INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
NPTEL
NPTEL ONLINE CERTIFICATION COURSE

Mechanical Operations
Lecture-16
Particle dynamics-1

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Welcome to the fourth week of mechanical operations course. And today we are starting lecture 1 of fourth week and which is on particle dynamics. This lecture 1, I will complete in two parts, in part 1, I will speak on what is particle dynamics and what is terminal settling velocity and its derivation.

In second part we will cover the effect of wall and effect of particle shape on terminal settling velocity and further we will discuss the hindered settling. So let us start the part 1 of lecture 1 that is the definition of particle dynamics and further topics related to this.

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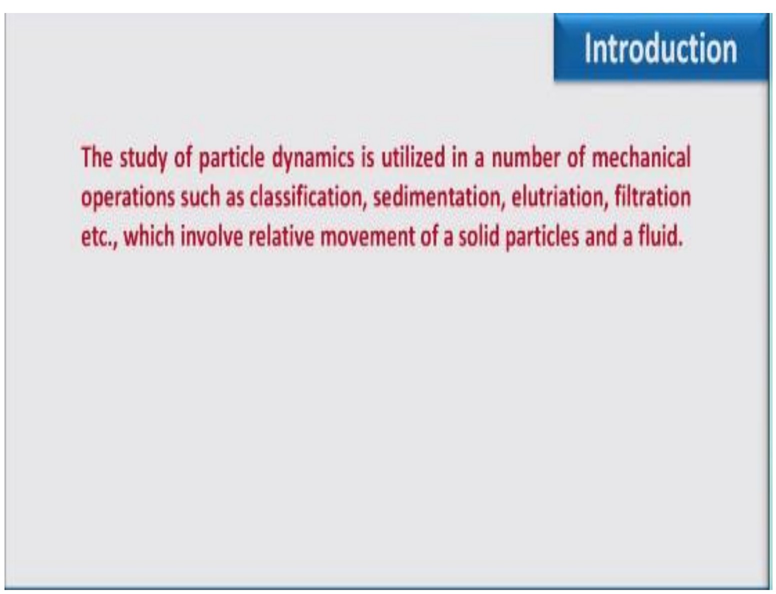

Now the study of particle dynamics is utilized in a number of mechanical operations such as classification, sedimentation, elutriation, filtration etcetera which involve relative movement of solid particle and a fluid. So wherever solid particle as well as fluid and their relative movement is involved that kind of study comes under particle dynamics.

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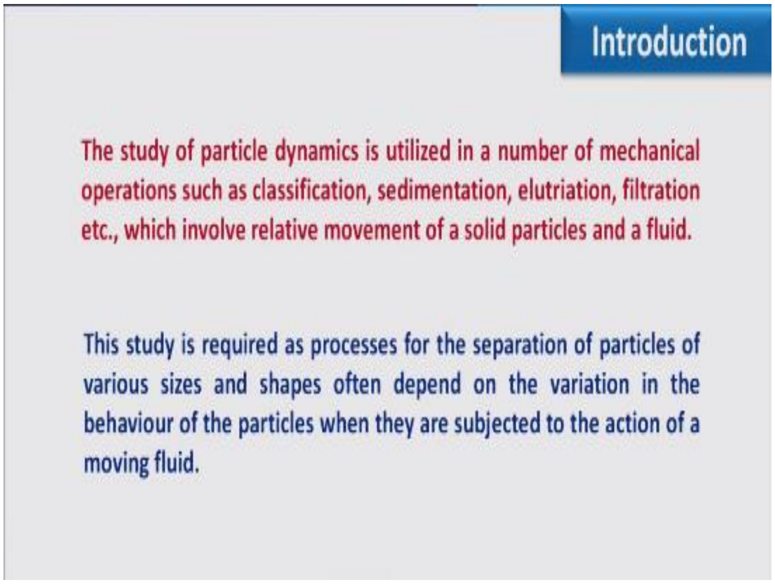

So this study is required as processes for the separation of particles of various sizes and shapes often depend on the variation in behavior of particle when they are subjected to the action of moving fluid. So basically in particle dynamics, in using this study we will find out the distribution or the separation of particles of different sizes and shape, but main point is those particles should be involved in fluids.

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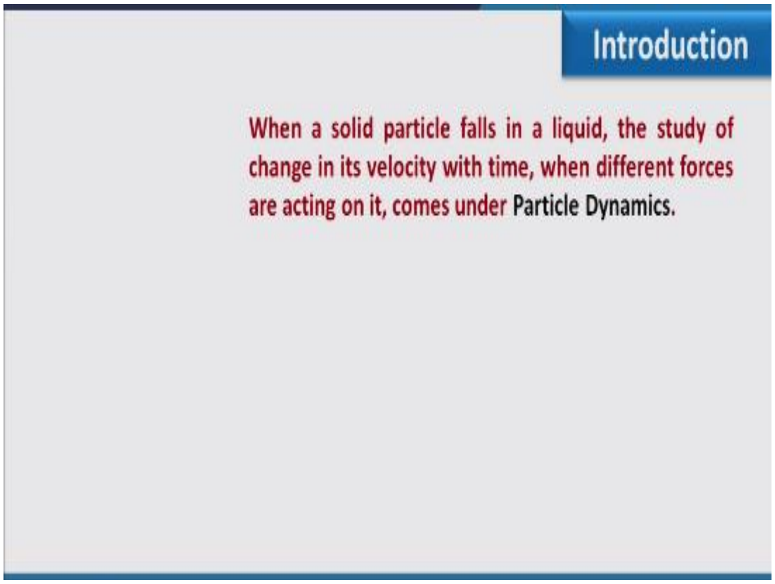

So when a solid particle falls in a liquid, the study of change in its velocity with time when different forces are acting on it comes under particle dynamics. So you can understand particle means solid body and dynamics means its movement in fluid and when we consider the movement of fluid we consider the velocity and its changes with time. So all this study comes under particle dynamics.

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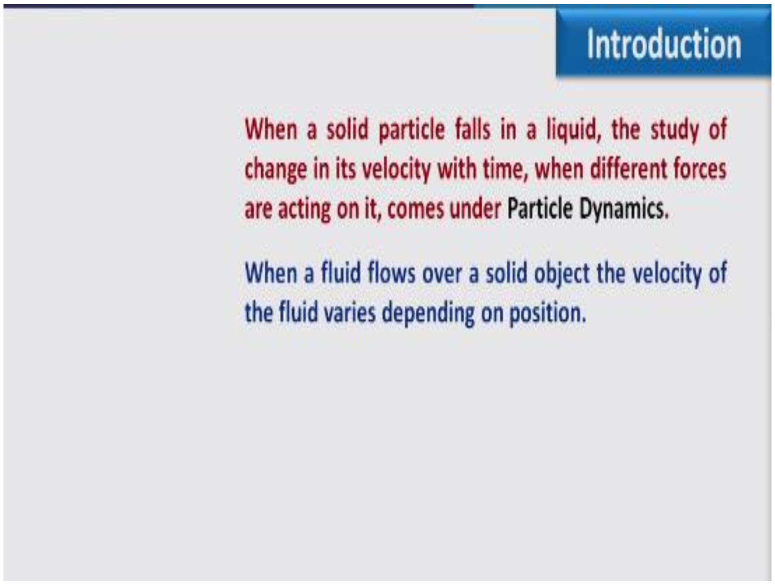

When a fluid flows over the solid object the velocity of fluid varies depending on the position. So what we can say that the velocity over the particle is not uniform, it will depend on from what side fluid is coming and what are the different sections, different surfaces, which are in direct contact of fluid or in not direct contact with fluid.

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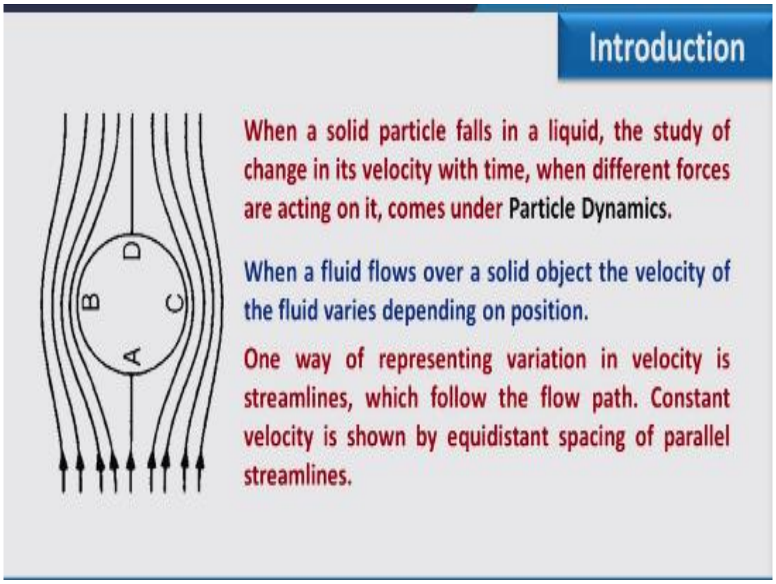

For example, if this is the solid object we are having and fluid is moving around this, obviously fluid cannot move through this, fluid will move around this. So when we consider the velocity it will change at different position of the particle. So one way of representing variation in velocity is streamlines.

I guess you already know about the streamline in fluid dynamics and that we will use to represent the variation of velocity which follow the flow path. So constant velocity is shown by equidistant spacing of parallel streamlines. So when streamline which are moving parallel to each other and when the distance between these lines are constant then we can say that velocity is constant.

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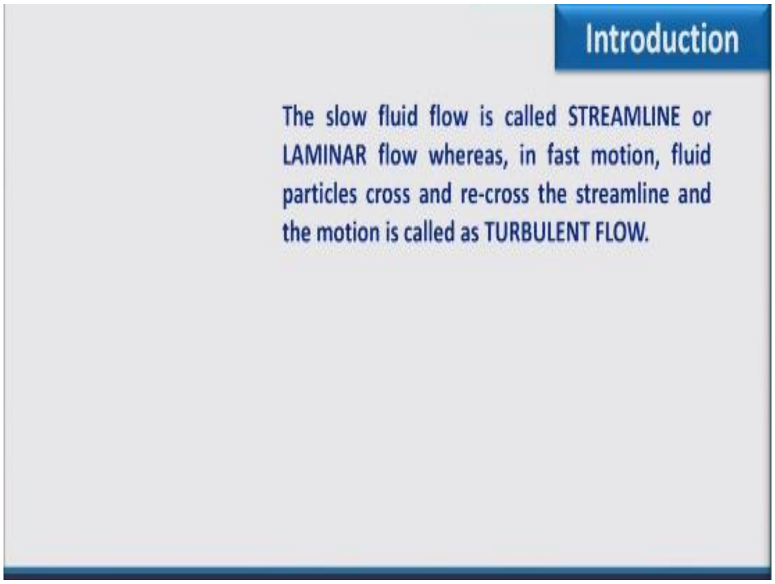

So the slow fluid flow is called the streamline or laminar flow. You understand what is the streamline movement in laminar flow that they move in a suggested pattern or in a uniform pattern. So when they move in a uniform pattern they do not cross each other then we say that this type of movement as laminar flow.

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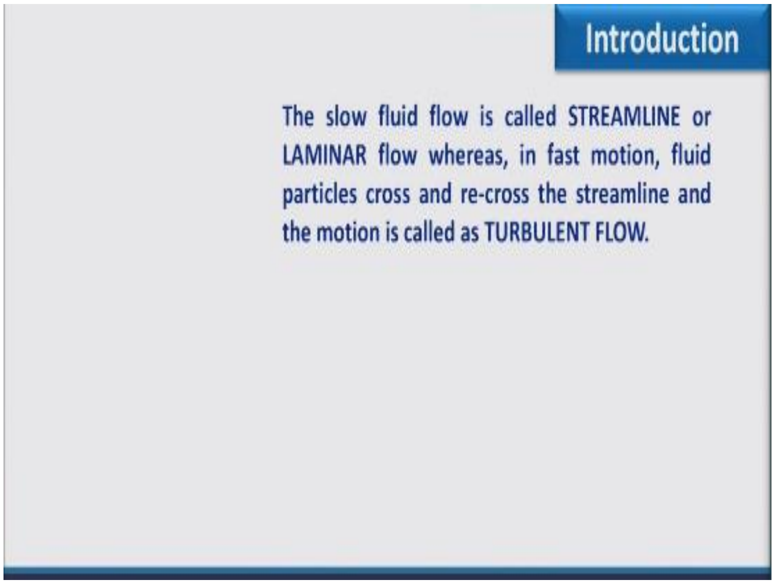

However, in fast motion fluid particles cross and re-cross the streamline and the motion is called turbulent flow. So in laminar flow fluid is moving in a particular streamline or fluid is moving in a streamline whereas when we considered the turbulent flow particle will cross the streamline. So the streamline will not be uniform anymore. So the same example we are considering here also.

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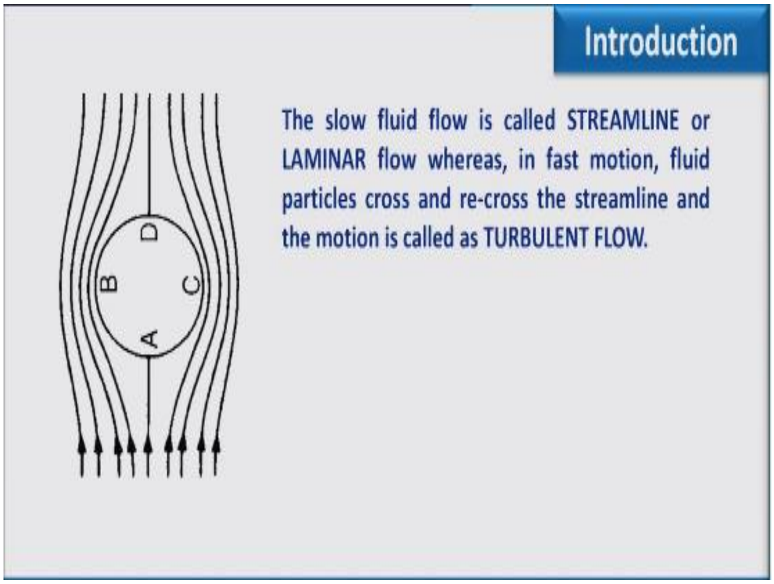

Now what happens when we consider the velocity at A, B, C, D point.

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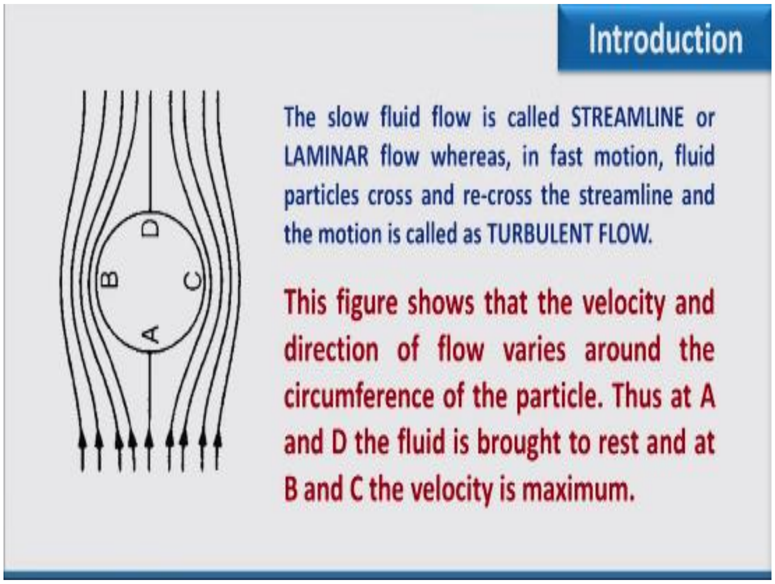

So we can say that the velocity and direction of flow varies around the circumference of the particle. So at point A as well as point D, if you see this point A as well as point D, the fluid is almost at rest at that point. Whereas when we consider point B as well as point C the velocity is maximum.

So A and D are the point where the water remains or fluid remains at still position, at B and C it will have the maximum velocity. So you can understand that as the movement of or as the particle will move or the fluid is moved over the particle it will not have uniform velocity throughout the particle surface. So as far as dynamics of single particle is concerned.

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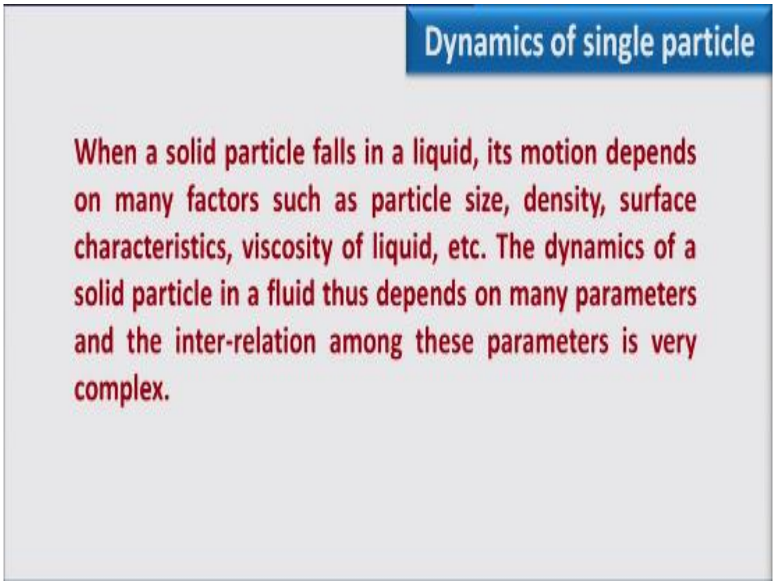

When a solid particle that is single particle falls in a liquid its motion depends on many factors such as particle size, density, surface characteristic, viscosity of liquid etcetera. And dynamics of a solid particle in a fluid thus depends on many parameters and their interaction or inter-relation when we study that is very complex.

Therefore, to study the dynamics of single particle we have taken some of the assumptions and these are.

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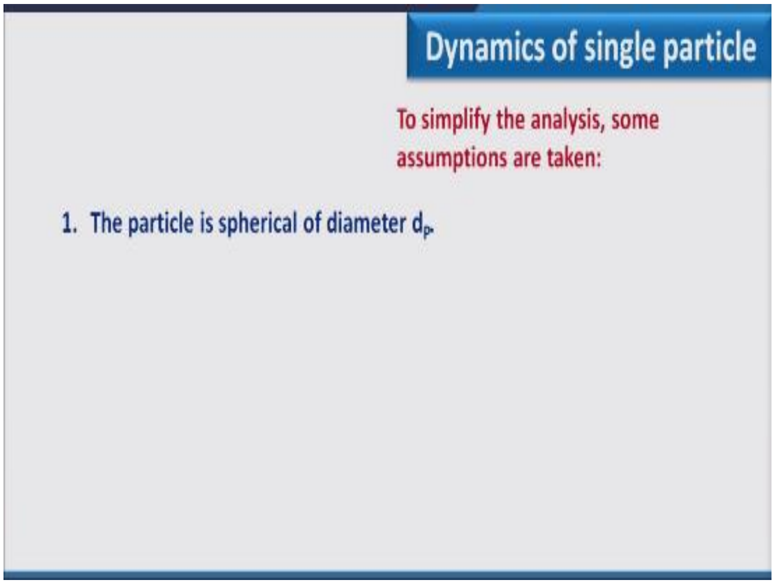

First assumption is particle is spherical in diameter and diameter we have shown with d_P. So we are considering spherical particle.

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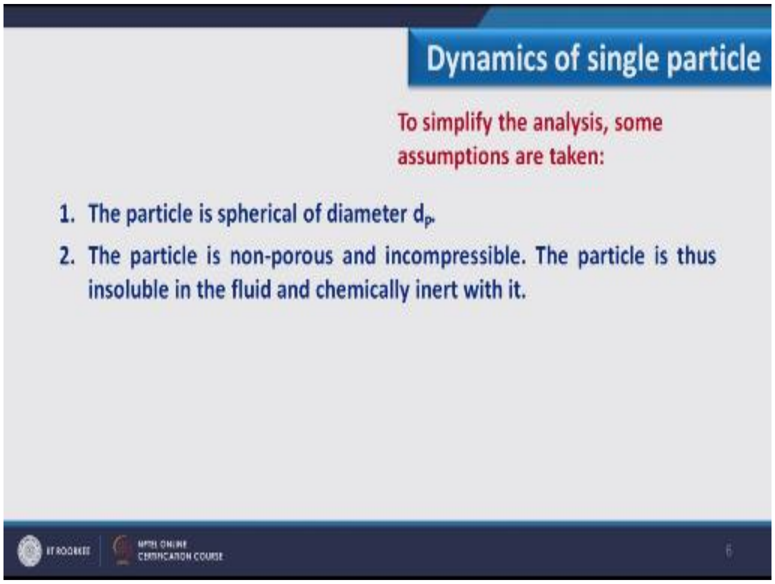

Second assumption is particle is non porous and incompressible so particle is thus insoluble in the fluid and it will be chemically inert with the fluid, density and viscosity of fluid is constant.

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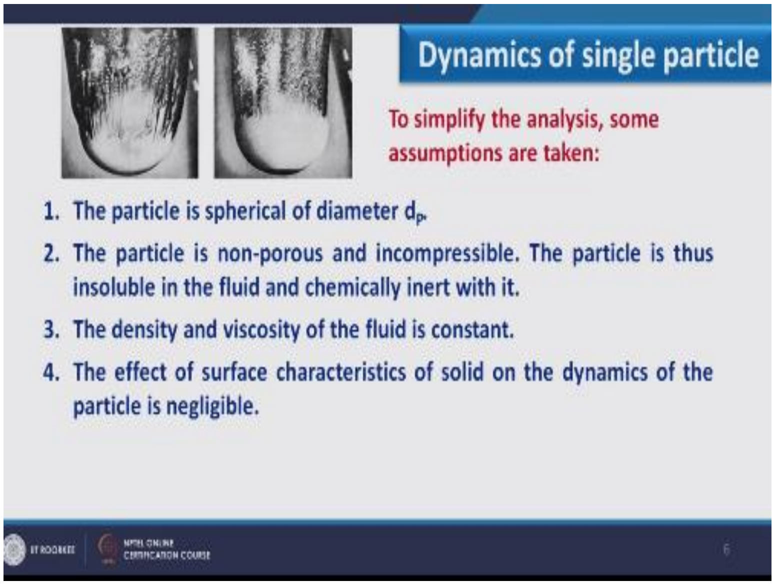

The effect of surface characteristics of solid on the dynamics of particle is negligible so here if you see this figure this figure basically shows a smooth particle and you can understand that fluid is moving very smoothly from the edges of this particle. On the other hand if I consider rough particle then water which is or fluid which is moving over this it will be quite scattered or quite spreaded, so we are considering that surface is smooth and therefore the movement of fluid through the particle is streamlined.

The particle is freely settling under gravity, what is the meaning of this that in this case we are considering that particle is single there is no other particle which can put hindrance in the movement of this particle, so single particle which is moving freely under the action of gravity and final assumption we have.

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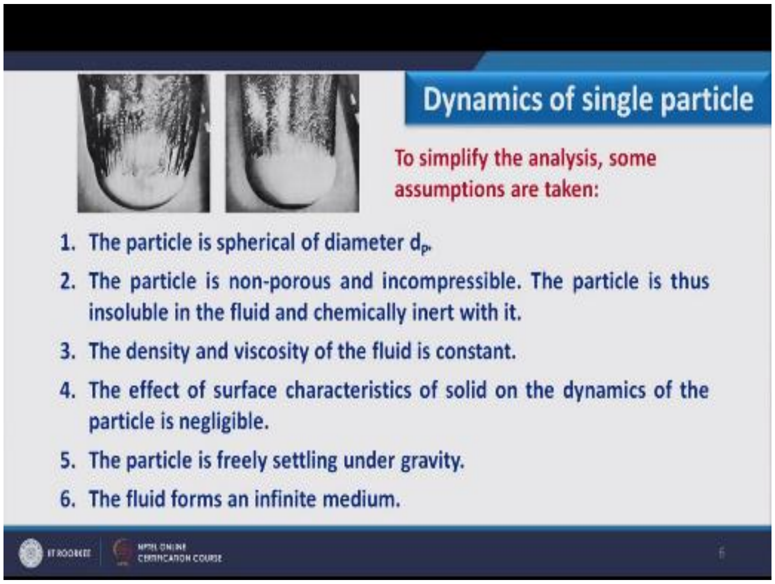

The fluid forms an infinite medium, what is the meaning of this that if we are considering a cylinder of very small diameter and when particle is moving through this the effect of wall comes into the picture so here we are considering that fluid is moving in infinite medium, it means cylinder diameter in which particle is falling in a liquid is significantly large in comparison to diameter of particle, so here significant assumptions we have taken which some of these assumption will be realized in subsequent slides.

But at this time we are considering all these assumptions. Now as far as dynamic of particles are concerned here we are going to cover different topics, first is terminal settling.

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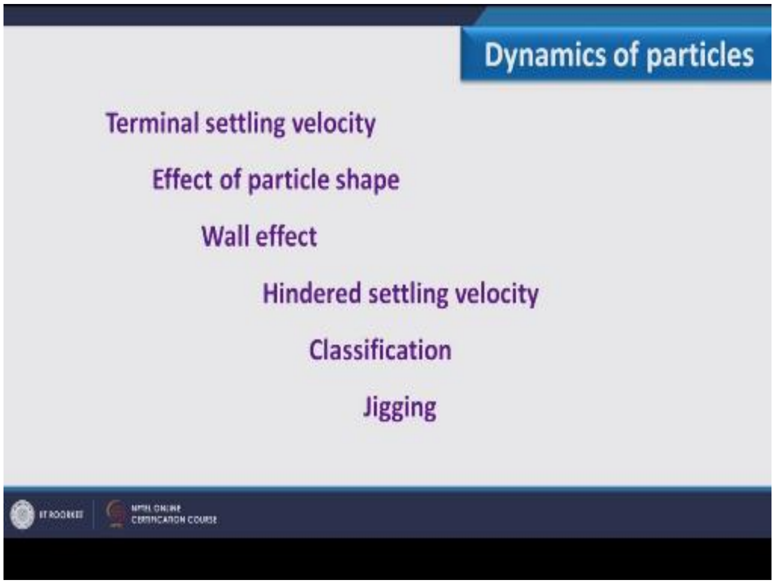

Velocity, second effect of particle shape and third is wall effect and apart from this we will discuss hindered settling velocity and classification, and finally we will discuss jigging so all these six topics will come into particle dynamics, now if we consider top three.

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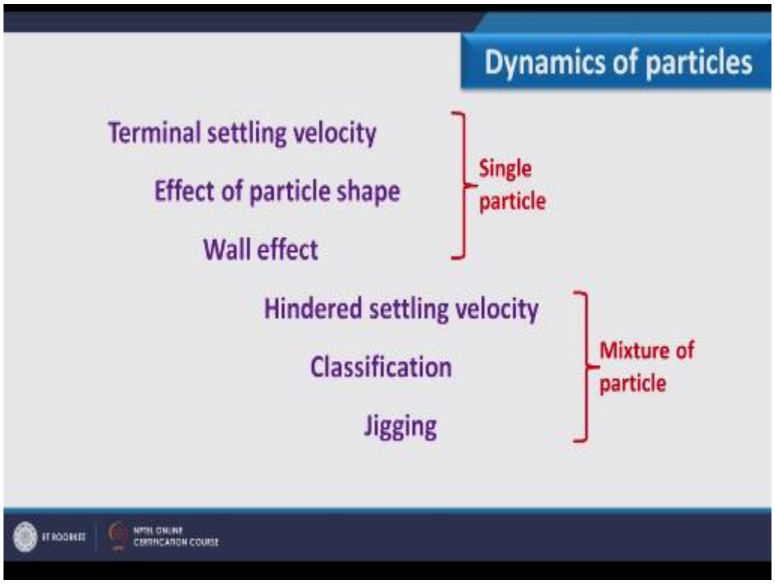

Topics that is terminal settling velocity, effect of particle shape and wall effect all these are corresponding to single particle whereas next three topics are correspond to mixture of particles when we have mixture or when we have suspension when we have slurry. So let us start the first topic of this that is terminal settling velocity. Now to make to make you understand what is the concept of terminal settling velocity I have taken one big container you can see.

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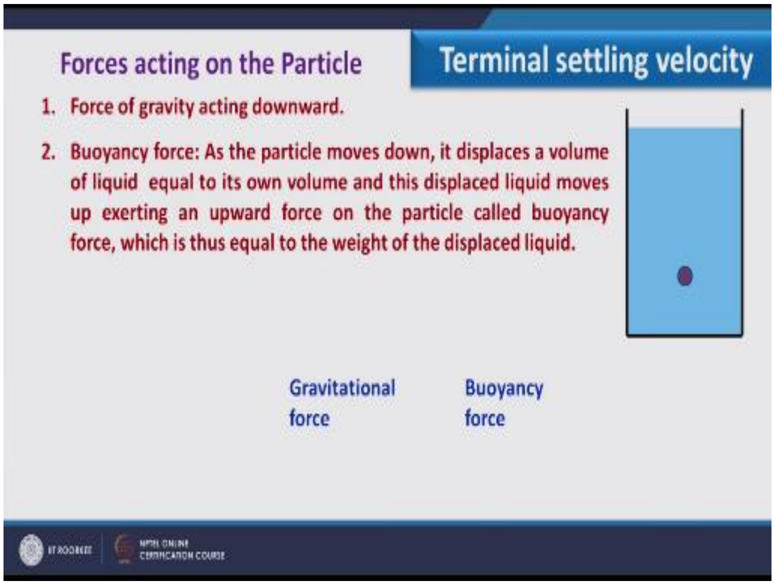

Over here in which fluid is filled and when particle is falling in this fluid what are the forces which are acting on the particle, the very first force is the force of gravity which is acting downward, that we can denote as gravitational force, second is the buoyancy force, now what is buoyancy force I guess you understand what it is as the particle moves down it displaces a volume of liquid equal to its own volume, so when particle is moving into the fluid it displaces the volume of its own volume towards upward and this displaced liquid moves upward and therefore it exert an upward force on the particle and that force we call as buoyancy force.

Which is definitely equal to the weight of displaced liquid so weight of displaced liquid means mass of displaced liquid into g, now apart from these two another.

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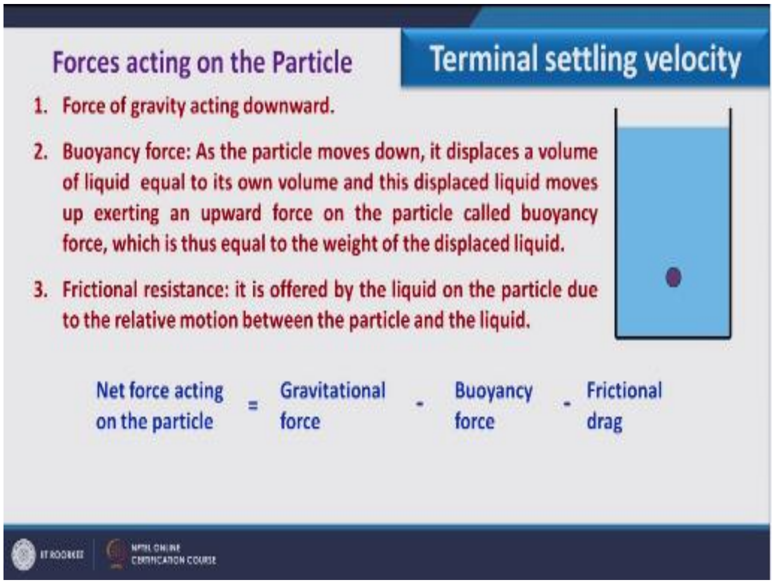

Force which will act on the particle is frictional resistance, it is offered by the liquid on the particle due to the relative motion between particle and the liquid. Now here we have three different forces and net force which is acting on the particle is the gravitational force minus buoyancy force minus frictional force, gravitational force is acting downward, buoyancy and frictional force will act upward.

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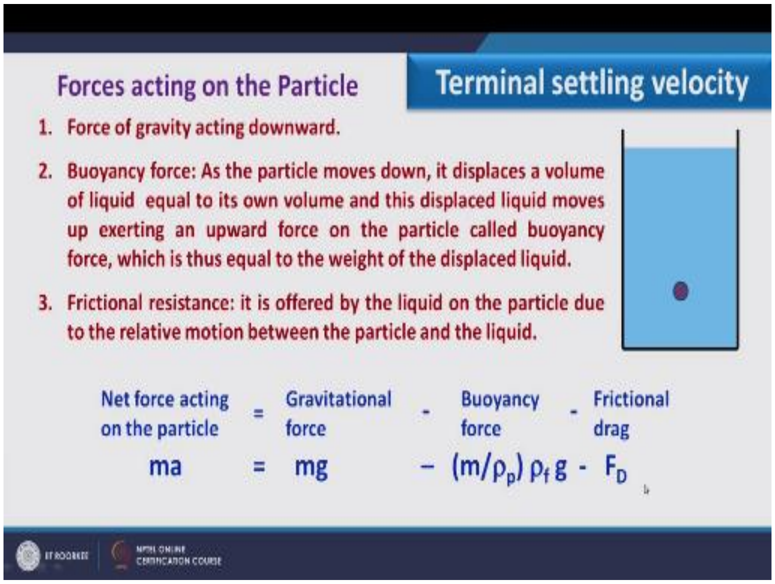

And mathematically we can represent net force acting on the particle that is ma=mg that is gravitational force minus $m\rho_p$, what is $m\rho_p$ is the volume of particle and when we multiply this by ρ_f we can get mass of the liquid which is displaced into g so that is the weight of displaced liquid, and F_D we have used for frictional drag so F_D is nothing but the kinematic force.

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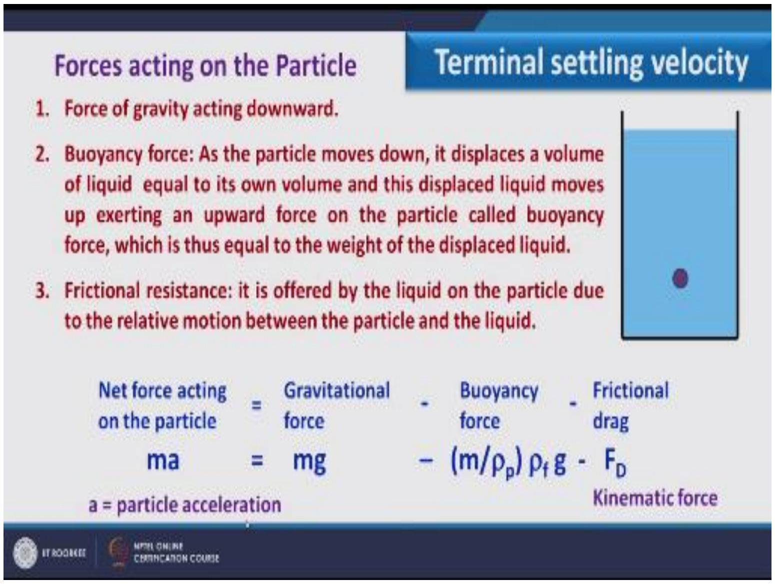

And a is basically particle acceleration. Now when we study this equation in detail what it shows, it shows the mechanism of a particle movement when it is falling in a liquid, now when the particle start falling in the liquid its velocity is very less, so when velocity is very less the frictional drag which is putting hindrance in the movement of liquid that will also be negligible, so when this F_D is negligible at that time that magnitude of ma will increase and as m is constant a will increase which is nothing but the particle acceleration.

So when acceleration will increase it compels the particle to move with faster velocity so when the velocity will increase so again the drag, frictional drag which is acting in upward direction it will increase so it will put resistance in the movement of particle. So when FD will increase again the acceleration will keep on decreasing so at some point what situation will occur that whatever force is acting on the particle that is mg will be equal to the buoyancy force plus frictional drag. At that time there will not be any net force which is acting on the particle and particle will move with 0 acceleration or we can say it will move with the constant velocity, so that time when the velocity is constant this we can define as terminal settling velocity of the particle. So what is the condition that at terminal settling velocity ma would be 0 so that we will take as a base of the derivation this was the equation which we have.

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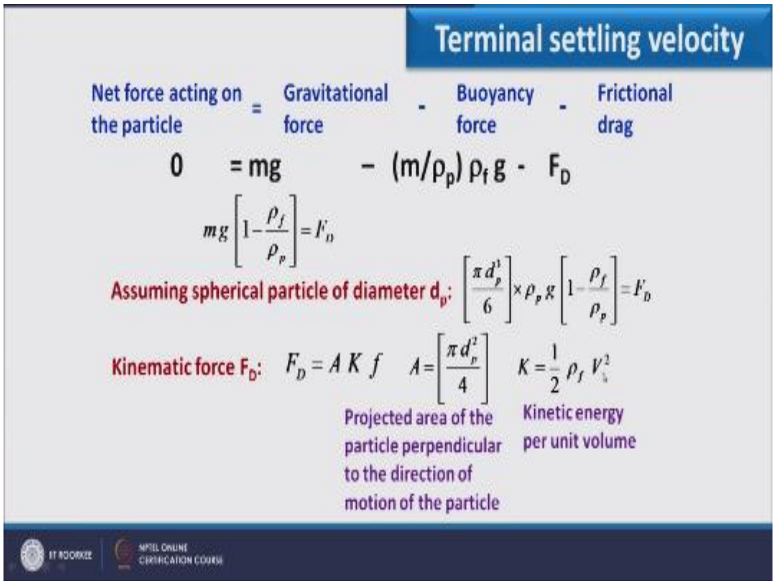

Discussed in the last slide where ma is 0 because particle is reach to terminal settling velocity, so considering this remaining expressions we can rewrite the expression in this form mg in 21- ρ_f \/ρ_p = $F_{D.}$ Now we have to write the expression for $F_{D,}$ further we have already assumed that the particle is spherical with diameter so m can be represented in terms of particle diameter, if you consider this the expression is лd_p^q /6 x ρ_p x g x 1- ρ_F/ρ_p = F_D so what is this лd_p^q /6this is nothing but the volume of a particle into ρ_p so that would be the m of mass of the particle.

So this m we can represent as volume x density so rest of the expression will remain same. Further we can consider kinematic force which we have represented as F_D so F_D we can write as AK x f, now what is this AK x f A is the projected area of particle perpendicular to the direction of motion of the particle so A we can represent as the area of spherical particle which is лdp^2 / 4 and we can write K which is nothing but the kinetic energy per unit volume and it is = ½ $\rho_F V^{2,}$ so that is nothing but ½ and V^2 but per unit volume is there that is why we have use the density term.

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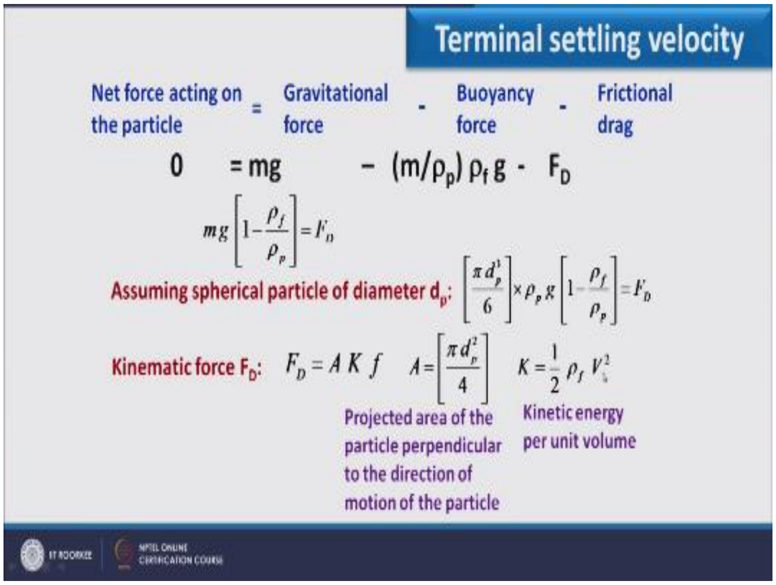

And f is nothing but a factor f_D small f_D, f_D is the kinematic force F we can represent as small f and subscript with f_D that is the drag force, so using the expression of A and K we can write the f_D.

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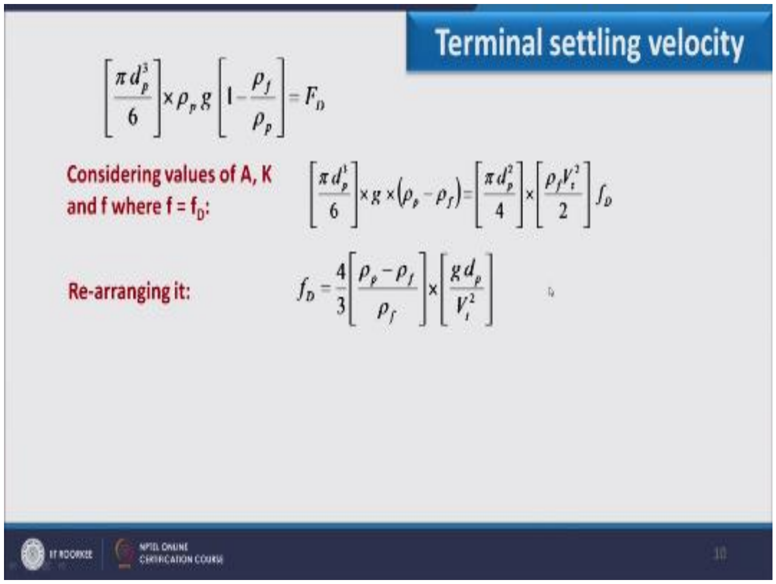

So considering value of AK and f where f = f_D we can rewrite this equation where this expression will remain same, however the right hand side that is A we can write as area perpendicular to the motion into a kinetic energy per unit volume into fD, so considering all these factor we can write the equation which we can resolve or rearrange. Now once we rearrange we can calculate the value of f_D where which comes as 4/3 $\rho_p – \rho_f / \rho_f$ into $g^d{}_p / V_t^2$ so if you see this expression.

Here we can write V_t so this V_t would V_t will represent the velocity of particle and that comes under kinetic energy term so from here this equation we can write f_D and further.

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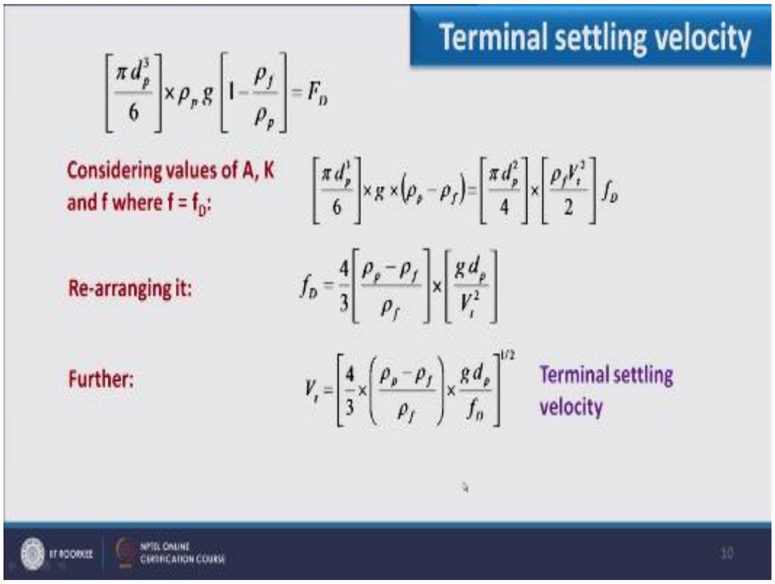

We can calculate, we can derive, we can find the value of Vt from this expression in terms of fD so this is the expression for terminal settling velocity. Now what happens over here that once I am having this expression it depends on, it is directly proportional to particle size that is dp as well as particle diameter so what is the meaning of this that velocity will increase when we are dealing with larger particle and further velocity will increase when we consider the heavier particle which has more density.

So this is the generalized expression for terminal settling velocity.

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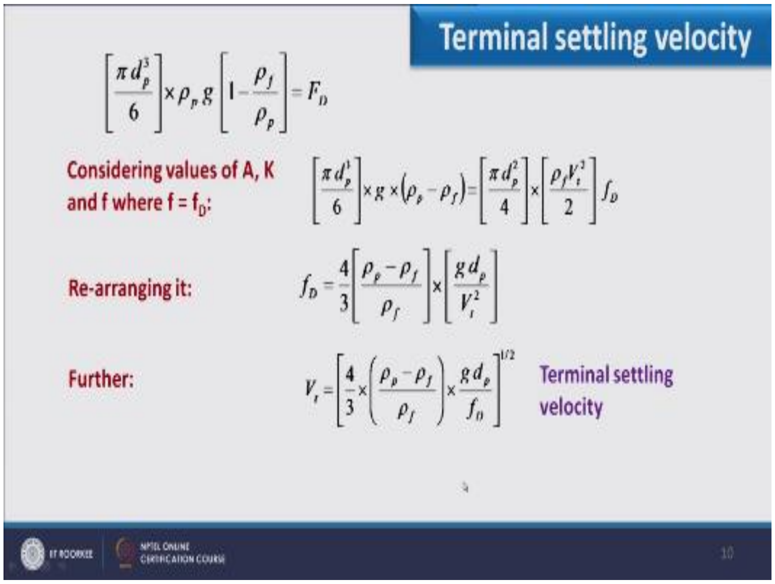

We will further consider this equation to derive the equation for different regions of flow.

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Now to understand different regions of flow we can have the graph between Drag coefficient vs Reynolds number this is the.

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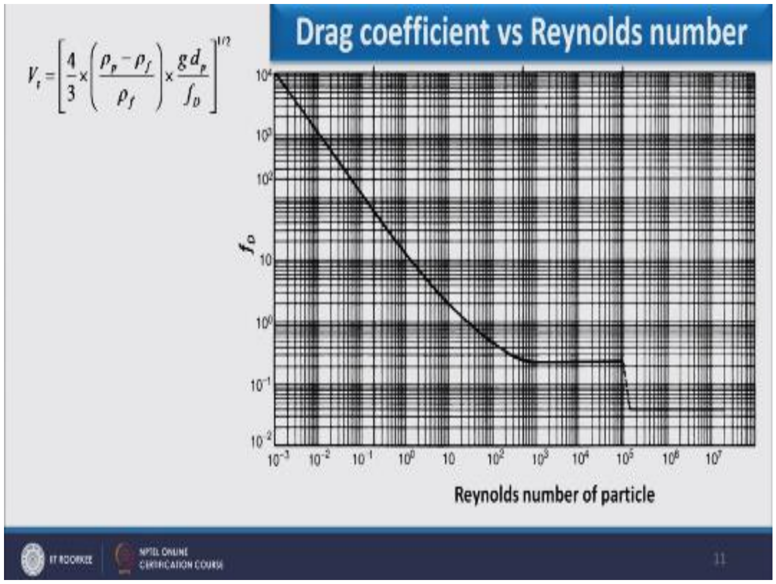

Very known well known graph here we have this f_D that is Drag coefficient and here we have the Reynolds number and this is the profile by which f_D is changed, then we change the Reynolds number, so this is the well known graph, now we will consider different regions in this graph and here we have the generalized equation of terminal settling velocity and further we will discuss the specific equation which can be used for particular region, so first of all we will consider.

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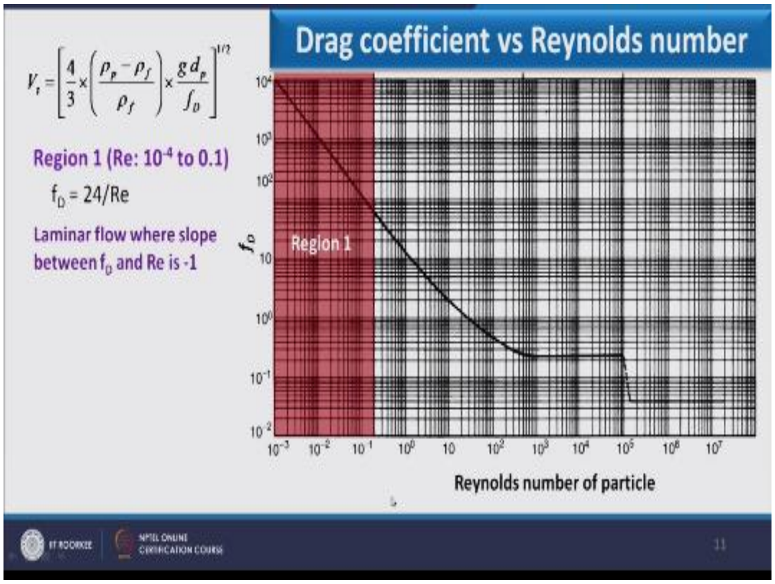

Region1 where Reynolds number vary from 10 raise to minus 4 to 0 point 1 so up to here the region1 is available which we have shown with the red color in this graph that is region one, and now if you consider this region 1 here we have almost linear variation of fD with respect to Reynolds number, so in this region fd can be defined as 24/Re and it is basically laminar region where the slope between fD and Re is – 1 so all these regions are pre defined and you can study about this in fluid dynamics in detail.

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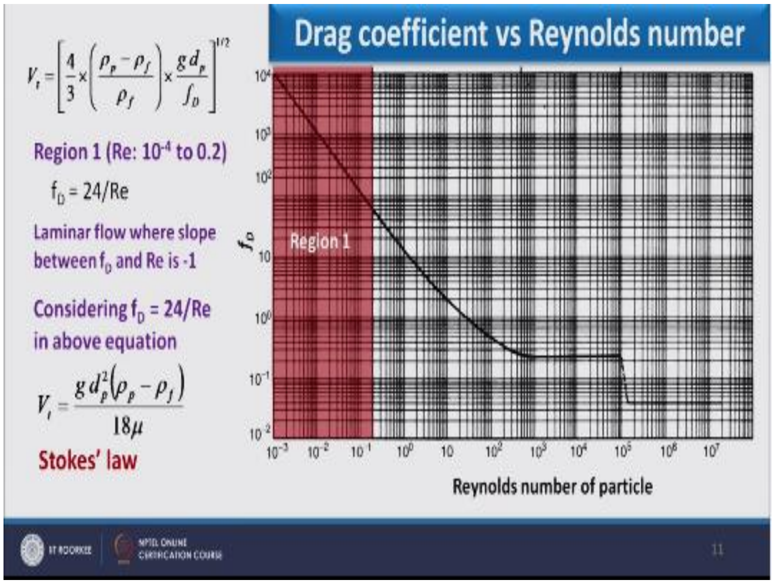

So this graph we have used between f_D and Re where in region1 the slope between fD and Re is – 1. Now once I consider the f_D factor which is 24/ Re in generalized equation of terminal settling velocity then we can rewrite the expression of Vt that is g dp2 ρp – ρF/ 18 so this expression is basically used for laminar flow and we call it the Stokes' law, so a Stokes' law is used to calculate terminal settling velocity of particle when it is falling in laminar zone. And now in this slide we will discuss the application of Stokes' law, you can understand that it, it is applied in very less velocity or in the less Reynolds number or in laminar region. So Stokes' law is important to understand.

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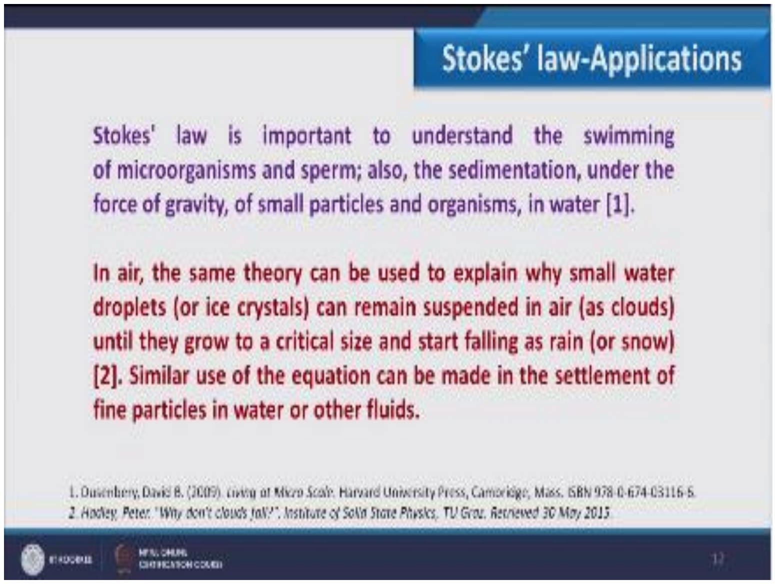

The swimming of microorganisms and sperm, also the sedimentation under the force of gravity of small particles and organisms in water, now what is the meaning of this, that when we consider laminar flow, laminar flow can be achieved when the particle diameter is very less. If you remember the expression of Reynolds number that is $d v \rho/\mu$ so in that case if diameter of particle that is d is very small it is more likely that it will follow, it will fall in the laminar region.

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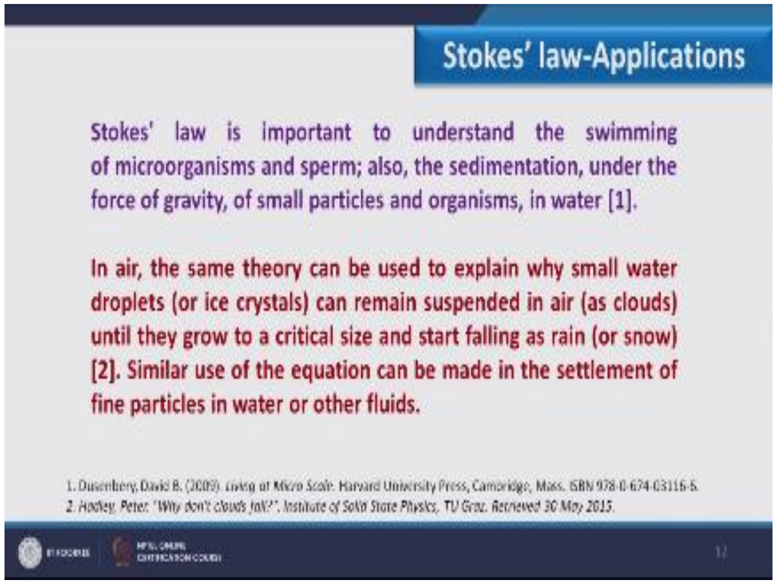

So Stokes' law basically demonstrate or basically applicable to compute the velocity of microorganisms and sperm which are very small as far as diameter is concerned. Another application is.

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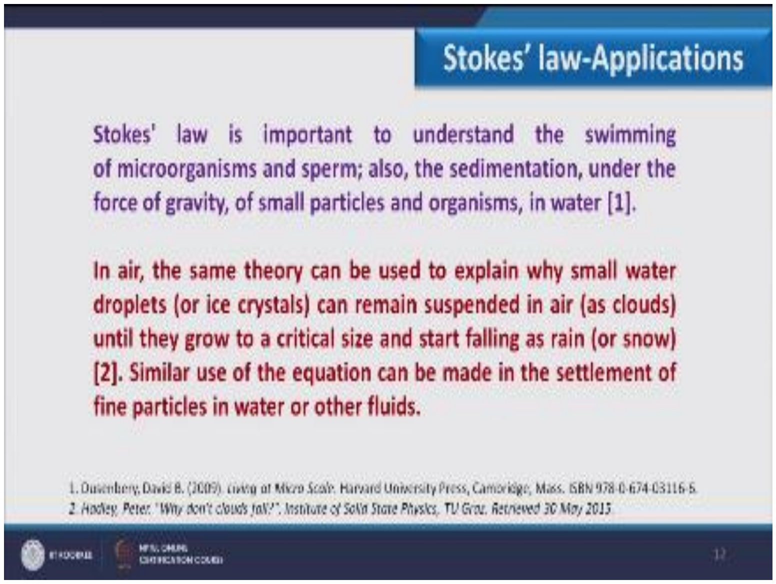

In air the same theory that is Stokes' law can be used to explain why small water droplets or ice crystals can remain suspended in the air as cloud until they grow to a critical size and start falling as rain or snow. So similar use of equation can be made in the settlement of fine particle in water and other fluid, so what is the main point of this statement is Stokes' law is applicable when we are considering particle size of very small diameter. So this two application I have taken from these references.

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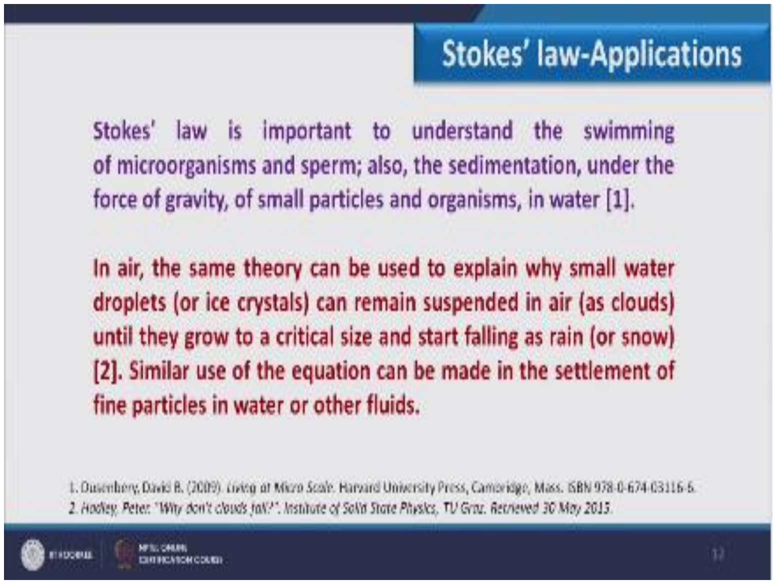

You can go through these references for to study about this in detail.

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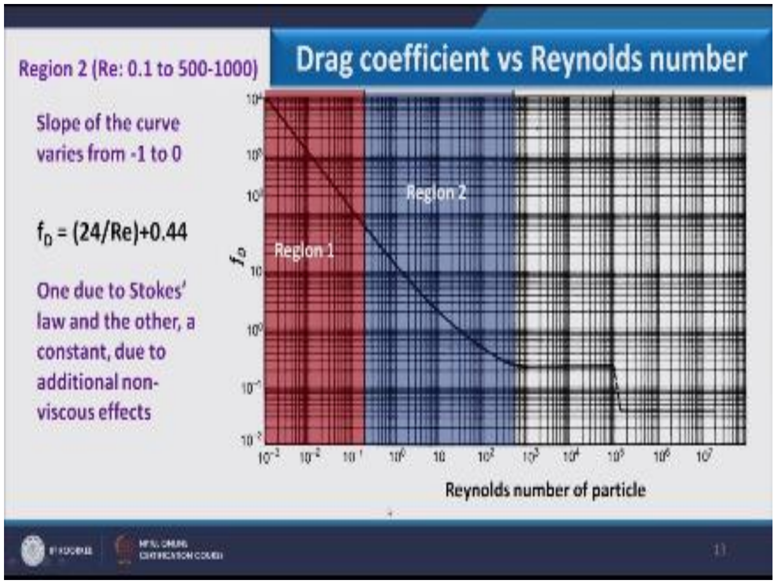

So first region we have already covered, now we are dealing with, we are finding the second region, second region varies from 0.1 to 500 to 1000, so this is the second region where it stops up to 500, so 500 to 1000 range we can take here we have taken as 500. Now in this case slope of the curve varies from 0.1 to 0 because at this, at the end of this region the slope is reaching to 0 value. So the slope in this region two varies from 0, varies from 0.1 to 0. In this case drag coefficient is defined as (24/Re)+0.44.

So first expression of this f_D equation is due to the Stokes' law and other that is 0.44 which is a constant and it is due to the additional non viscous effect. So this f_D we can use to calculate the terminal settling velocity falling in this region, so there is basically we call it a transition region because here from -1 to 0 the slope is reaching and therefore we call it a transition region.

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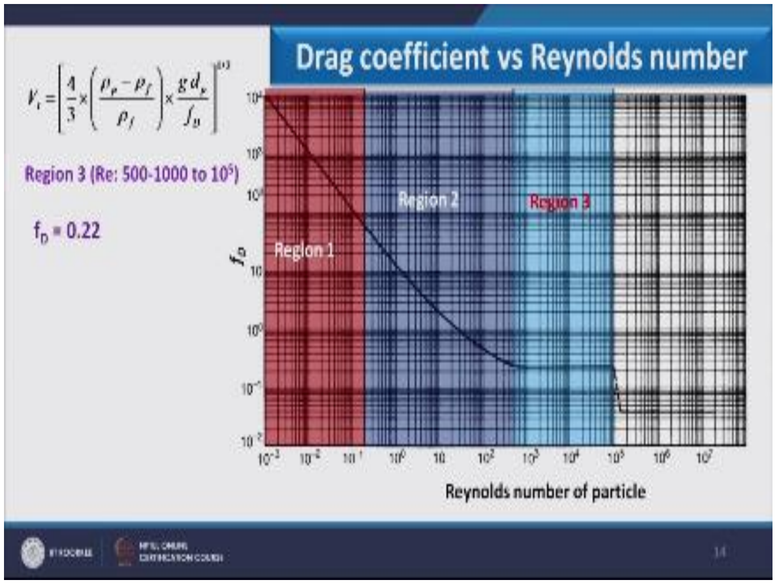

So two regions we have already discussed, now as far as third region is concerned this is the generalized equation which we have discussed already. And region 3 where the Reynolds number varies from 500 to 1000 to 10^5, so this is the region we will call region 3. In this region if you see the variation of f_D with respect to Reynolds number is almost constant. So in this case it is approaching to turbulent flow where the velocity is so high that it will not change with the surface significantly so that will move over the particle and it will not put significant drag or significant resistance in the path of the flow of the particle.

So therefore in region 3 we have considered f_D as constant which is equal to 0.22, you can see the value from this graph also.

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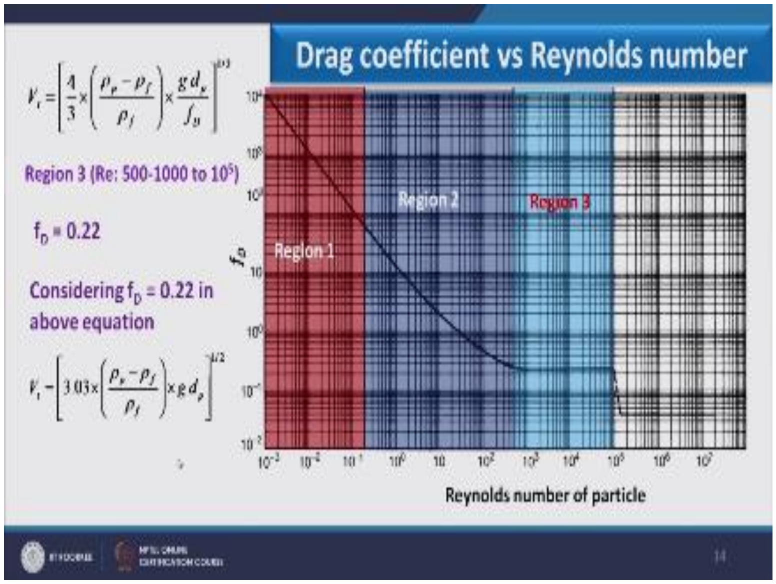

Which is available over here, now once I use the f_D 0.22 in this equation what we can say that, we can have equation in this form which is 4/3 and that is divided by 0.22 so 3.03 is the factor which is coming and rest of the parameter will remain same. So you can see this is the terminal settling velocity equation when we are considering f_D is almost constant, this is more or less a turbulent region and this law we call as Newton's law.

So you can understand for laminar zone we have taken, we have defined Stokes' law, for turbulent region we can use Newton's law. Now beyond this further when Reynolds number will increase that we call as developed turbulent region. So all these three region we have already covered.

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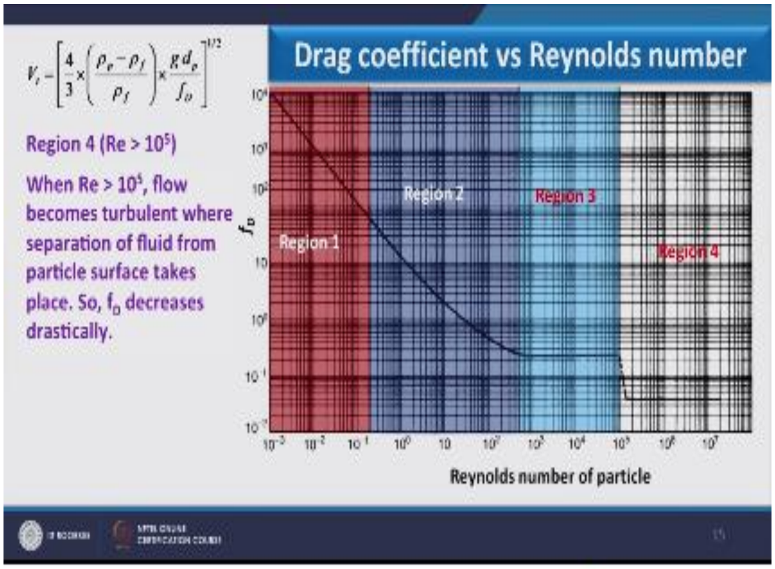

Now here what we have is the same expression, generalized expression of V_t that is terminal settling velocity as a function of f_D. And now we are considering region 4 where the Reynolds number is greater than 10^5. Now what happens over here, if you consider this particular region so here in this region you see the drag coefficient is decreased significantly and then it becomes constant. Now why it is so, when the velocity of liquid velocity of fluid is very high what happens, when it pass over the fluid it will basically passed with a distance from the particle surface.

It means it will not touch the particle surface it move away from the or beyond the particle surface because particle, because velocity is significantly high, so it will for example, when I am considering particle like this it will move like this, so it remains separated from the particle surface. When it remains separated from the particle surface it will not put any drag into the particle and therefore drag coefficient at that time will be significantly decreases.

So when we consider Reynolds number greater than 2*10/5 the f_d value decreases significantly and in this region f_d value is found as 0.05, that you can also read from this graph. So considering all these regions we can define different equations for terminal settling velocity, in the same line for region 4 we can

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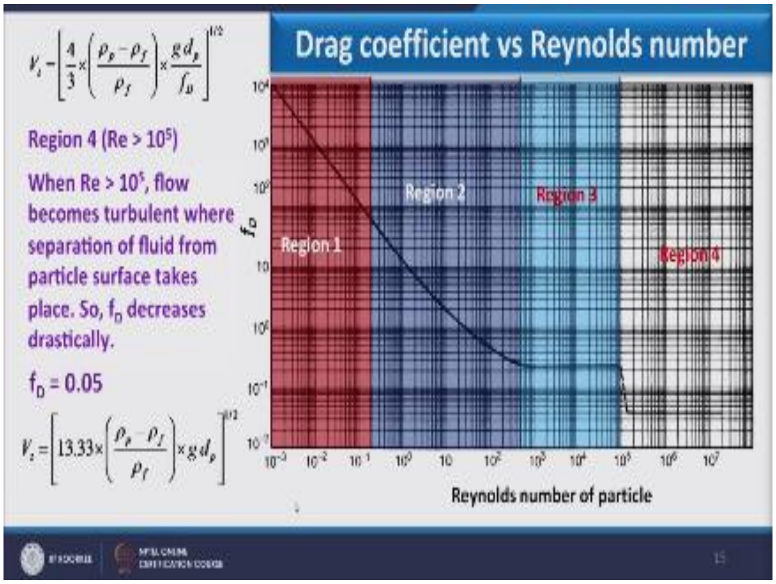

Write the expression of terminal settling velocity while considering $f_{d} = 0.05$ and that 0.05 we can put over here and for the resolve this, so this is the final expression of terminal settling velocity when the particle is moving in developed turbulent zone where Reynolds number is greater than 10^5, therefore for different region we have defined different equations and these are the equation and all these equation.

If you see all these equations are proportional to d_p as well as ρ so the equations show about for terminal settling velocity of the particle is, and this terminal settling velocity of particle increases with the increase in particle size as well as particle density. So when we are considering different size particles and different density particles the heaviest and the largest particle will settle in less time in comparison to others because that is terminal settling velocity is directly proportional to d_p as well as ρ.

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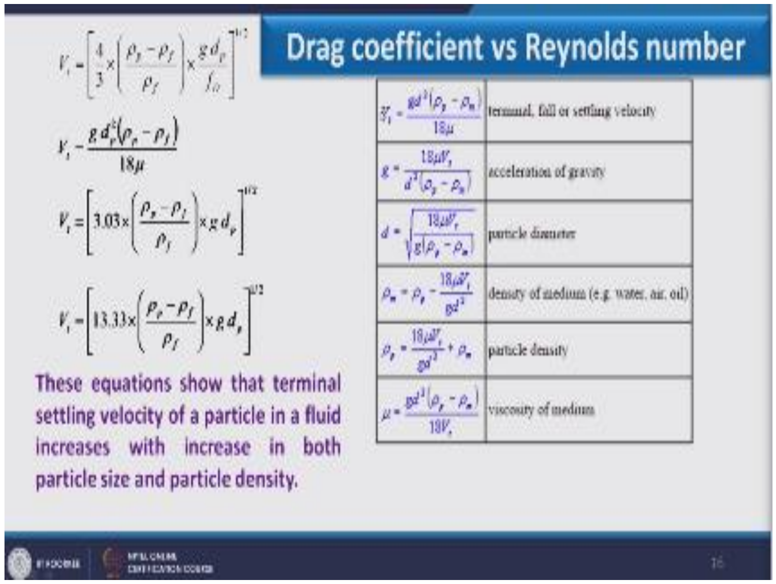

Further if you see this table what this table shows here this table is representing V_t and expression if you remember, this is the expression of Stokes' law, so here we can calculate settling velocity when the particle is falling in laminar zone and the same equation we can use to calculate other parameters associated in this expression, for example if I want to calculate the acceleration of gravity then we can have $g = 18\ \mu vt$ by $d\sqrt{\rho_p - \rho_m}$ where this ρ_m

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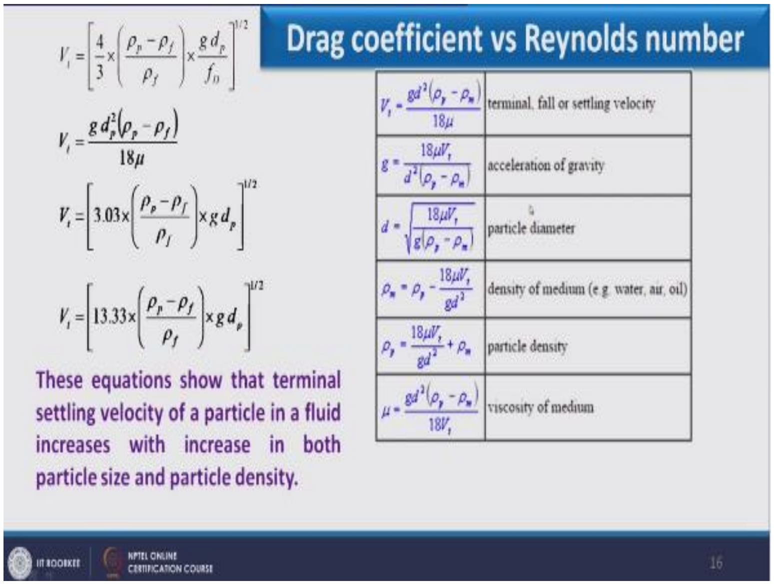

We have taken as ρ_f that is density of fluid or we also call it density of medium. Further particle diameter can be known once I know the terminal settling velocity and here we can find the density of the medium as a function of density of the particle, particle density also we can calculate using the Stokes' law and viscosity of medium we can also find using the Stokes' law. So single equation can be used to find different parameters which are associated with this, so here you see we have taken $\rho_m = \rho_f$ and this is the link of where this table is taken.

So if you want to study more about this you can study here. Now in this particular part of lecture 1 we have covered the derivation of terminal settling velocity and we also have seen equations of terminal settling velocity when the particle is moving in different region, so that is all for now, we will continue in next part of the same lecture, thank you.

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