole equation of f_D as a function of V_t, therefore to calculate the velocity we should calculate the Reynolds number and as the standard graph available to us is drawn between f_D as well as Reynolds number so we will find the expression of f_D as a function of Reynolds number not as a function of terminal settling velocity. So here you see Reynolds number of particle is d_V V_t ρ_f / μ_f, so here again we have calculated, we have considered d_V that is volumetric dia insisted of d_P and from this expression we can write the expression of V_t like Reynolds number μ_f / d_V ρ_F.

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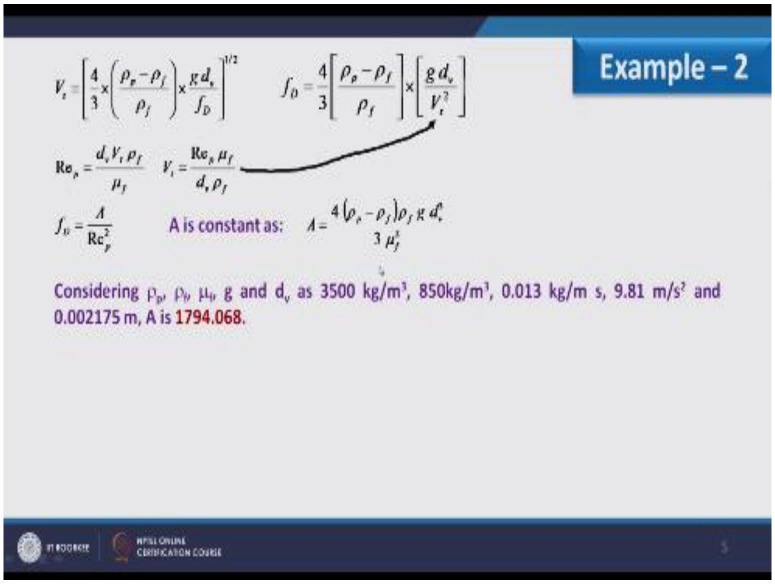

So this V_t expression we will put over here and final expression of f_D we can get as a function of Reynolds number and if you see the final expression we have f_D equal to A/ Re^2 now what is A. A is the constant where we have put all values which are constant like particle density, fluid density, particle diameter, tha is volumetric dia and fluid viscosity, all these are constant so we have represented all these value as single parameter and that is A.

So $f_D = A/ Re_p^2$ that is the expression we have got so considering value of known parameters that is ρ_p ρ_f μ_f g and d_v as 3500 kg/m^3, 850 $kg/m^{3,}$ 0.013 kg/ms and 9.81m/s^2 and 0.002175 m as volumetric dia, so considering all these value we can calculate value of A which comes out as 1794.068, so this expression we have.

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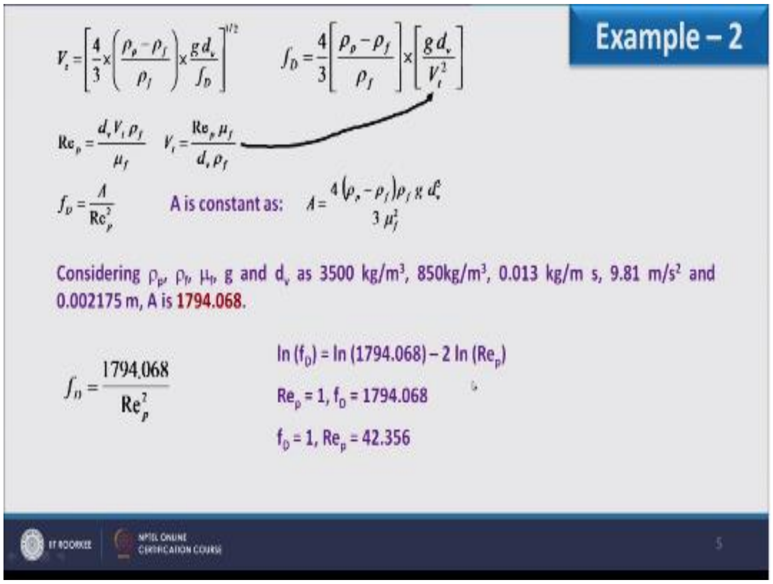

Already known as far as its value is concerned so now we have expression of f_D as 1794.068/ Re^2. If we take the log of this expression so l and f_D that is natural log f_D = ln (1794.068 – 2ln (Re) number, so if you see here we have f_D as well as Reynolds number based on logarithmic form.

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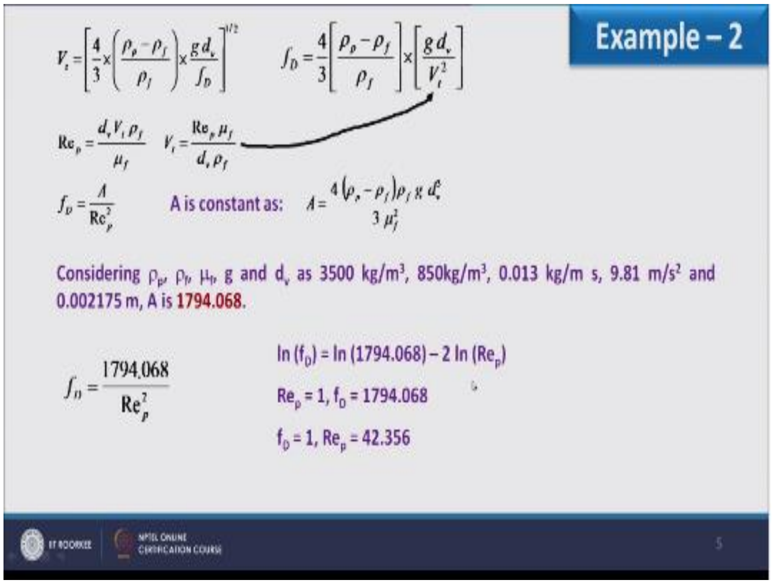

And for further calculation we have taken Reynolds number as 1.

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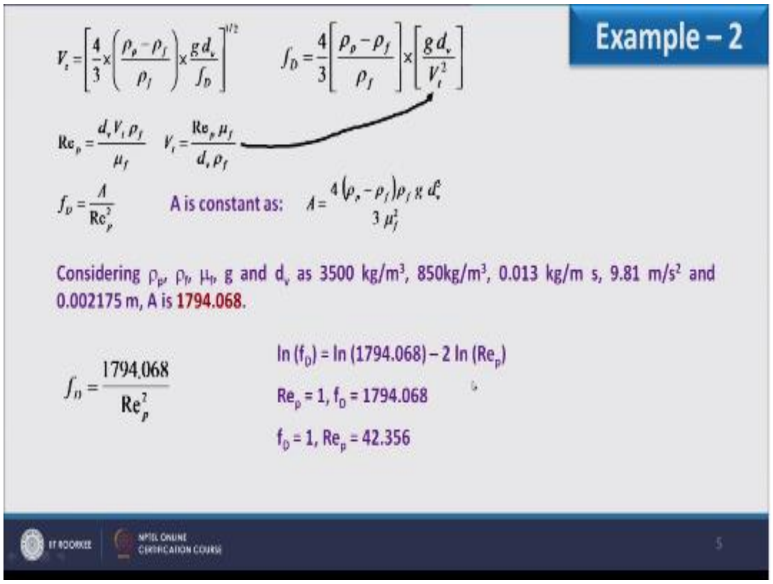

Because once I take the Reynolds number as 1 this complete expression would be equal to 0 so when Reynolds number is one f_D value.

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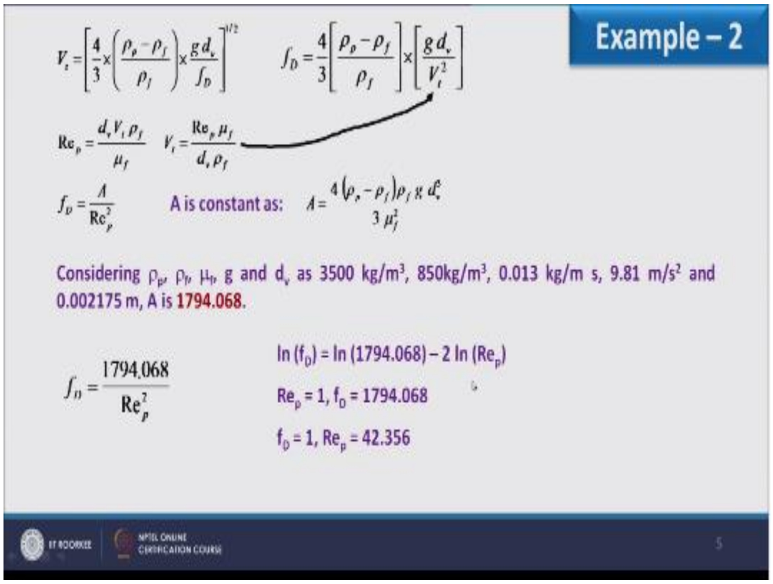

Is 1794.068 and when we take f_D as 1 then this expression would be 0 Reynolds number comes out as 42.356. So these value will be used to calculate the Reynolds number of particle, so you see here we have the value of Reynolds number.

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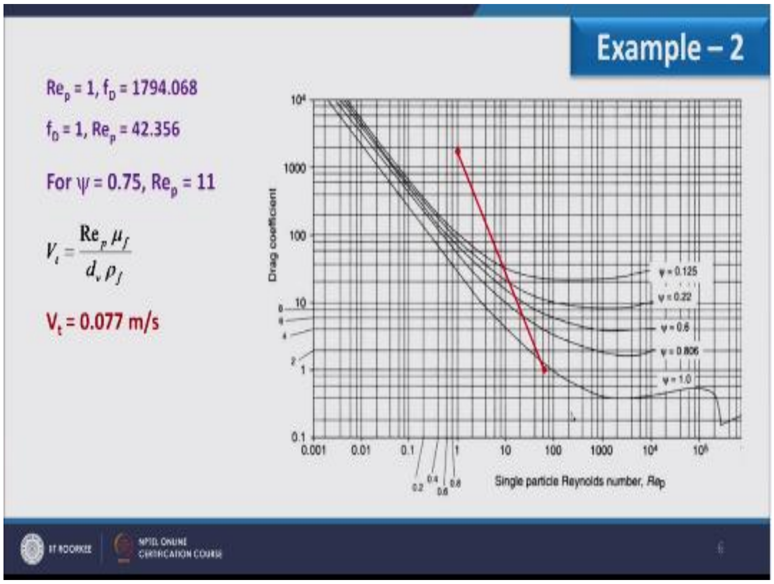

And f_D corresponding to Reynolds number 1 value f_D is coming as 1794.068 and f_D = 1 the corresponding value of Reynolds number is 42.356, so using these values we will calculate the Reynolds number. How we will calculate this, we have already computed what is the sphericity of the particle and it is coming out as 0.75. Now if you see all these plots here we have this is 0.125 and the third plot is of 0.6 and 4^{th} is 0.806 so between this we can have the value of 0.75, now to point out that value.

First of all we have to point out these points, for example correspond to Reynolds number one the value of f_D is coming as 1794 which is this point and correspond to f_D value 1 we have the Reynolds number 42.35, so joining this two point we can basically find the governing equation governing line in which Reynolds number.

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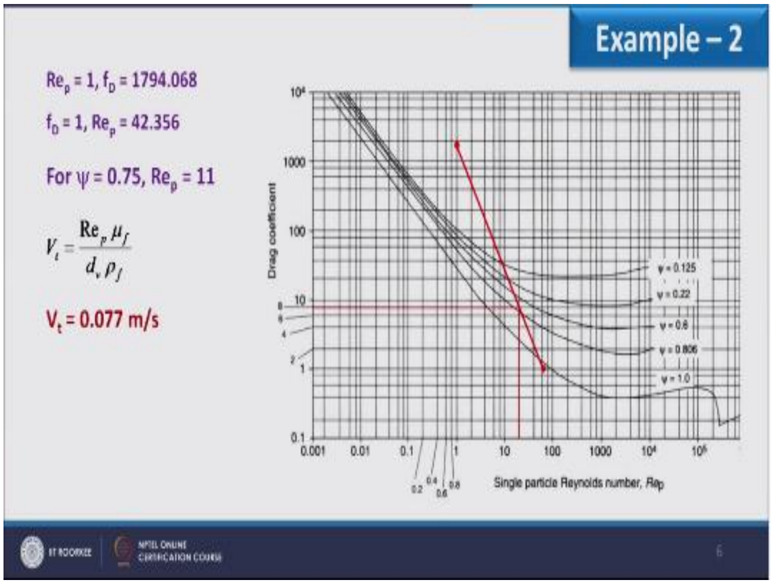

Is falling and if you see between 0.6 and 0.8 sphericity we can have the value of a sphericity as 0.75 so if you see between value 0.6 and 0.8 of Ψ we can have the value of Ψ = 0.75, and whatever the plot of 0.75 cross this line that we can consider as the point of consideration. So here you see this point we have considered as 0.75 because if we have the plot of 0.75 sphericity it will fall the, it will intersect the red line at this point only so considering this point we can have this drag coefficient.

As well as the value of Reynolds number, so following this point we can have Reynolds number a 11 which we can see from this graph very easily so once I know the Reynolds number value we can calculate the value of terminal settling velocity which comes out as 0.077m/s. So once I do not know the region where particle is falling and when it is of non spherical shape by following this method we can calculate the velocity of particle, so that is example two. Now we are considering example 3, in this example we have to calculate.

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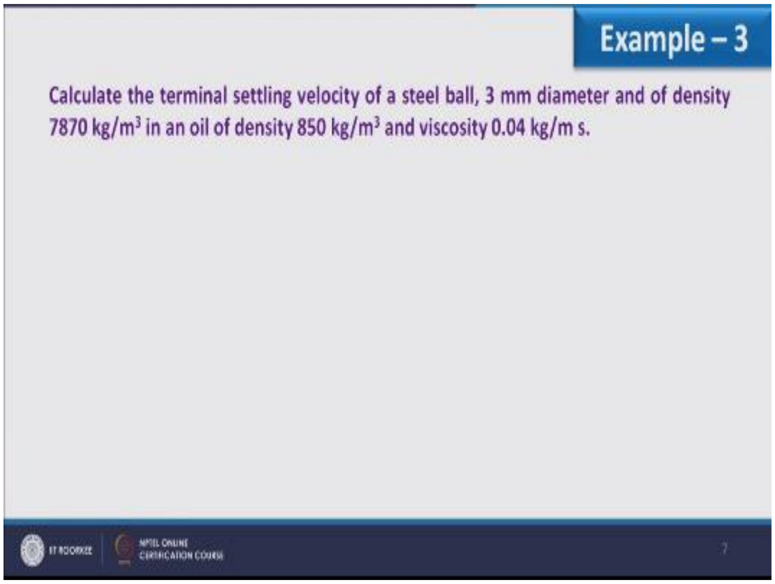

Terminal settling velocity of a steel ball having 3mm diameter and density 7870kg/m^3 which is falling in oil of density 850kg/m^3 and viscosity 0.04kg/ms. Now it's solution we will start, if you see this particle.

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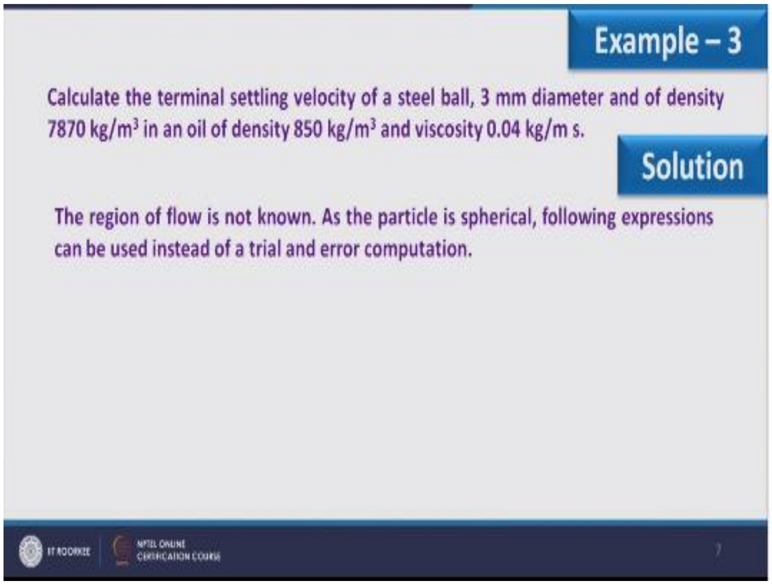

We are considering as steel ball so that can be considered as spherical particle, in this problem also region in which particle is falling is not known however as particle is spherical in shape we can use direct method using Galileo number instead of using trial and error method which we have used in example 2. So in this problem we have to calculate another settling velocity so as far as solution is concerned first of all we have to find the region in which particle is falling and if you read the problem region is not defined over here.

And we also did not know the x what expression we have to use, and to know the expression first of all we should know the region and to know the region we should know the Reynolds number for that we need the terminal settling velocity so that is trial and error method, but in this particular problem we are considering steel ball and this steel ball is, we can consider, we can assume as spherical in nature so instead of using trial and error method we can use direct method using Galileo number. Now as far as this approach is concerned here we have.

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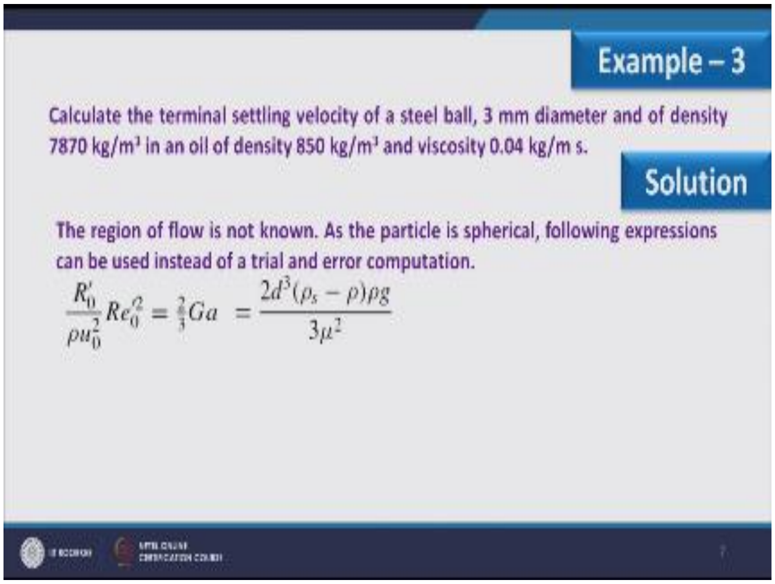

The expression $R_0'/\rho u_0^2\ Re^2$ which is equal to 2/3 Galileo number, and as far as Galileo number expression is concerned it is like this $d^3(\rho_s - \rho)\rho g/\mu^2$ so that is the Galileo number expression in which all parameters are known to us.

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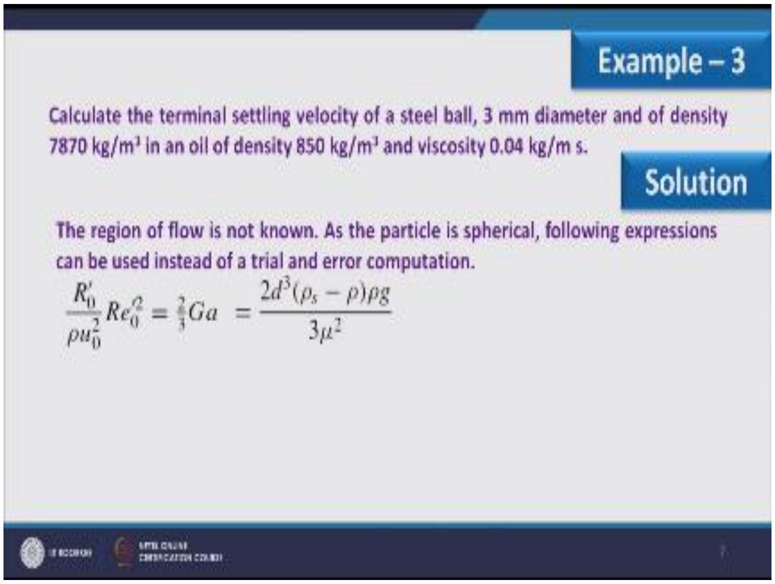

So first of all we will calculate the Galileo number.

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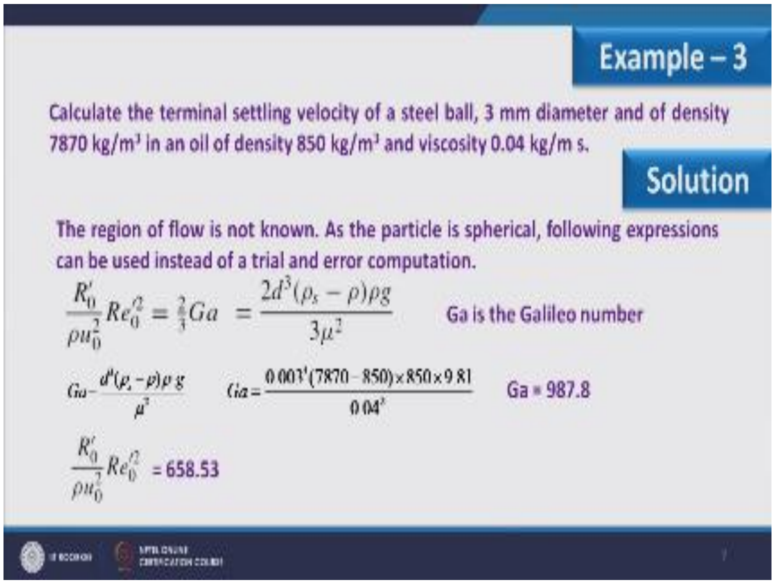

For this example, so considering value of diameter as 0.003 density of particle 7870 and that is of all 850 and considering viscosity of fluid 0.04 we can calculate Galileo number which comes out as 987.9, so once I have the Galileo number I can find the value of this particular expression that is equal to 2/3 into Galileo number, so 2/3x987.8 it gives the value 658.53. So this expression value we already know so how we will compute the terminal settling velocity. First of all we will use the graph for computation purpose, this is the expression which has value 658.53.

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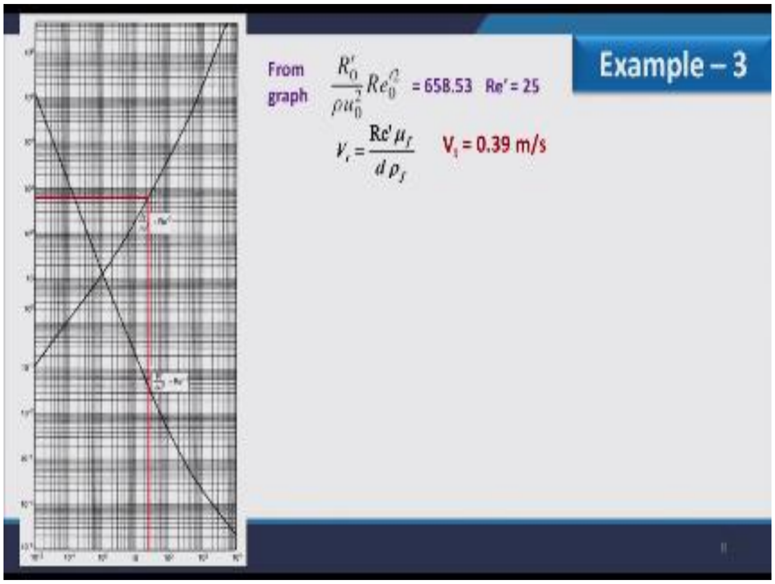

And this is the graph, if you see this y axis is basically this particular parameter and x axis has the Reynolds number value. So you see here 658.53 we have to draw and this will fall somewhere here and when it cut this curve and corresponding value of Reynolds number we can trace from the x axis. So when we do this Reynolds number that is Re′ comes as 25. Now for this Reynolds number we can calculate terminal settling velocity of particle which is coming as 0.39 m/s.

So this is one way for computing the terminal settling velocity of particle, here we have used the graph. Further the same computation.

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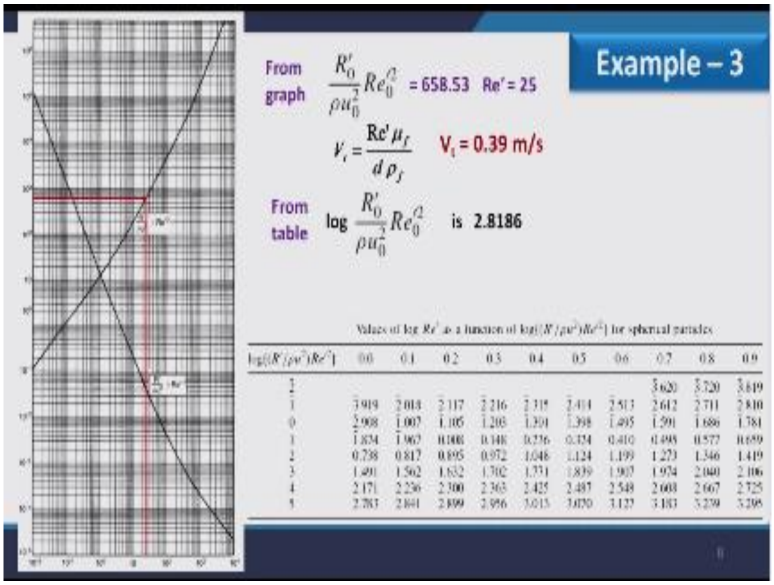

We can done using the table and if you remember the table has the value in terms of log. So first of all we will calculate log of this value which is basically log of 658.53 and it is coming out as 2.8186, and the data is available in this table if you see this here we have the data of log $R_0'/\rho u^2/Re'$ and here also we have the data in decimal form and all these data which are shown in this table are basically the log of Reynolds number values.

So here we have to find the value of Reynolds number corresponding to 2.8186.

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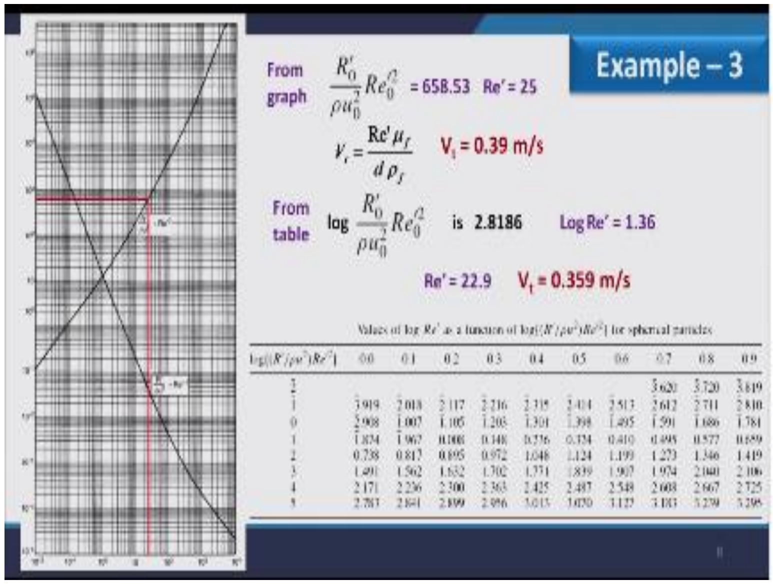

So if you see here we have the value of 2 and 0.8 will lie over here so we will use this parameter. Further it is 2.8186 so again we have to find the value correspond to 2.9, so 2 is here and 9 is here so value is 1.419. So value of Reynolds number corresponding to 2.8186 will lie between 1.346 and 1.419 and how we can find this value, by interpolation so when we do the interpolation then log Re′ we can find as 1.36 and corresponding Reynolds number value is coming as 22.9 and therefore we can calculate velocity as 0.359.

So in this way we can calculate the velocity once, I do not know the region we can follow this direct method but it specifically used for spherical particle. If we have to calculate the settling velocity of non spherical particle without knowing the region we can follow example 2. Now we have example 4, where.

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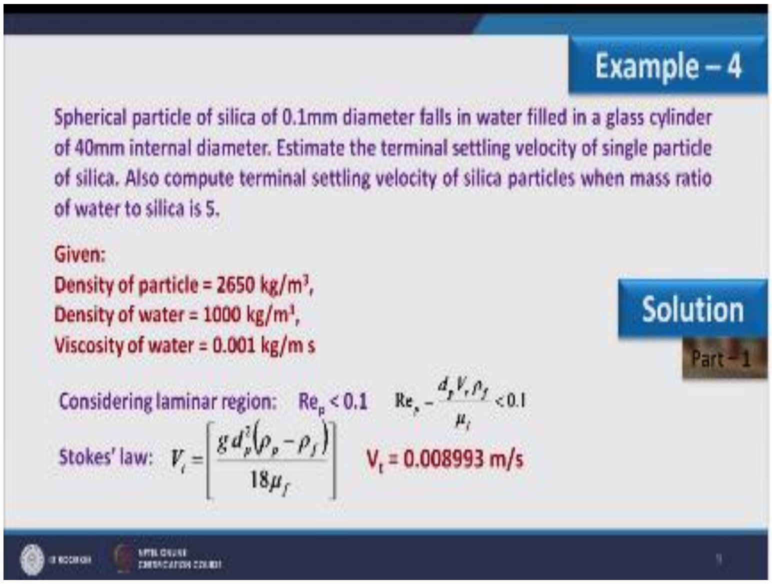

Spherical particle of silica of 0.1mm diameter falls in water filled in the glass cylinder of 40mm internal diameter. Estimate the settling velocity of single particle of silica and here mass ratio of water to silica is given as 5. These are the parameters for solution, first of all we will estimate the terminal settling velocity of single particle which is considering in part1. For example, if I am considering laminar region where particle is falling so Reynolds number should be less than equal to 0.1.

And this is the expression Reynolds number which must be less than 0.1, so in this case Stokes' law will be applicable so putting the values in this expression we can calculate terminal settling velocity which is coming out as 0.8993m/s.

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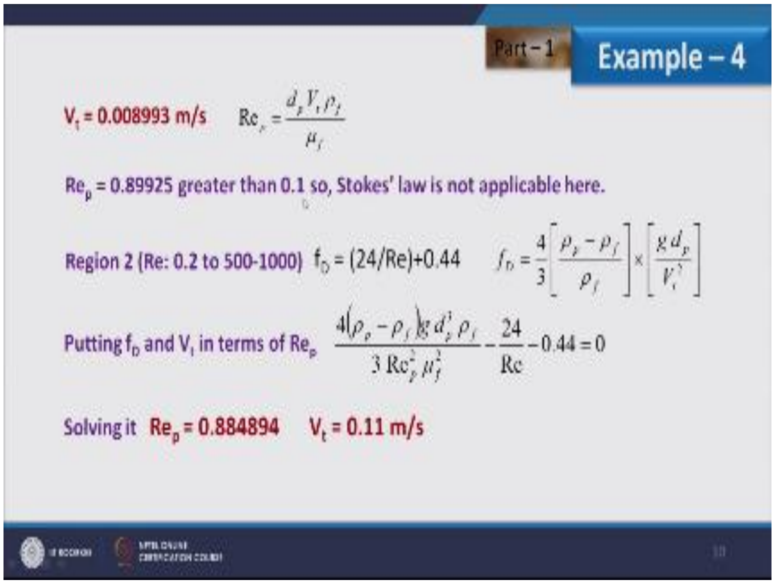

Now once I have this value further we will calculate the Reynolds number for verification and if we calculate Reynolds number it is coming out as 0.89925 which is greater than 0.1 so Stokes' law is not applicable here. Let us consider second region, where f_D is defined as 24/ Re + 0.44 and this is the generalized expression for f_d so putting f_d over here we can have the expression in terms of Reynolds number. This is the final expression and when we solve it Reynolds number is coming out as 0.88489 which is falling between 0.2 and 500.

So this region is correct and consequently we can calculate value of velocity that is 0.11 meter per second. In this case as the particle is falling in a container of 40mm diameter we can have the wall effect also, now wall effect if you remember the expression of C_f it is defined for turbulent as well as laminar region, but here it is falling in transition region so we will calculate C_f using both expression and this value we are getting for laminar flow and this is for turbulent flow, among this we will select the larger value which is 0.9963.

Considering this factor C_f in terminal settling velocity which we have found in last slide we can calculate final terminal settling velocity which is coming out as 0.1096 m/s, so that is the part one of this.

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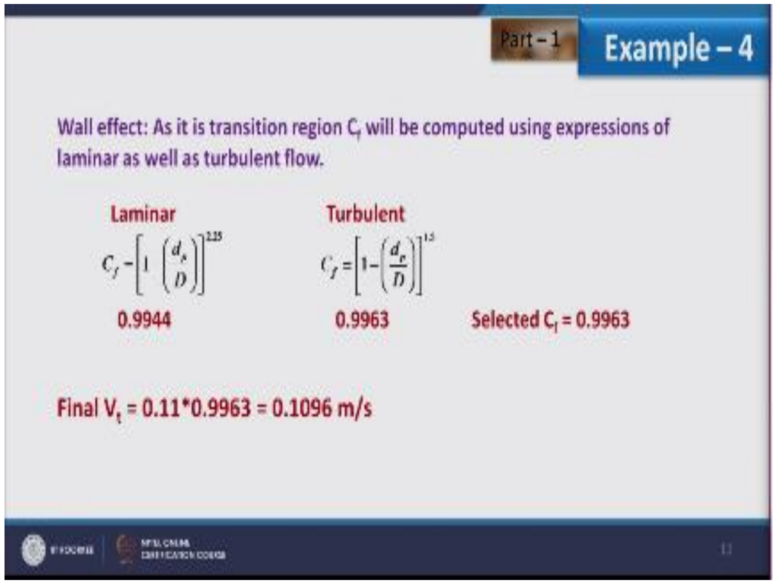

Now part two is here we have

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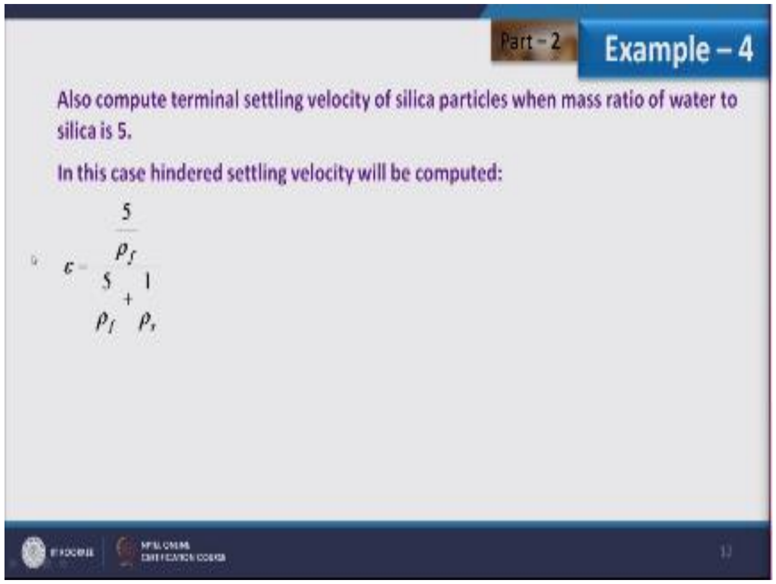

To consider the mass ratio of water to silica, it means here we have amount of silica available in water and it means that there are number silica particles and therefore we will follow the hinder settling velocity expression here. So first of all we have to calculate volume fraction that is $5/\rho_f$ $/5/\rho_{f+}$ $_{1/}\rho_s$ because five fraction of water and one fraction of silica, considering this we can calculate epsilon has 0.9298, once I have epsilon we can calculate F_s that is settling factor by this expression which in coming out as 0.6443 and after computation of F_s we can calculate hinder settling velocity which is the terminal settling velocity x Female Speaker:, so finally hinder settling velocity is coming as 0.0706m/s.

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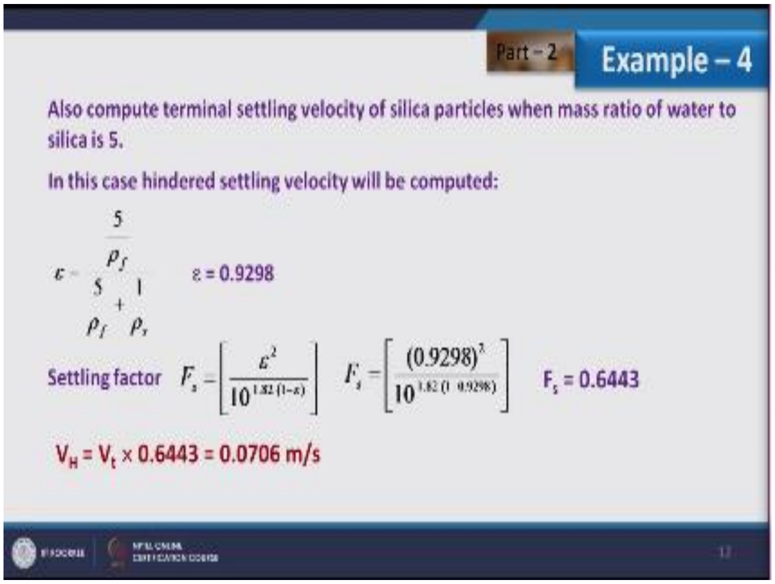

Now here we have fifth example in which bacteria of five micrometer

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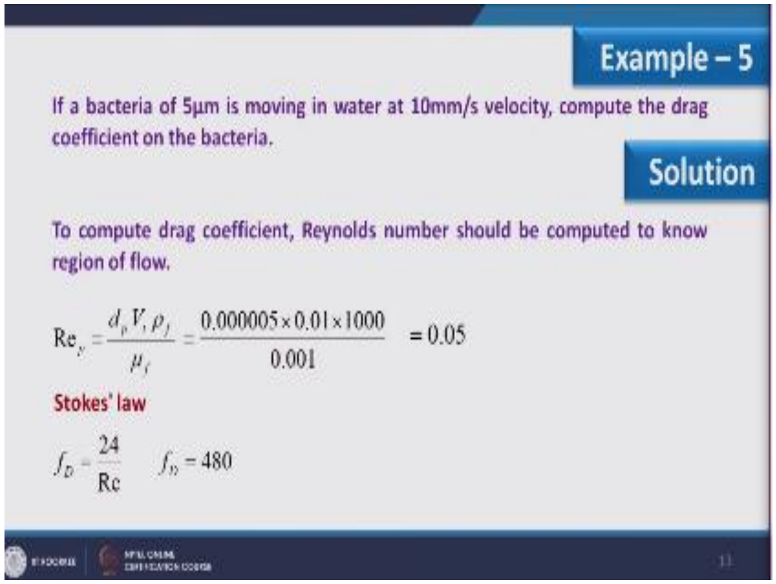

Is moving in a water in 10 mm per second velocity so what is the drag coefficient on the bacteria that we have to calculate. Now to calculate drag coefficient Reynolds number should be computed to know the region of flow. So Reynolds number we can calculate because we know the velocity so it can be directly calculated which is coming out as 0.05 so it is less than 0.1 so Stokes' law will be Applicable over here where F_d is 24/Re so F_d we can calculate which is coming at 480 so that is the drag coefficient value on the particle.

So that is the drag coefficient on the bacteria and here we have another simple example so in this example

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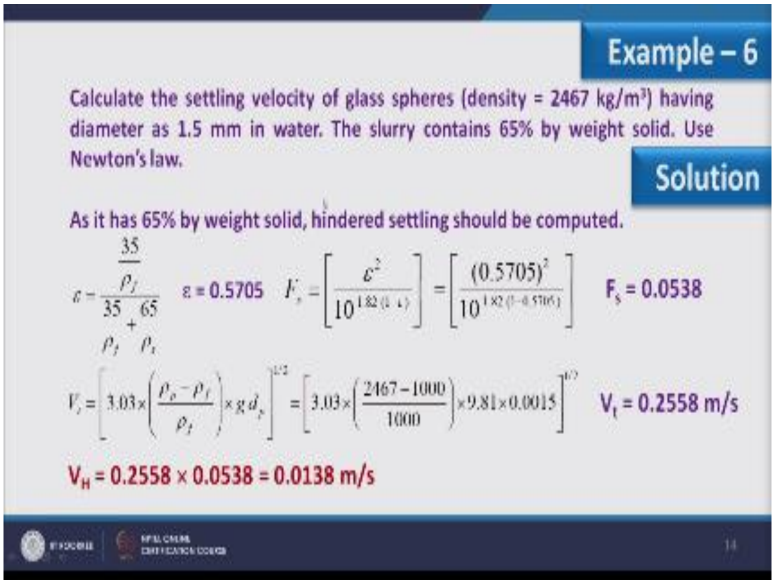

We have to calculate settling velocity of glass sphere having density 2467Kg/m^{3}and diameter 1.5mm and this is falling in water, so slurry contains 65 % by weight of solid and we have to use the Newton's law, so region is already given to us that it is falling in turbulent region and we have to use the Newton's law as 65% weight is available obviously we will calculate the hinder settling instead of terminal settling. So first of all we should know epsilon value that is if 65% solid is available

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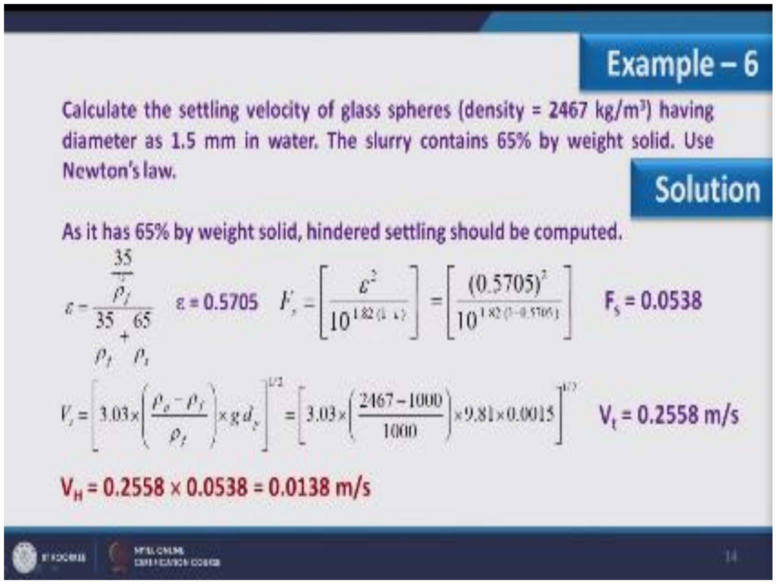

So 35% water will be there so 35/ ρ_f /35/ ρ_f+65/ ρ_s so epsilon value is coming as 0.5705, considering this we can calculate the F_s value which is coming at 0.0538, once I know this I can calculate the hinder settling velocity but for that we should know the terminal settling velocity. As Newton's law we can apply which has this expression by putting all values in this we can have terminal settling velocity as 0.2558 and when we multiply this with the F_s value the hinder settling velocity or final velocity of the particle is coming as 0.0138m/s, so here we have the summary of this lecture, in this lecture different work examples.

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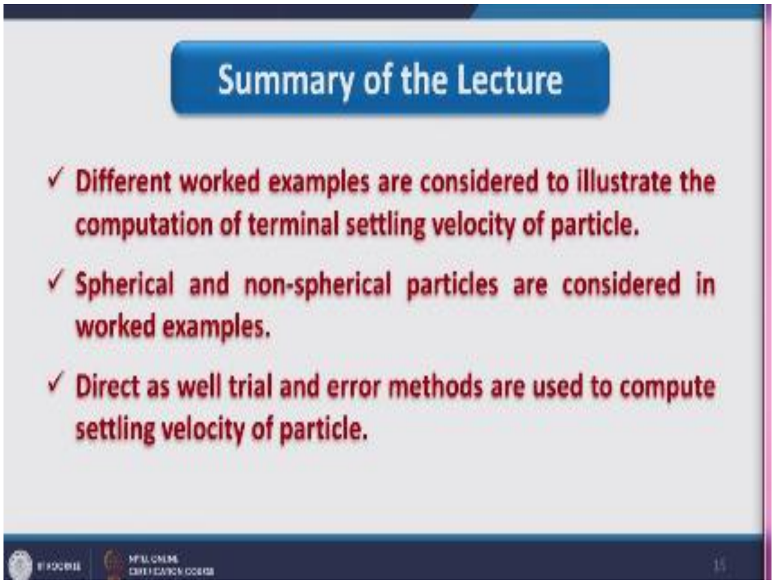

Are considered to illustrate the computation of terminal settling velocity of particle, spherical as well as non-spherical particles are considered in work examples, direct as well as trial and error method are used to compute settling velocity of particle.

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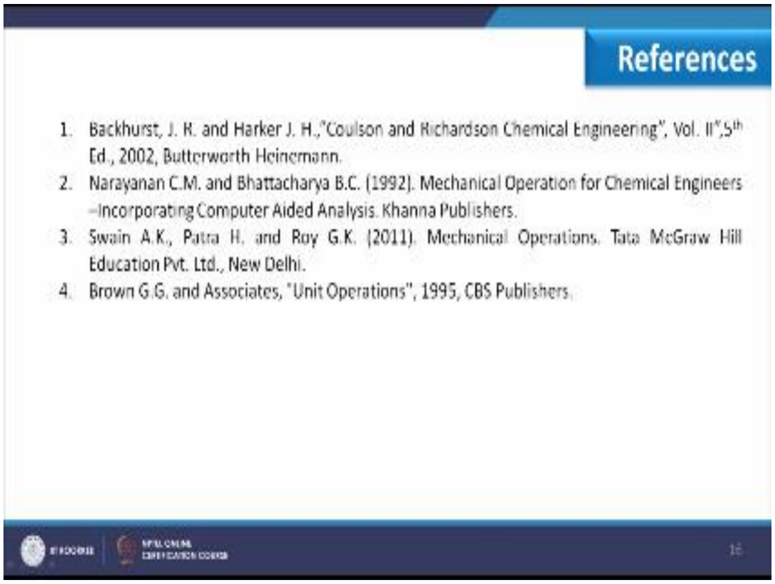

And these are the references that is all for this, thank you.

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