INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
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Mechanical Operations
Lecture-17
Particle dynamics-2

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Welcome to the second part of lecture 1 which is on particle dynamics. So if you remember the first part of this we have derived the expression of terminal settling velocity and we also have seen different equations of terminal settling velocity falling in different region.

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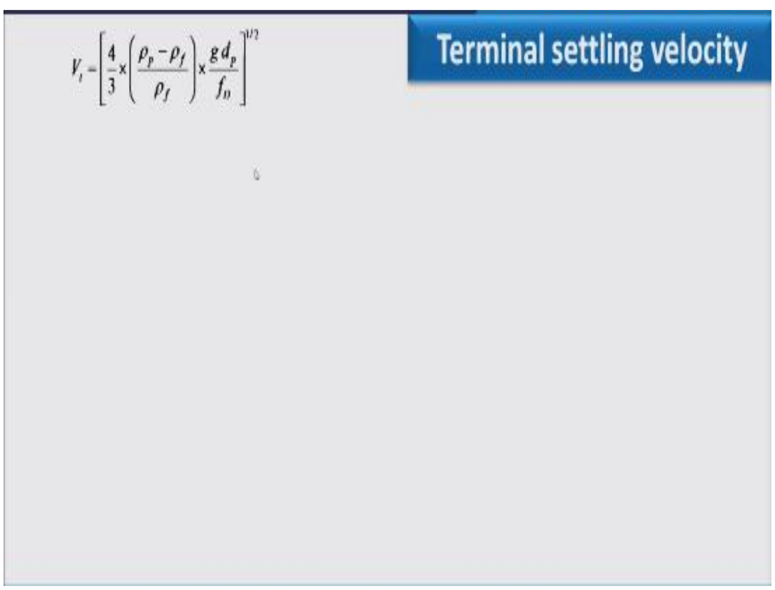

And if you see this equation, this is the generalized equation of terminal settling velocity. And here V_t is a function of f_D. Now what happens when we do not know the region in which particle is falling, then I cannot take the value of f_D. For example, if I do not know whether the particle is falling is laminar region or any other region, so how I can take value of f_D. If it is falling in laminar region then only I can take the value of f_D as $24/r_E$.

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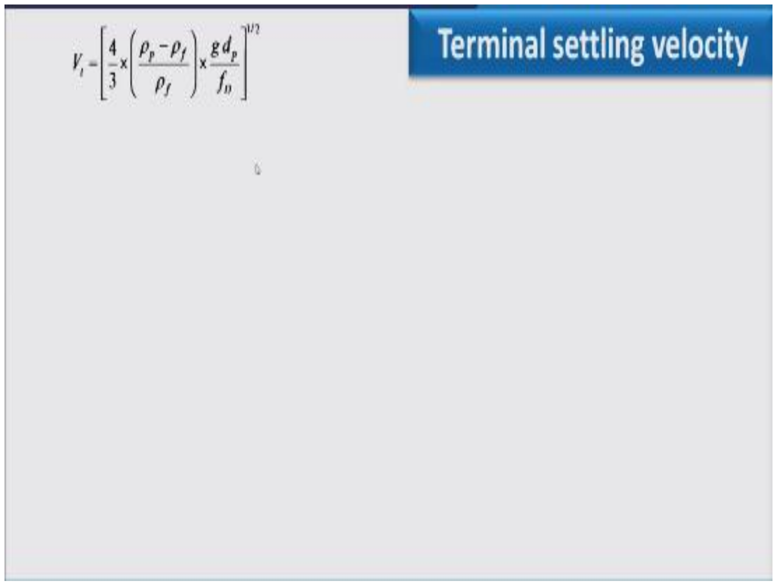

Now to know the value of f_D we should know the value of Reynolds number. And to calculate Reynolds number again we need the value of terminal settling velocity that is V_t. Therefore, this f_D as well as Reynolds number calculation is trial and error based. It is not directly based until or unless I know the region in which particle is falling.

So to handle such situation we have another expression of another way to calculate terminal settling velocity and about this you can study in detail in this reference.

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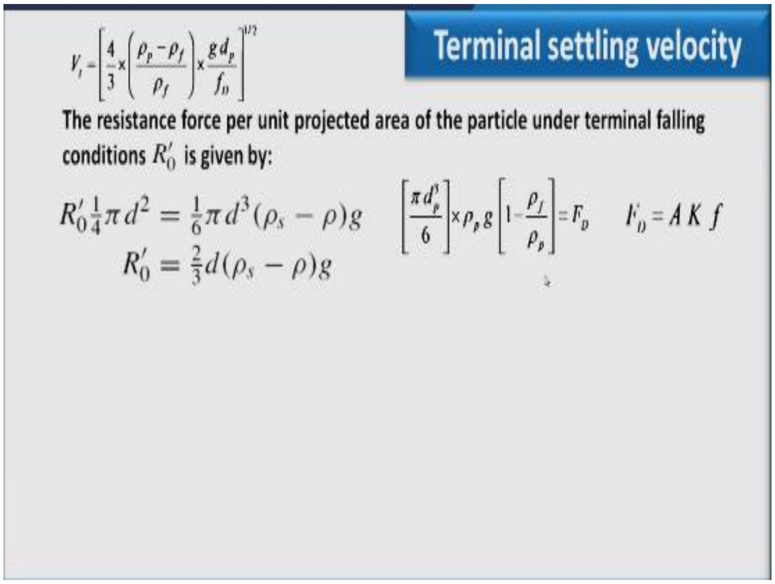

So in this method what we have taken that, what we have considered that resistance force per unit projected area, the resistance force per unit projected area of the particle under terminal falling condition R_O' and it is given by this expression. So R_O' is the resistance force and the resistance force per unit area and here we have multiplied it with projected area of particle.

So R_O' x $1/4 \pi d^2 = 1/6\ \pi d^3\ (\rho_s - \rho)\ g$. Now what is this right hand side expression. If you remember where we have started derivation of terminal settling velocity. So here this complete expression is equal to f_D and if you match this equation with this it is equal, because here we have d_P and that d_P will be replaced at the place of 1.

So when we resolve this equation this expression the same expression we will get. And that is equal to f_D. So f_D basically A x K x f and if you see the parameter R_O' x A, so what is R_O' is, R_O' is nothing but the multiplication of K x f. So same equation you can use over here, but instead of K x f we have to write the parameter R_O'.

And if you see this expression as well as this expression some of the parameter are denoted differently. For example, here we have taken diameter of particle as D previously we have taken this as d_P and in this case we have taken ρ_s and that is the density of solid and here we have taken density of particle ρ_P and here this is the density of medium or fluid and in this, in previous expression we have represented this with ρ_F.

So here you can say parameters are same, but their notation are different, but I have already defined, so you can resemble the parameters with the previous definitions of parameters.

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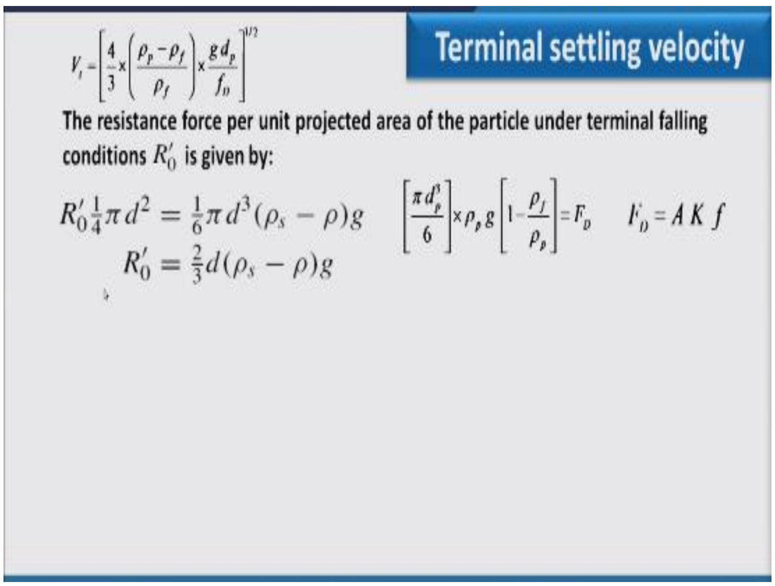

Now here using this particular expression we can find the expression of R_O' which is 2/3 d $(\rho_s - \rho)$ g. So this is the expression of resistance force on the particle.

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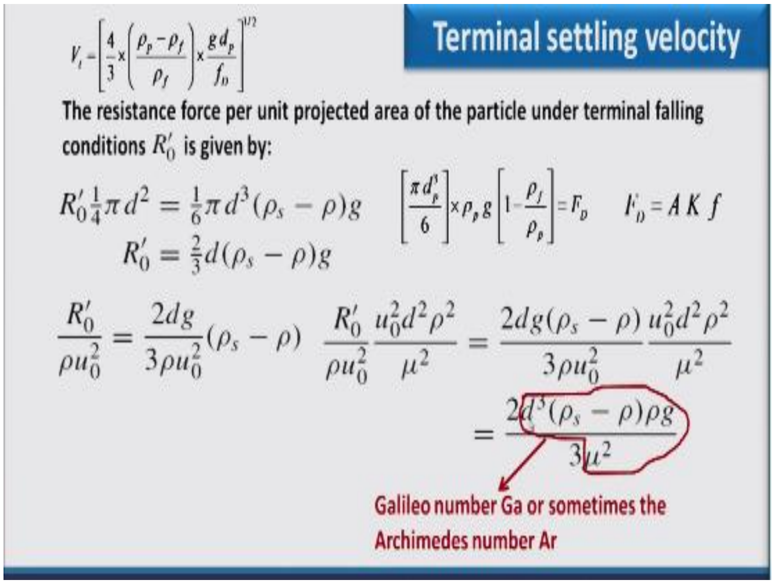

And further we will consider this expression and both side of this equation we can divide by ρu_0^2, here left hand side also as well as right hand side also, we have divided it by ρu_0^2 where u_0 is nothing but the terminal settling velocity that is V_t we have represented previously. So if you consider this expression, further we will multiply both side of this equation by Reynolds number square.

If you see this expression that is $u_0^2d^2\rho^2/ \mu^2$ this is nothing but the Reynolds number square. So R_o'/u_0^2 x Reynolds number square. It is equal to this expression into Reynolds number square. So when we resolve this equation and in this side if we see that u_0^2 and here u_0^2 is cancelled out and we can further write the expression which is present to right hand side like this $2d^3(\rho_s - \rho)\ \rho g/3\mu^2$.

Now if you consider this particular set which we have encircled that is $2d^3(\rho_s - \rho)\ \rho g/3\mu^2$ if you consider this expression this we have denoted with Galileo number that is Ga or sometimes we also call it Archimedes number Ar. So if you consider the Galileo number all parameter which are available in this number are known to us.

For example, if we have to calculate terminal settling velocity of a particle I will aware with the diameter of particle, I will aware its density μ of fluid as well as density of fluid both parameter we will know and g value also know. So we can calculate Galileo number once I know the parameters.

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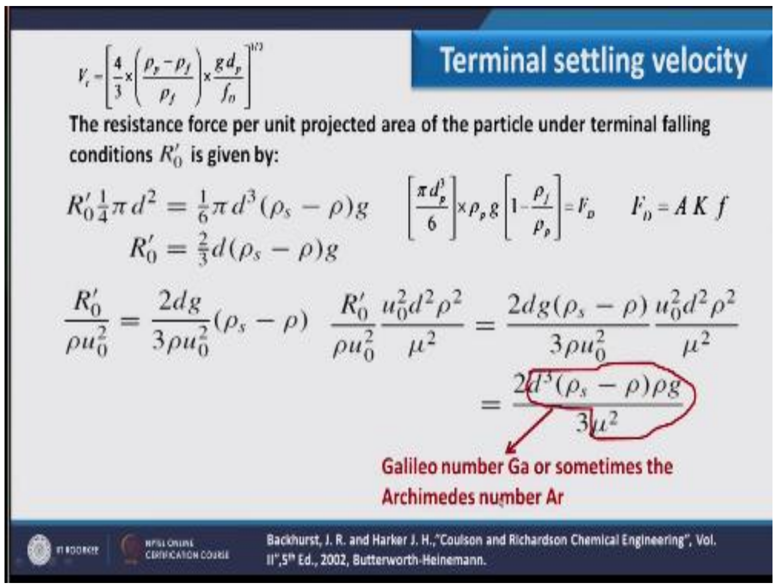

And these are usually known to us. So if you consider this particle expression that is $R_o'/\rho u^2$ into Reynolds number square this we can equate with 2/3 into Galileo number. So this expression is a function of all known parameters to us. So here we have Reynolds numbers as well as this expression as Galileo number we can find.

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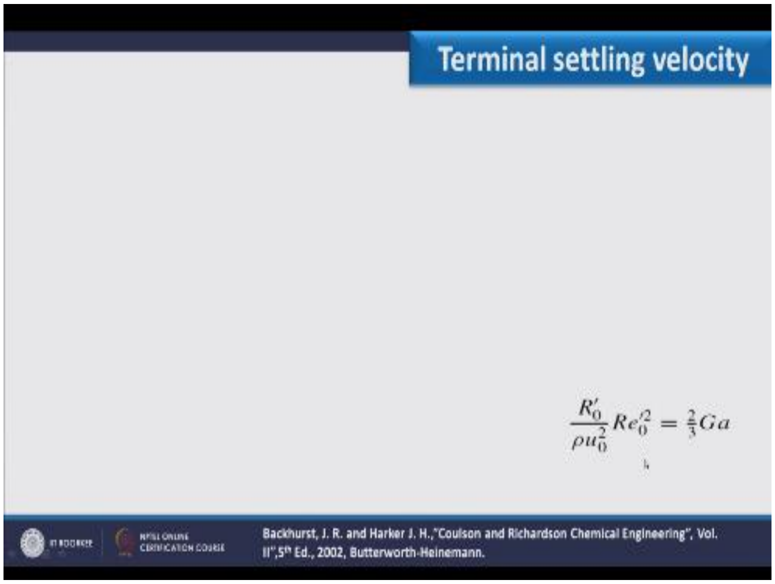

Further we have this expression, so how we can calculate terminals setting velocity using this once I know the parameter I will calculate Galileo number and 2/2 will be equal to this parameter so the value of this hole parameter can be calculated.

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Terminal settling velocity

Values of log Re' as a function of $\log\{(R'/\rho u^2)Re'^2\}$ for spherical particles

$\log\{(R'/\rho u^2)Re'^2\}$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\bar{2}$								$\bar{3}.620$	$\bar{3}.720$	$\bar{3}.819$
$\bar{1}$	$\bar{3}.919$	$\bar{2}.018$	$\bar{2}.117$	$\bar{2}.216$	$\bar{2}.315$	$\bar{2}.414$	$\bar{2}.513$	$\bar{2}.612$	$\bar{2}.711$	$\bar{2}.810$
0	$\bar{2}.908$	$\bar{1}.007$	$\bar{1}.105$	$\bar{1}.203$	$\bar{1}.301$	$\bar{1}.398$	$\bar{1}.495$	$\bar{1}.591$	$\bar{1}.686$	$\bar{1}.781$
1	$\bar{1}.874$	$\bar{1}.967$	0.008	0.148	0.236	0.324	0.410	0.495	0.577	0.659
2	0.738	0.817	0.895	0.972	1.048	1.124	1.199	1.273	1.346	1.419
3	1.491	1.562	1.632	1.702	1.771	1.839	1.907	1.974	2.040	2.106
4	2.171	2.236	2.300	2.363	2.425	2.487	2.548	2.608	2.667	2.725
5	2.783	2.841	2.899	2.956	3.013	3.070	3.127	3.183	3.239	3.295

$$\frac{R'_0}{\rho u_0^2} Re_0'^2 = \frac{2}{3} Ga$$

Backhurst, J. R. and Harker J. H.,"Coulson and Richardson Chemical Engineering", Vol. II",5th Ed., 2002, Butterworth-Heinemann.

And this table if you see it is having the data of log Re as a function of log R/ ρu^2 / Reynolds number square. So if you see this is the same parameter which we have used over here and log of this is equal to 2/3 Galileo number, so I know already the Galileo number, so I know the law I know this parameter and I can calculate log of this. So once I calculate log of this by referring this table we can calculate the value of log Reynolds number.

For example, if log of this is coming as 2.15 so you can see here we have the value of log R'/ ρu^2 x Re'^2. So here we have the value 2 and I have to find the log Reynolds number value correspond to log of this expression as 2.15. So here we have to 2 value and point 1 is available over here. So I will consider this particular value and further 2.2 that is 2 is available here and 0.2 is available here so further I will collect this value.

Now making interpolation between these two I will calculate the value of log Reynolds number at 2.15. So once I know the Reynolds number I can calculate terminal settling velocity of particle. So instead of using trial and error method we can use this method direct, directly to calculate terminal settling velocity and here Re' that is Reynolds number ' is a Re that we have previously defined.

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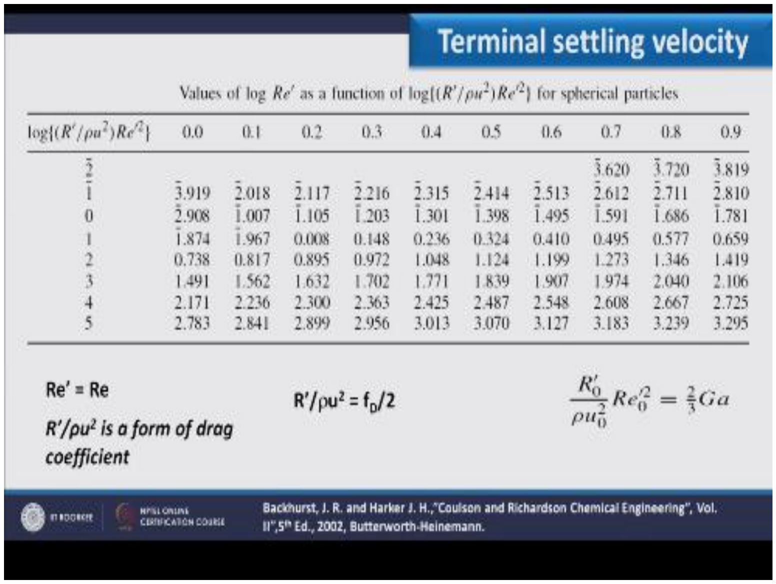

R' / ρu^2 is a form of drag coefficient and this value R'/ ρu^2 it is value is equal to $f_D/2$ so by this method we can calculate terminal settling velocity of particle directly and this is a specifically used for a spherical particles.

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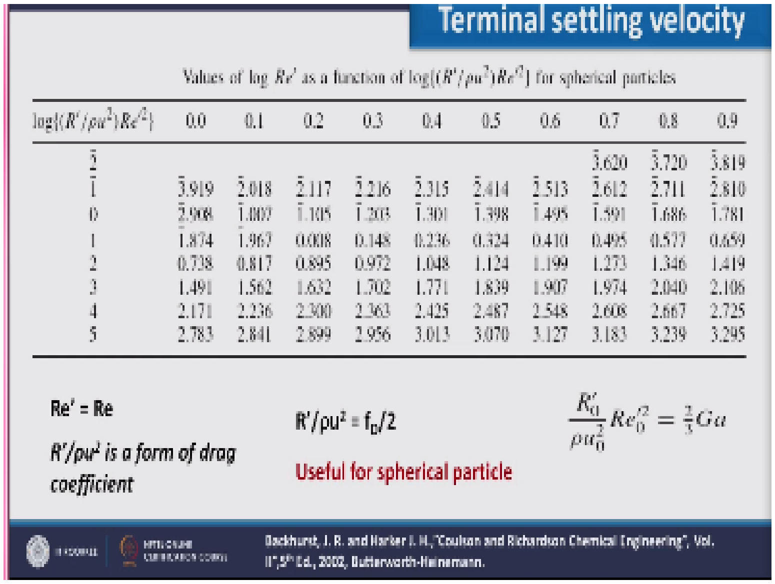

Values of log Re' as a function of $\log\{(R'/\rho u^2)Re'^2\}$ for spherical particles

$\log\{(R'/\rho u^2)Re'^2\}$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
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So if we have to calculate the terminal settling velocity of spherical particle I can calculate it directly without knowing that in which region particle is falling. So instead of using pervious laws that is a Stoke's law, Newton's law we can use this method directly to calculate the terminal settling velocity, but it is only applicable for spherical particle. If we have to calculate settling velocity of non spherical particle we have to follow the trail and error method there is no other way of doing that.

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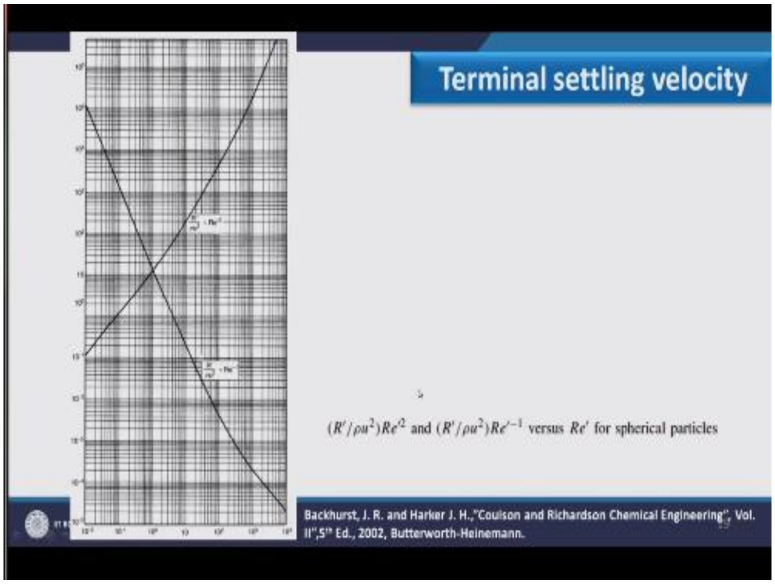

So this graph is again showing the same values here this is basically $R' / \rho u^2$ Re on this Y axis and X axis is basically Reynolds number ‘. So using this graph also we can calculate the terminal settling velocity here another graph is also plotted but we have to use this graph for spherical particles. Now as for us derivation of terminal settling velocity is concerned we have taken some of the assumptions like all 6 assumptions if you remember which we have discussed in part one.

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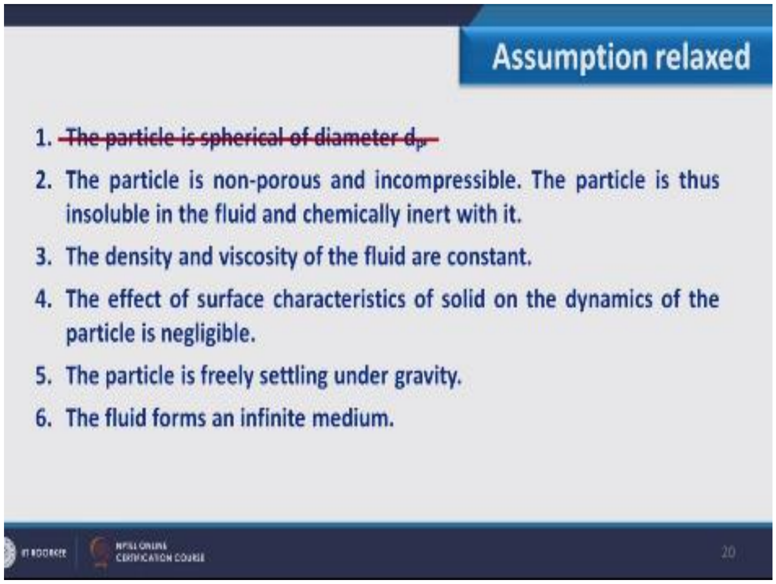

Now at this point we are going to relax some of the assumptions. So first is assumption we will relax is particle is spherical. Now we calculate, now we consider the expression when particle is non spherical in shape. So considering days we will study the effect of particle shape on terminal settling velocity.

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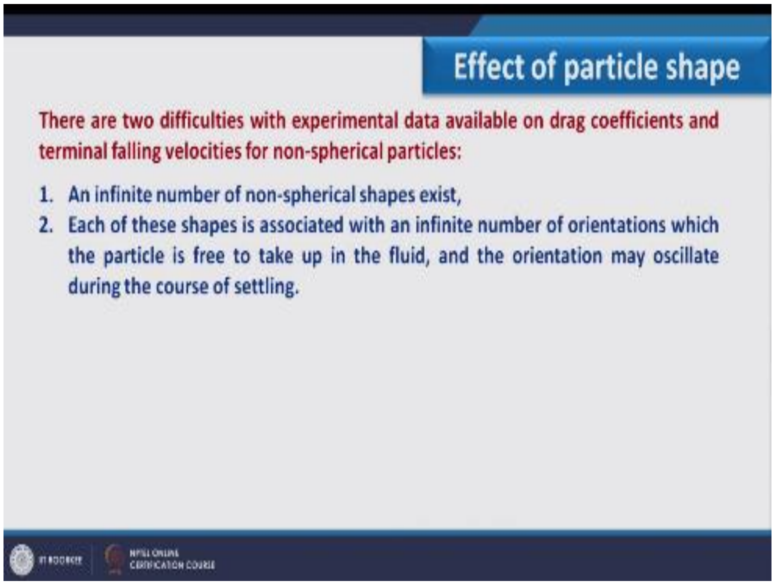

So there are two difficulties with experimental data available on drag coefficients and terminal falling velocity for non spherical particle. The first is an infinite number of non spherical shapes exist, and second each of these shapes is associated with infinite number of orientation which the particle is free to take up in a fluid and the orientation may oscillate during the course of settling. So for example when we consider irregular particle sometimes it will fall like this, sometime it may fall like this, sometime it may fall like this, and some time it may fall vertically also.

So orientation is continuously changing when the particle is falling in a fluid so that is another difficulty.

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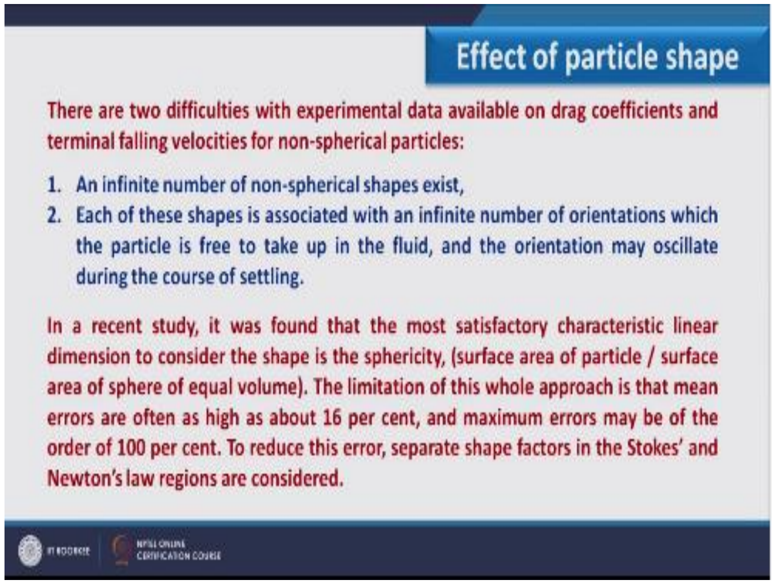

So in recent study it was found that most satisfactory characteristic linear dimension to consider the shape is sphericity. If you remember this sphericity we have discussed in the first week of this course which is basically the surface area of particle divided by surface area of a sphere of equal volume. And the limitation of this whole approach is that, if we calculate the terminal settling velocity using given equation, and if we carry out experimentation for non spherical particle.

So error used to come about 16% and maximum error may be of the order of 100%. So to reduce this error separate shape factors in Stokes as well as Newton's law region or considered. So you can see the effect of shape will be considered by considering another factor in Stokes' as well as Newton's law.

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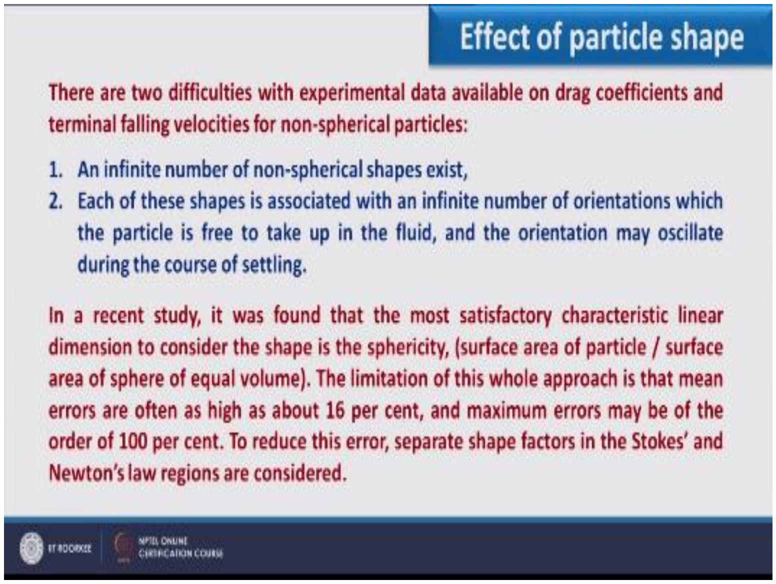

Now another problem in such condition is that.

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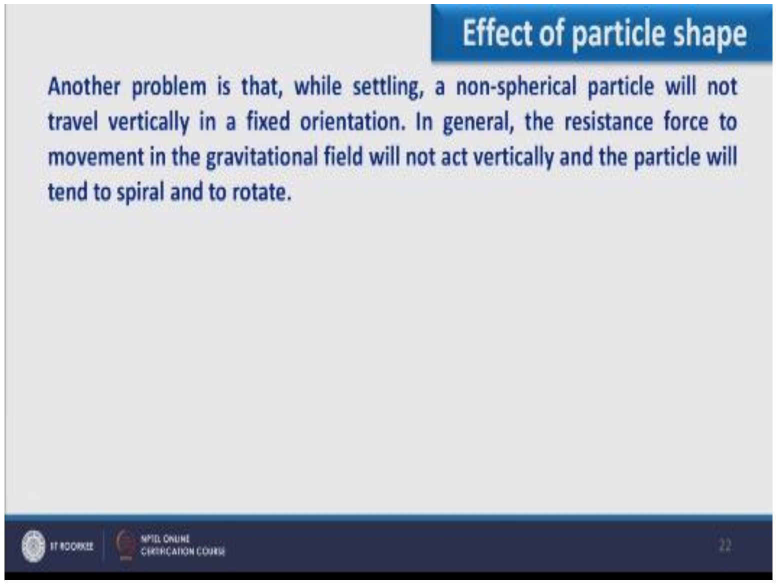

While settling non spherical particle will not travel vertically in fixed orientation. As we have discussed that continuously orientation is keep on changing when particle is falling in a liquid. In general the resistance force to movement in gravitational field will not act vertically and particle will tend to spiral and to rotate.

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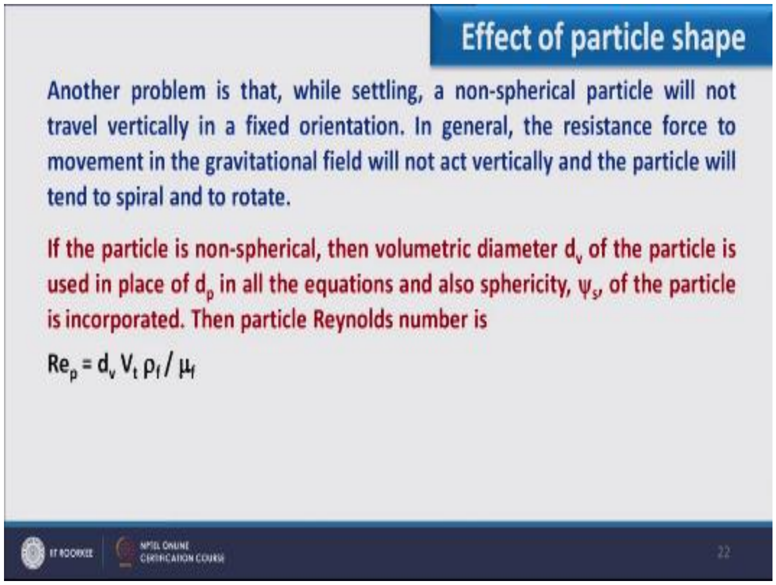

Therefore, if particle is non spherical, then volumetric diameter of particle is consider in place of d_p what is volumetric dia, I guess you remember this it is the diameter of a spherical particle of same volume as the particle.

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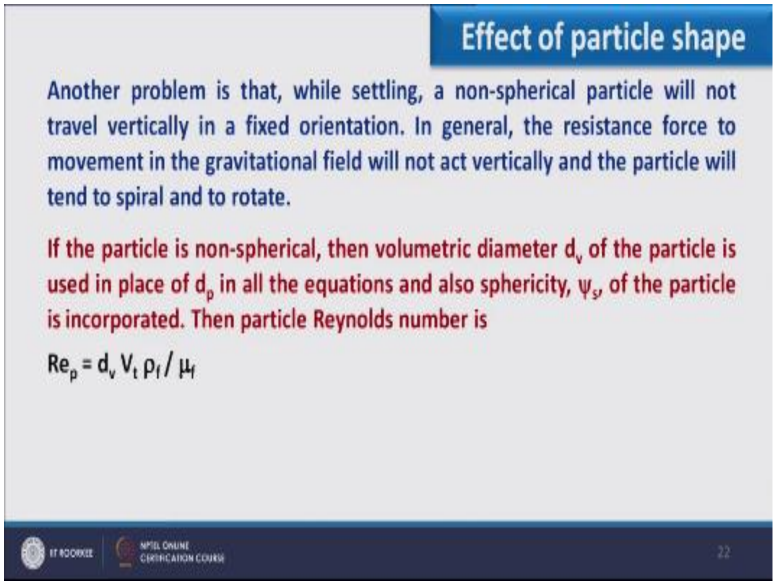

Therefore, to consider the non spherical shape of particle we consider d_v instead of d_p in all equation.

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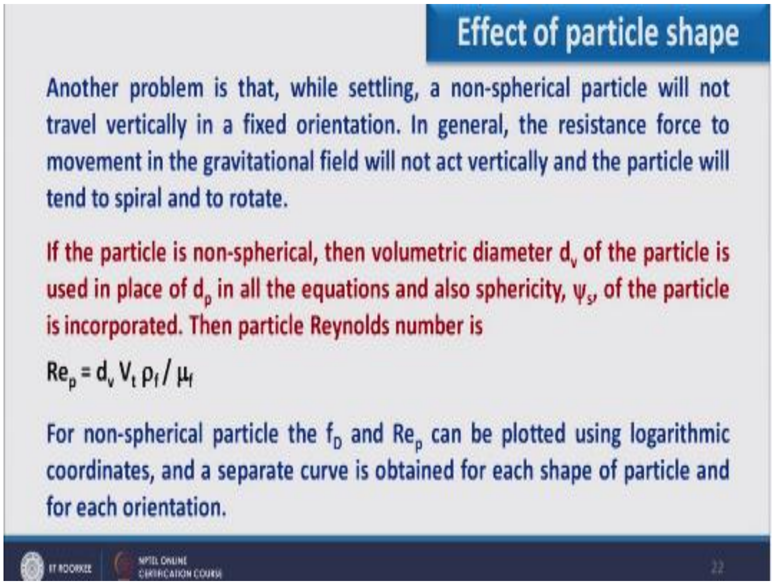

And along with this we will also consider 5s that is a sphericity of the particle to incorporate different shapes. Therefore, Reynolds number for irregular particle or non spherical particle is defined as $d_v V_t \rho f/\mu f$ previously we have taken this as $d_{p.}$ Now we will take diameter as $d_{v.}$ So for non spherical particle the f_D and Reynolds number Re_p can be plotted using log graph which we have seen previously also.

So same axis we will use for that is log, log axis to plot f_D and Reynolds number for non spherical particle and a separate curve is obtain for each shape of the particle and for each orientation.

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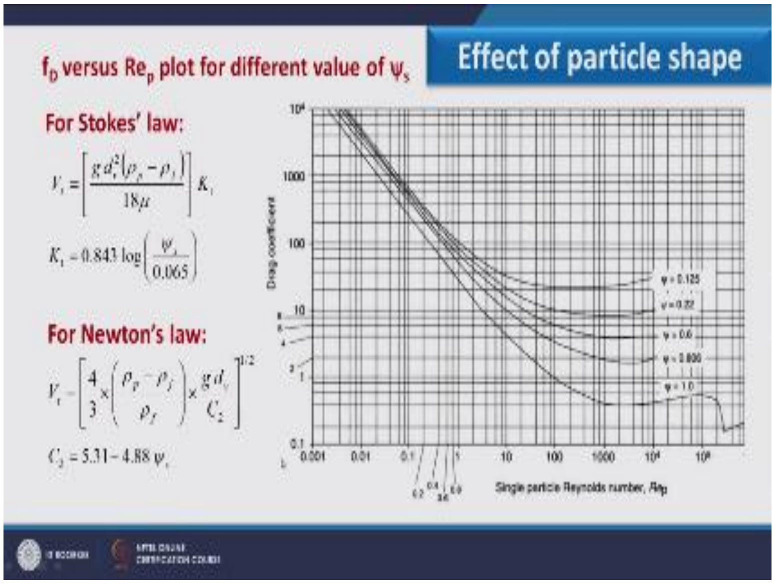

For example, if you see this graph, what this graph shows, you see the bottom most line we have discussed previously also and this is for spherical particle where ψ=1. So ψ is keep on changing when we move upward and we have different lines to calculate f_D based on Reynolds number and all these ψ will depend on different shape. So you see here we have f_D verses Reynolds number plot for different value of ψ_s.

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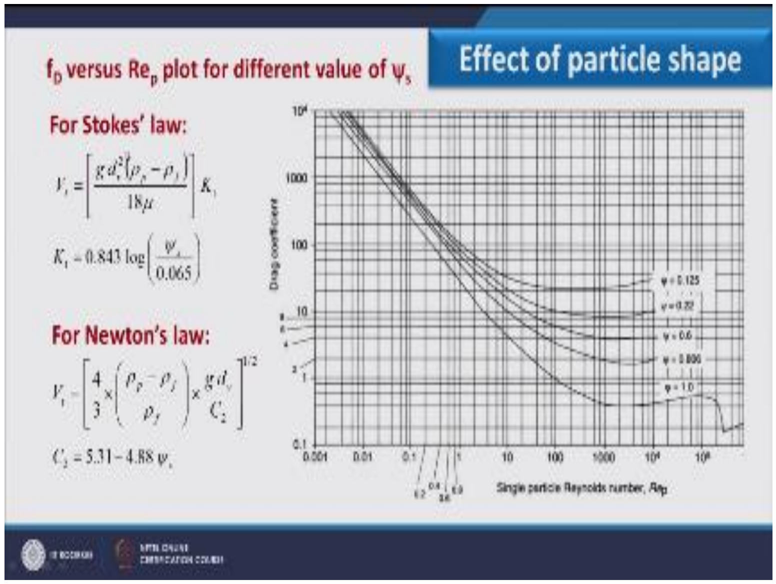

For Stoke’s law here we have the expression of terminal settling velocity up to here we have already seen in the previous part of this lecture. Now this expression is multiplied by another parameter that is K_1 where K_1 is defined as 0.843 log ψ_s/0.065. So ψ_s is basically considering the shape of particle and here we have taken volume at will die of particle instead of D_p so this is the revised expression for Stoke’s law when the particle is irregular and it is falling in laminar zone.

Similarly we can change the parameter for Newton’s law also here you see this is 4/3 x $\rho_p - \rho_f / \rho_f$ x Gdv /$c_2^{1/2}$ so this c_2 basically the correction factor when we are studying the effect of shape on terminal settling velocity. So this c_2 is defined as 5.31 – 4.88 ψ_s. So considering K_1 in Stoke’s law and c_2 in Newton’s law we can study the effect of particle shape on settling velocity. And again here we have consider all this assumptions in this up to here we have already relax the first assumption.

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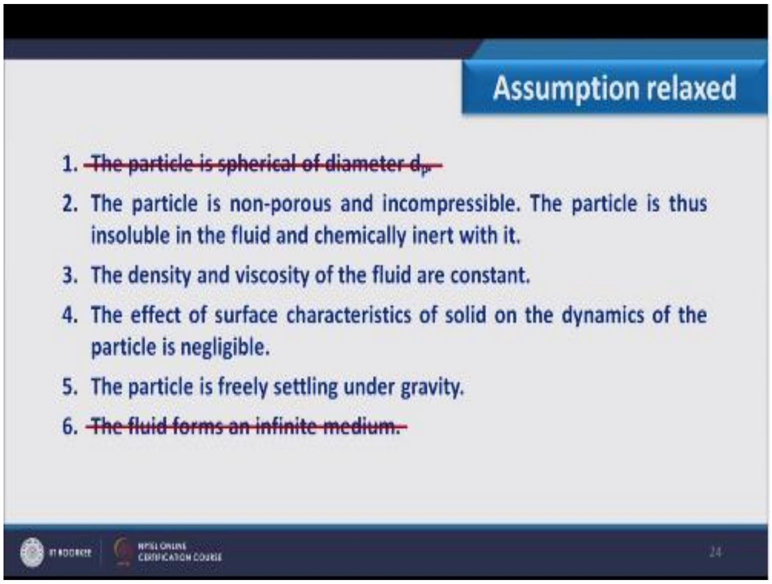

Now we are going to relax sixth assumption that is the fluid forms and infinite medium. So previously we have assumed that fluid is falling and infinite medium, it means it is falling in a cylinder filled with the liquid. So that cylinder diameter is significantly large in comparison to the diameter of particle. And therefore, it is therefore, the movement of particle will not be affected by the wall of the cylinder present near to us. Now we are relaxing this assumption to study the effect of wall on terminal settling velocity.

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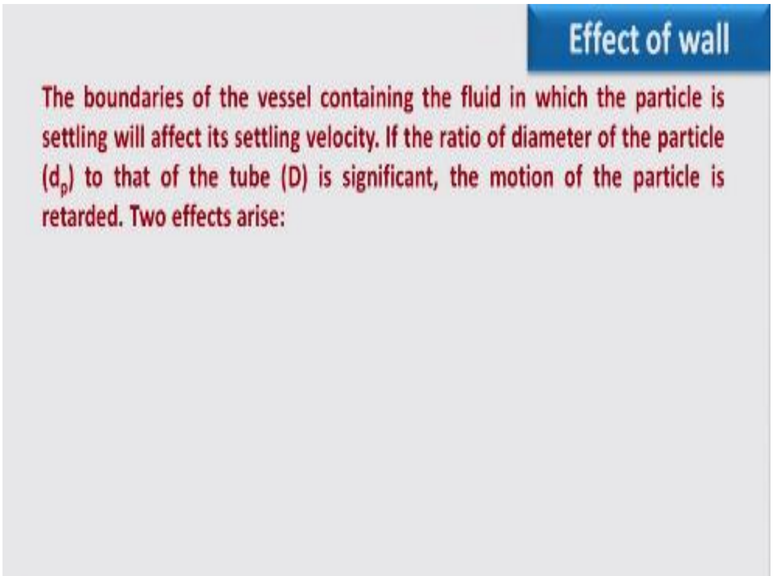

So the boundaries of the vessel containing the fluid in which particle is settling will affect its settling velocity. If the ratio of diameter of the particle d_p to that of the tube D were D is the inner diameter of cylinder or tube is significant. If the ratio of diameter of particle d_p to that of D is significant the motion of particle is retarded. Now why it is retarded, it has two different effects.

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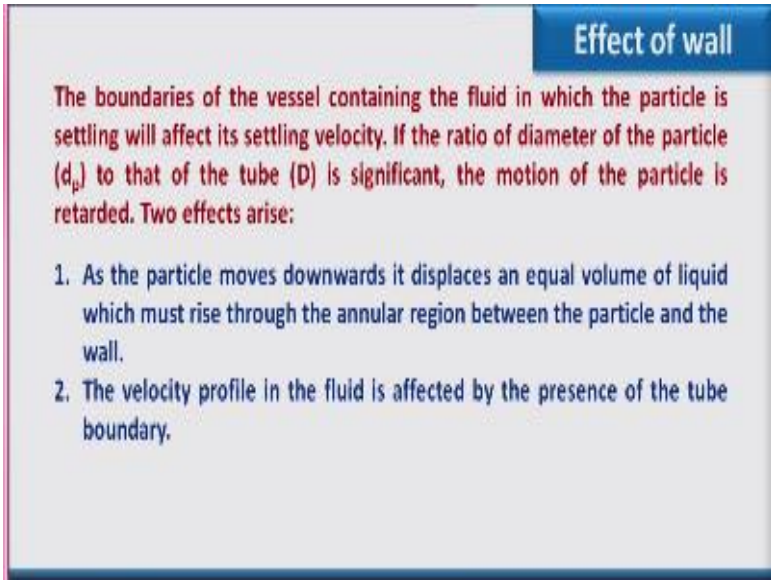

First as particle moves downward it displaces an equal amount of or equal volume of liquid which must rise through the annular region between particle and the wall. And second is the velocity profile in the fluid is affected by the presence of tube boundary. So these are the two affects well particle is falling in smaller diameter cylinder or tube.

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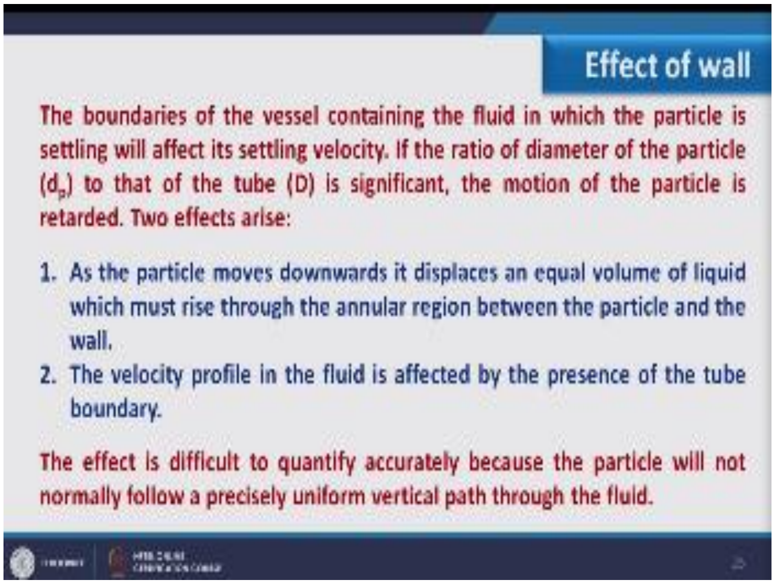

So the effect is difficult to quantify accurately because the particle will not normally follow a precisely uniform vertical path through the fluid.

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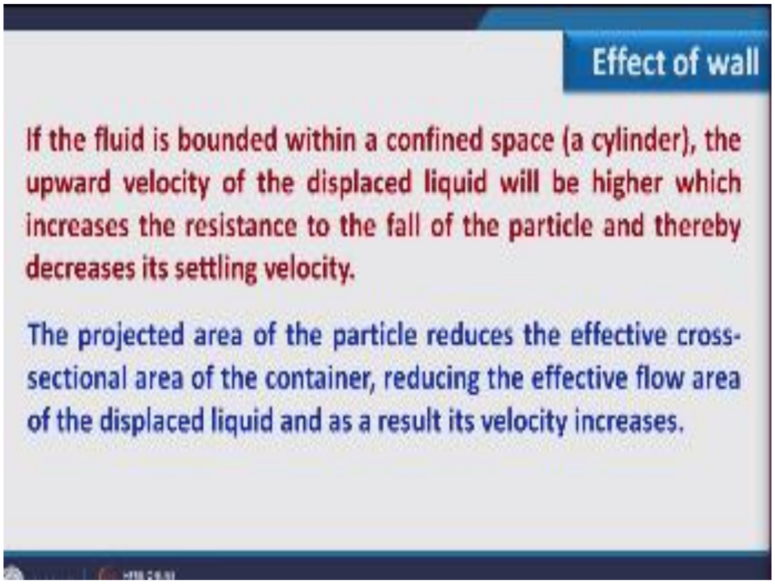

Therefore, if the fluid is bounded with the confined space that is a cylinder. The upward velocity of displaced liquid will be higher which increases the resistance to fall the particle and therefore, decrease it settling velocity. Now why it happens, why the, when volume goes, when the particle displaces the volume of liquid of its own volume, then it exert the resistance and why the settling velocity is decreased, because the projected area of particle reduces the effective cross sectional area of the container.

For example, initially I am having the container of this diameter and if particle is moving like this. So when particle is moving like this it displaces the liquid and if you see this is the diameter of cylinder and this is the diameter of particle, so effective area of the cylinder through which liquid will move will reduce. So it will reduce the effective flow area of displaced liquid and as a result velocity increases, so once velocity will increase.

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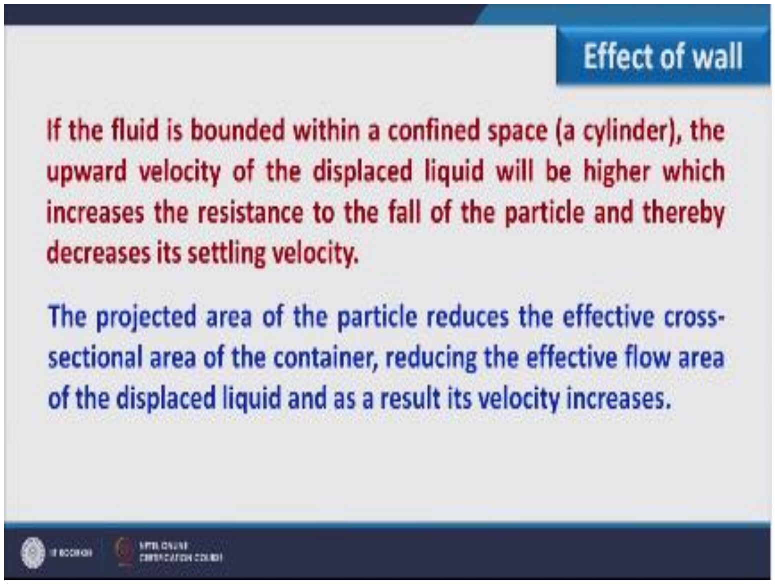

The resistance on the particle will increase and that is why the settling velocity of particle will decrease.

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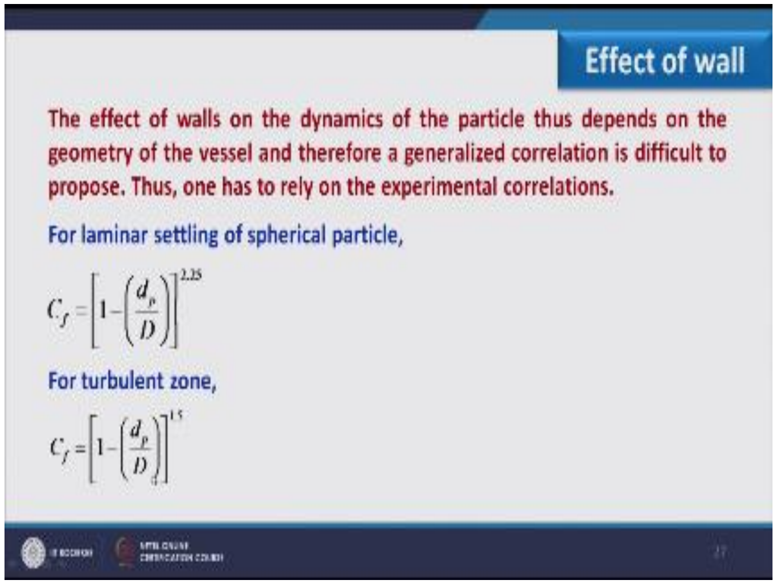

So the effect of walls on the dynamics of particle thus depends on the geometry of the vessel and therefore a generalized correlation is difficult to propose. And experimentally we can rely on the correlation for laminar settling of spherical particle, the correlation is C_f which is equal to $[1-(d_P/D)]^{2.25}$ so this is the correlation to count wall effect when the particle is falling in laminar zone. And similarly this expression we are having to account the effect of wall on particle movement when the particle is falling in turbulent zone. So once I know the value of C_f.

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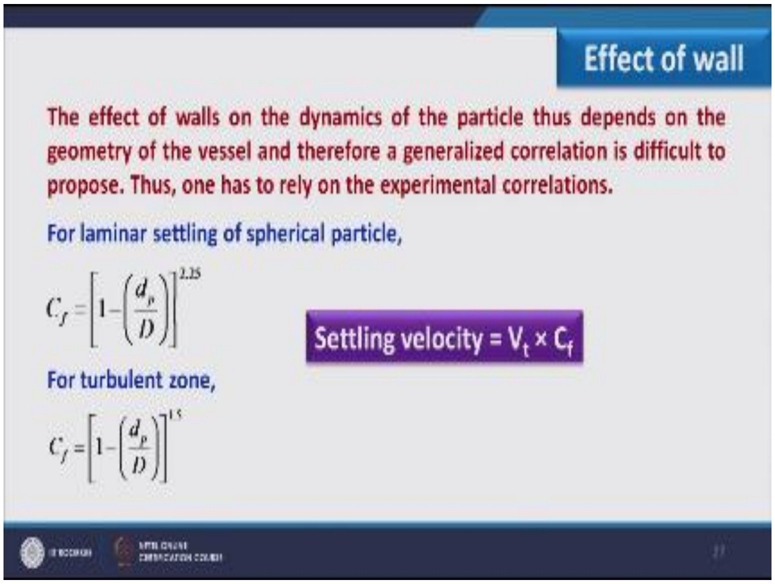

Then I can calculate the revised settling velocity of particle which is equal to $V_t x C_f$. So V_t you remember this is the settling velocity which we have derived in first part of this lecture and C_f we have just seen. So multiplication of these two will speak about the actual settling velocity of the particle, considering the effect of wall.

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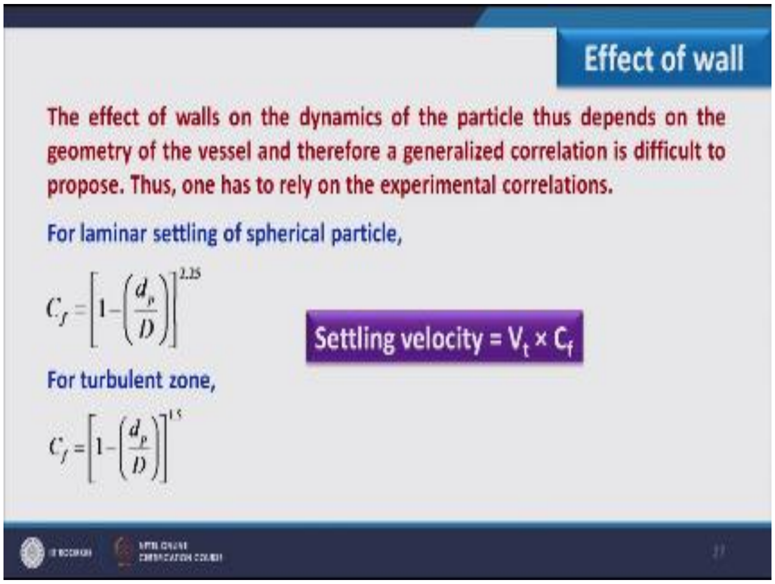

So and when we have consider irregular particle we can replace d_P with d_V that is volumetric dia. So in this way we can study the effect of wall. Further we have six assumptions in which two we have already relaxed.

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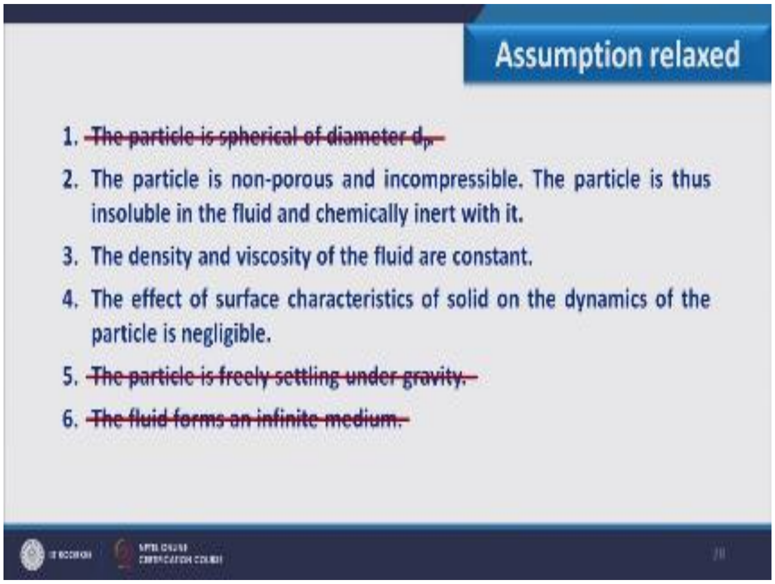

Now here we going to relax another assumption that is particle is freely settling under gravity. What is the meaning of this that, previously we have assumed that no other particle is available to put hindrance in the path of single particle. So single particle movement or dynamics we have discussed. Now here we are considering the number of particles because in usual condition we deal with the mixture of particles or number of particles not a single particle.

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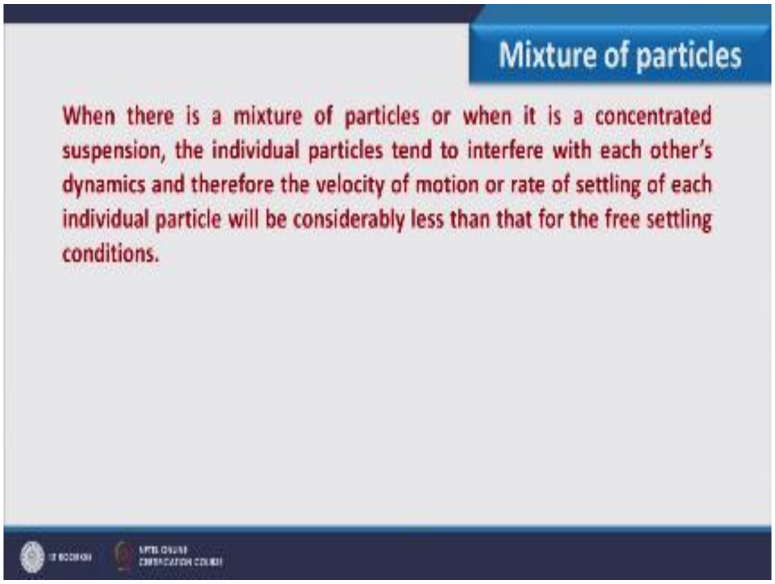

So when there is a mixture of particles or when it is concentrated suspension, the individual particles tends to interfere with each other dynamic, and therefore the velocity of motion or rate of settling of each individual particle will be considerably less than that for the free settling conditions. So that is the obvious situation then when, that when we are dealing with the number of particle the settling velocity of each particle will be decreased.

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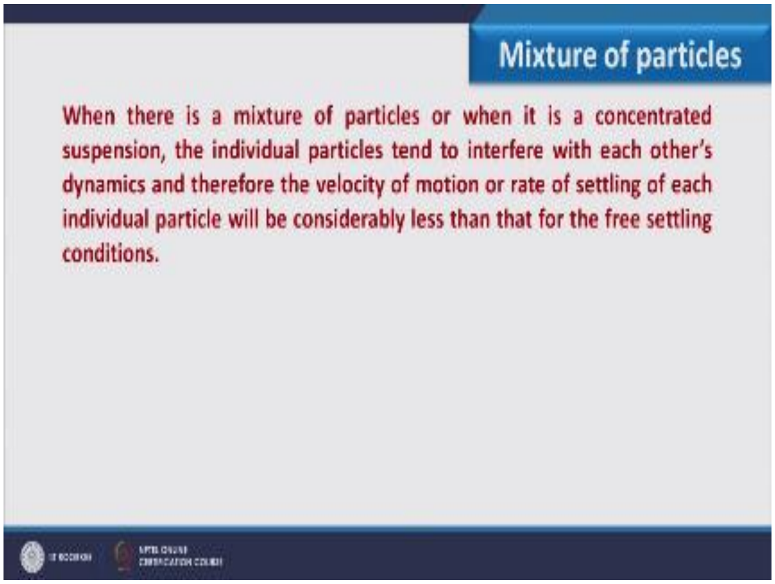

In comparison to when they are falling freely, so Stoke’s transitional and Newton’s settling expression.

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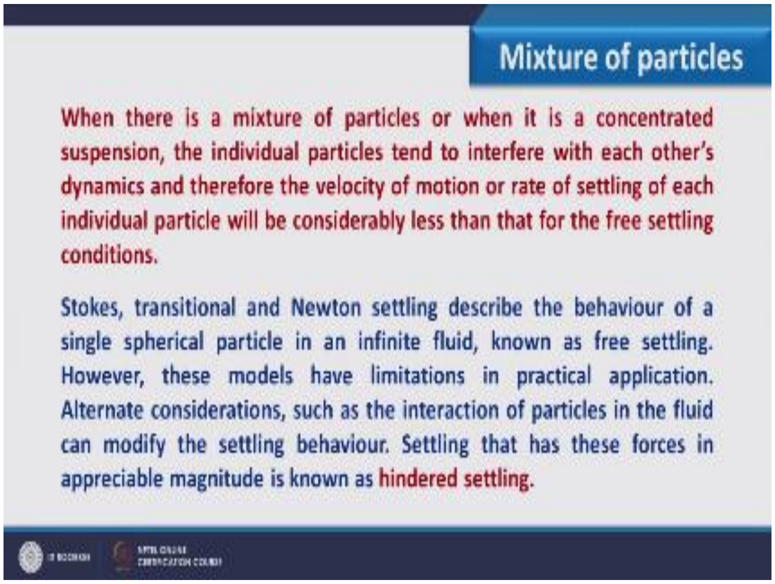

Which we have derived in part 1 of this lecture. Describe the behavior of single spherical particle in an infinite fluid and that is why we call it free settling. However, these models have limitations in practical applications, alternate consideration such as interaction of particle in a fluid can modify the settling behavior. Therefore, settling that has these forces in appreciable magnitude we call it as hindered settling.

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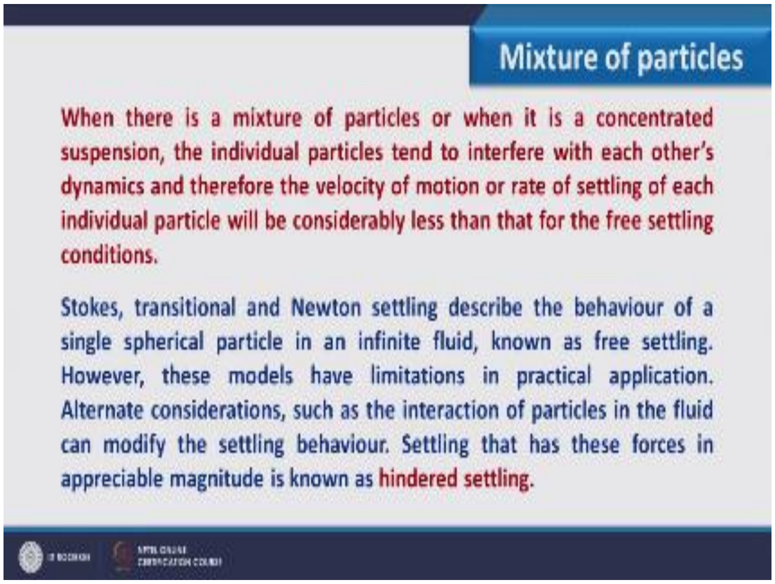

So when the particle is falling with the presence of other particle, we call it as hindered settling instead of free settling.

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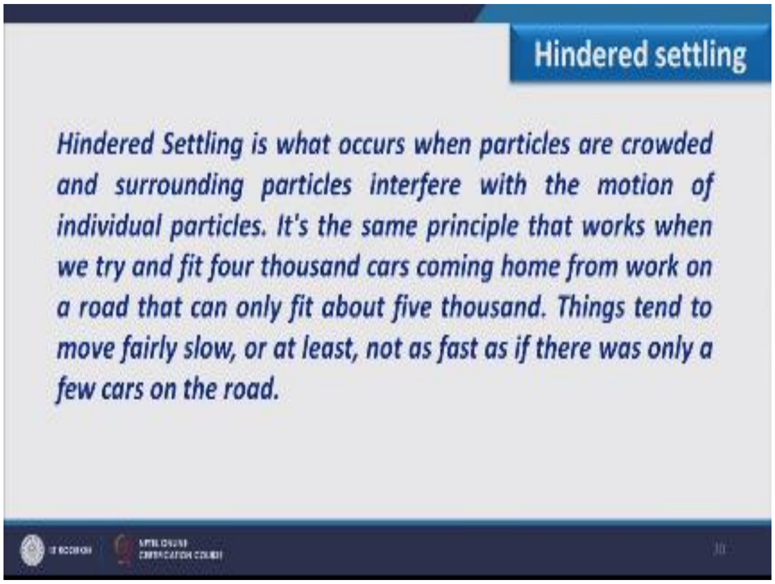

So in this slide we have shown one interesting example of hindered settling, and hindered settling is what occurs.

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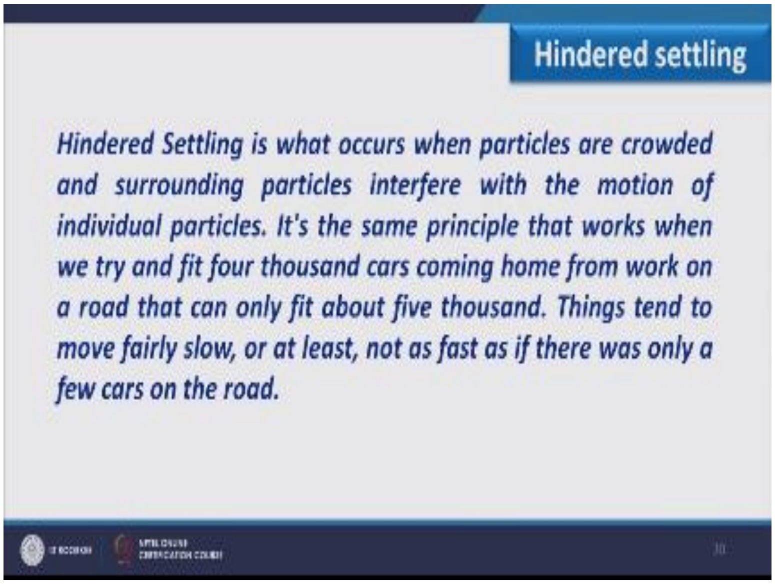

When particles are crowded and surrounded particles interfere with the motion of individual particle. So here up till now we have discussed in terms of number of particles. Now the same is applicable when we are discussing the movement of cars on road.

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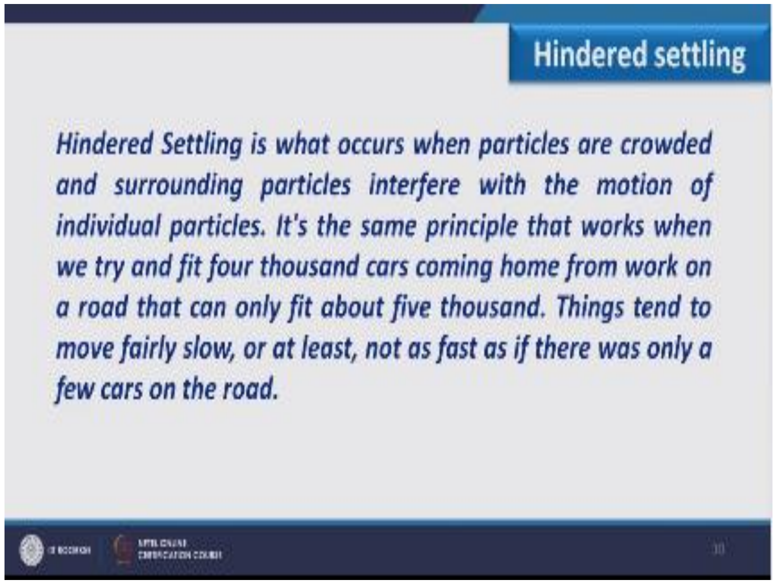

Now what happens when we usually come home from the offices almost at same time all offices are closed, so we used to have jam on the road. So the road has the capacity to accumulate 5000 cars and at that time we are having 4000 cars on road. So instead of free movement we used to move very slowly on the road and that is the perfect example of hindered settling which we usually see in our daily life.

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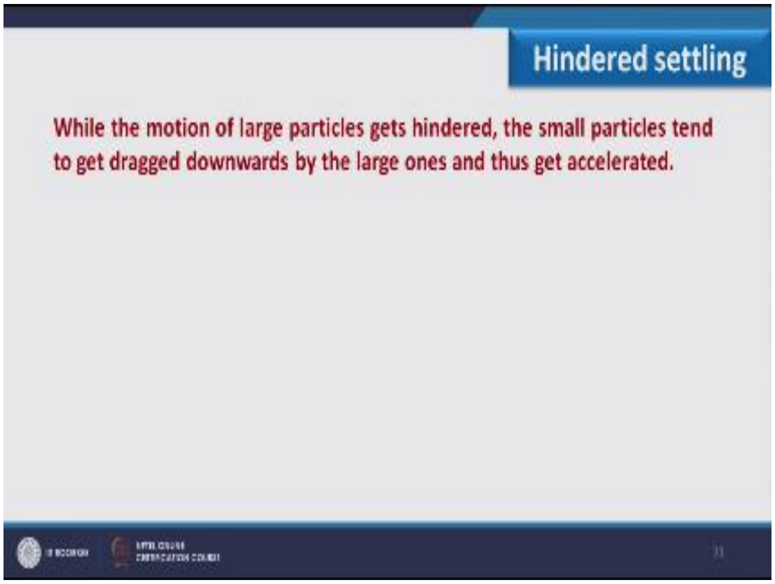

So the hindered settling is while the motion of large particles gets hindered the small particle tend to get dragged down by large one and thus gets accelerated. What is meaning of this statement that when a number of particles are falling in a cylinder then what happens, some small particle if I am considering it is settling velocity would be significantly less in comparison to large particle, as terminal settling velocity expression if you remember it is directly proportional to the diameter. So small particle will move very slow in comparison to large particle.

Now during hindered settling what happens when I am having the particle and when very large particle is falling above this then the small particle will not fall with it is own velocity, it will fall with the velocity of large particle and therefore the small particle is dragged down and gets accelerated .

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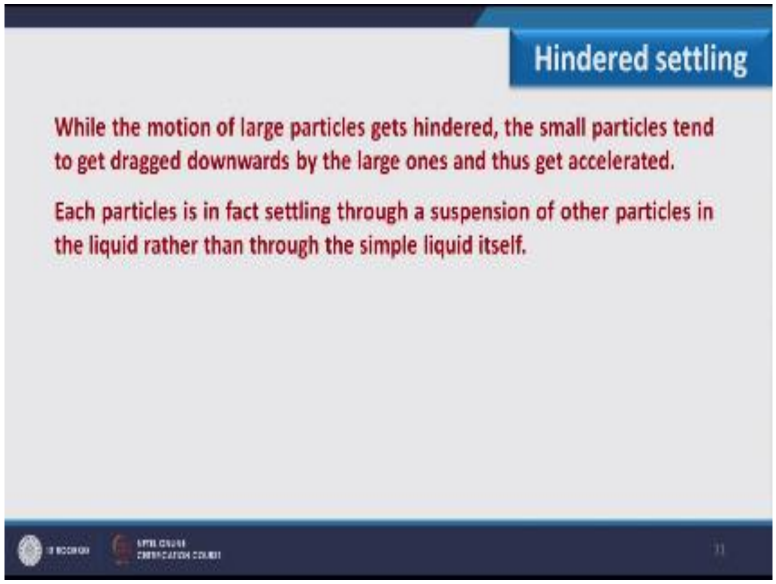

So this type of situation also occur while hindered settling each particle is in fact settling through a suspension of other particles in the fluid rather than through a simple liquid itself. So the particle is following through the suspension not in a clear liquid.

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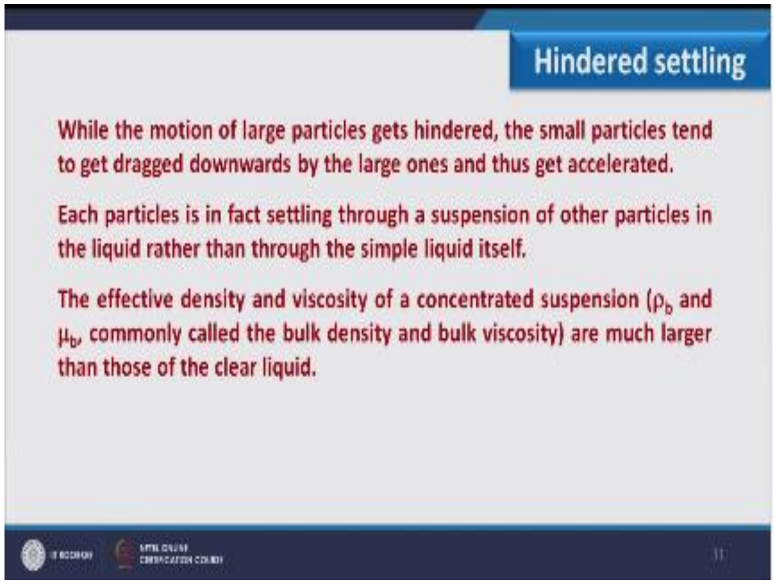

And in that case the effective density and viscosity of concentrated suspension that is ρ_b and μ_b which we commonly called as bulk density or bulk viscosity are much larger than those of the clear liquid. So when we are deriving the expression of hindered settling we can consider ρ_b and μ_b instead of ρ_f and μ_f.

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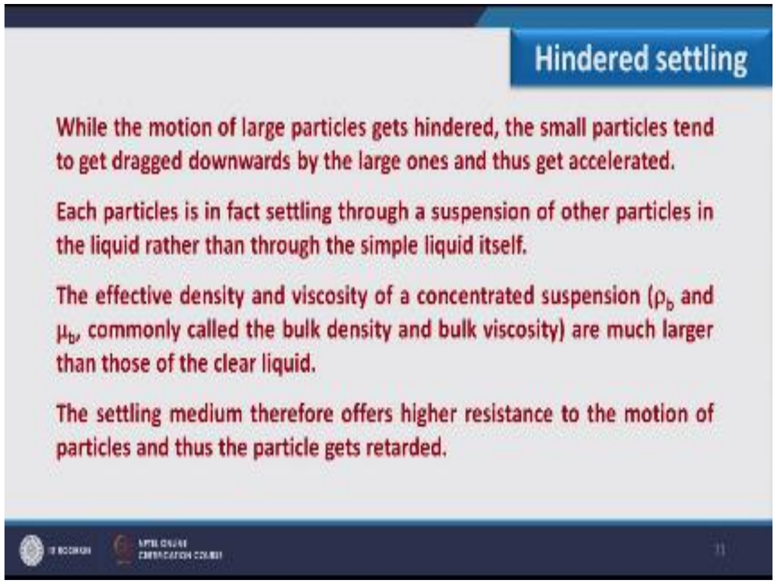

So settling medium therefore, offers higher resistance to the motion of particle and thus the particle gets retarded.

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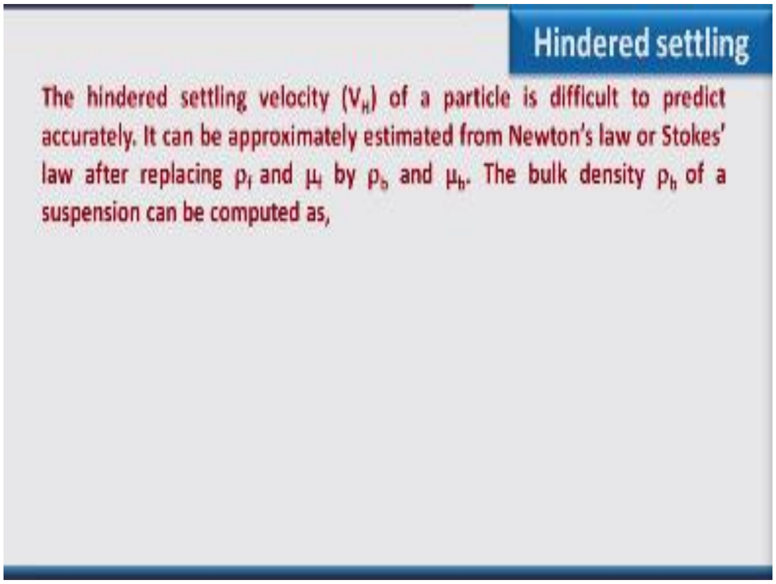

So here the hindered settling velocity that we have denoted with V_h of a particle is difficult to predict some approximation we can do by replacing ρ_f and μ_f by ρ_b and μ_b So first of all we have to defined what is ρ_b and what is $\mu_{b,}$

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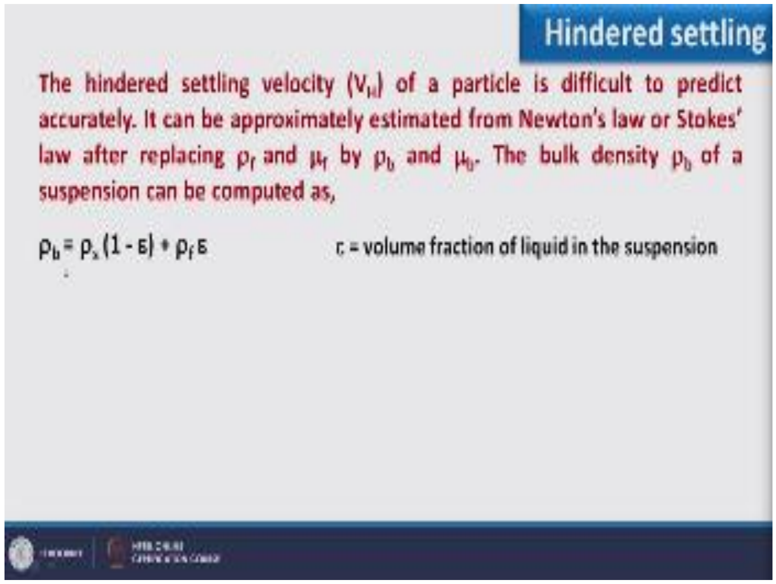

ρ_b is basically the bulk density which we can define as ρ_s (1- ϵ) + ρ_f ϵ. So ϵ is the volume fraction of liquid in the suspension, so 1 - ϵ is basically volume fraction of solid in the suspension. So that is ρ_s x volume fraction of solid + ρ_f x volume fraction of liquid. So in this way we can define ρ_b. And we can define bulk viscosity o that is μ_b using this expression, and if you remember the expression of terminal settling velocity this is the same expression, but we have written it for V_h that is hindered settling why because here ρ_f and μ_f is replaced by ρ_b and μ_b.

And here when we write the expression of ρ_b and μ_b over here further we can replace the expression in terms of ρ_f and μ_f and one additional term will appear. This additional term is basically represented as f_s. So hindered settling velocity would be terminal settling velocity in to f_s, f_s is the settling factor which is the ratio of hindered settling velocity to terminal or free settling velocity.

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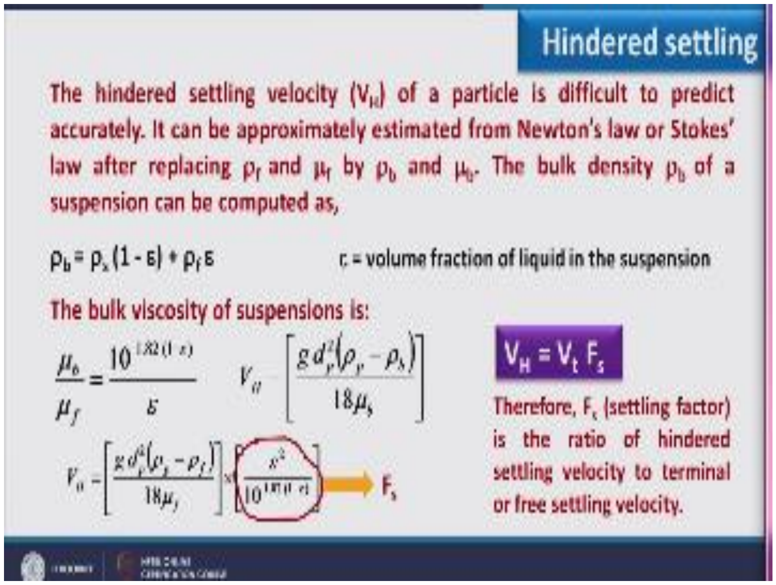

So in this way we can calculate hindered settling velocity.

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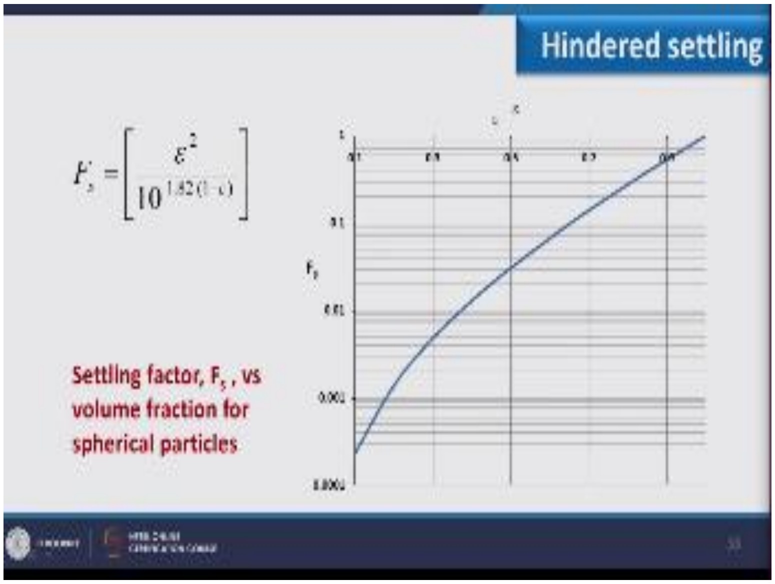

If you see this is a plot of f_s versus ϵ, ϵ will increase the f_s value will increase and that is very obvious like when we have increment in ϵ that is when we have more volume fraction of liquid we will have less volume fraction of solid. It means solid is available in less quantity. Therefore, the particle will fall freely. So when we increase ϵ we will increase f_s and f_s is multiplied with the V_t and we can have the expression, we can have the value of V_h.

Therefore, ϵ increment will give f_s increment and then terminal settling velocity or hindered settling velocity basically will increase. So that is all about this now here we have the summary of this lecture.

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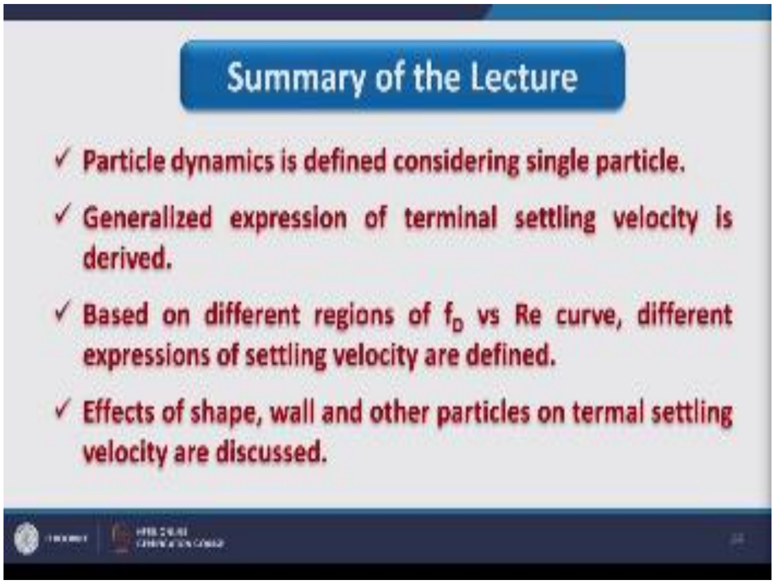

In this lecture the particle dynamics is defined considering single particle, generalized expression of terminal settling velocity is derived. Now based on different legends of f_d vs Reynolds number plot different expressions of settling velocities are defined, effects of shape wall and other particles on terminal settling velocity are discussed. So that is all about lecture 1.

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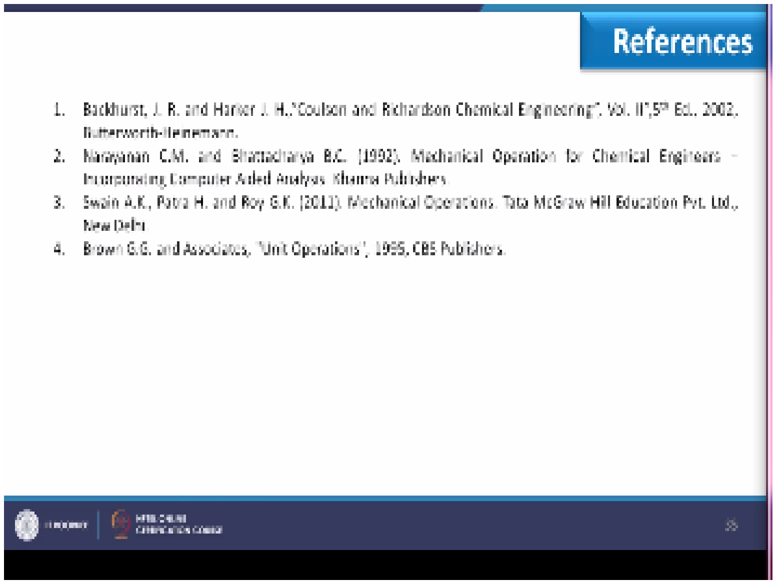

And here we have the reference, thank you.

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