INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Mechanical Operations

Lecture-11

Laws of comminution

With

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Welcome to the third week of mechanical operations course, today we will start lecture 1 of week 3 which is on laws of comminution, if you remember the fifth lecture of week 2 there we have discussed the size reduction, its definition, objective, and breakage pattern, mechanism, and mode of operation. Now here we will discuss the laws of comminution to calculate energy consumption in size reduction.

So it is all most impossible to find out the accurate amount of energy requirement in order to affect size reduction of a given material.

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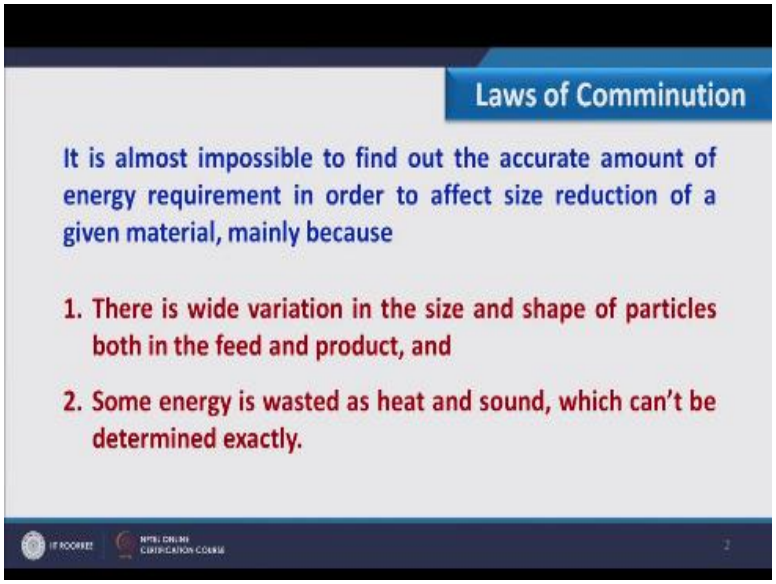

And this is because there is wide variation in size and shape of particles both in feed as well as product, and second point is some energy is wasted as heat and sound which cannot be determined exactly, therefore in very accurate way we cannot calculate the energy requirement for size reduction. The laws of comminution proposed by different.

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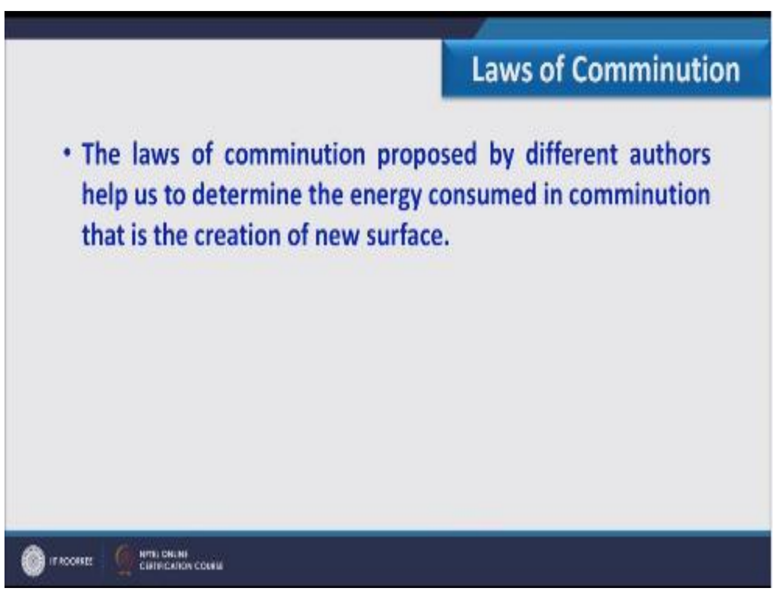

Authors help us to determine energy consumed in comminution that is the creation of new surface. Now what is basically is a comminution or size reduction, if you remember the last lecture that is lecture 5 of week 2 there we have discussed the purpose of comminution is to increase the surface area there and therefore we call it the creation of new surface. So the laws of comminution proposed by different authors, by using those law we can calculate energy consumption.

When the new surface of material is created.

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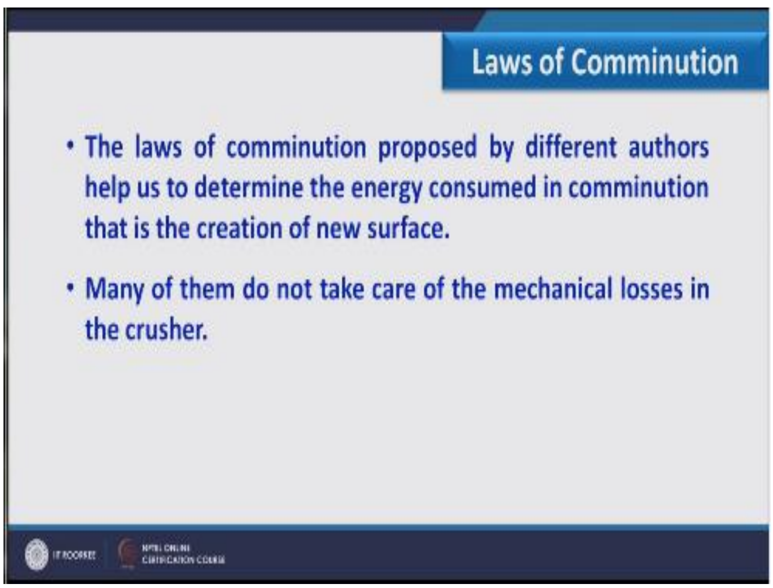

Many of them do not take care of mechanical losses in the crusher, they only consider the creation of new surface but the losses occur in crusher they do not consider, the method do not consider this.

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Most popular comminution laws are first, is proposed by Kick, second is Rittinger and third is Bond, so we are having three laws of comminution and we call it Kicks law, Rittingers law, and Bonds law. In subsequent slide we will discuss details about these laws.

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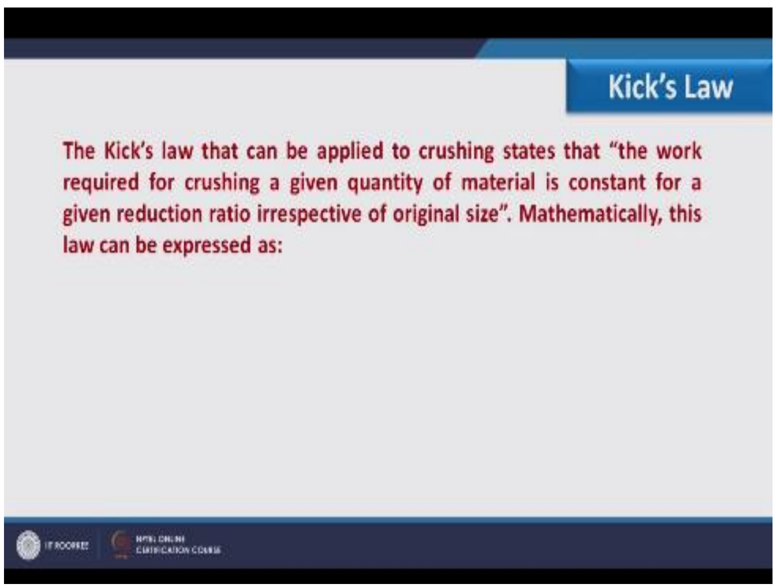

Let us start with the Kicks law, the Kicks law says, the Kicks law can be applied to crushing and it says that the work required for crushing a given quantity of material is constant for given reduction ratio irrespective of original size, so what is the meaning of this statement? That if I am having the feed and if we reduce the size of this feed whatever energy is required for this, whatever work is required for this will depend on size of feed as well as size of product.

But and that size of feed and product ratio we have defined as the reduction ratio, so therefore work required is proportional to the reduction ratio, it does not vary with the original size of the feed. If we define the Kicks law mathematically.

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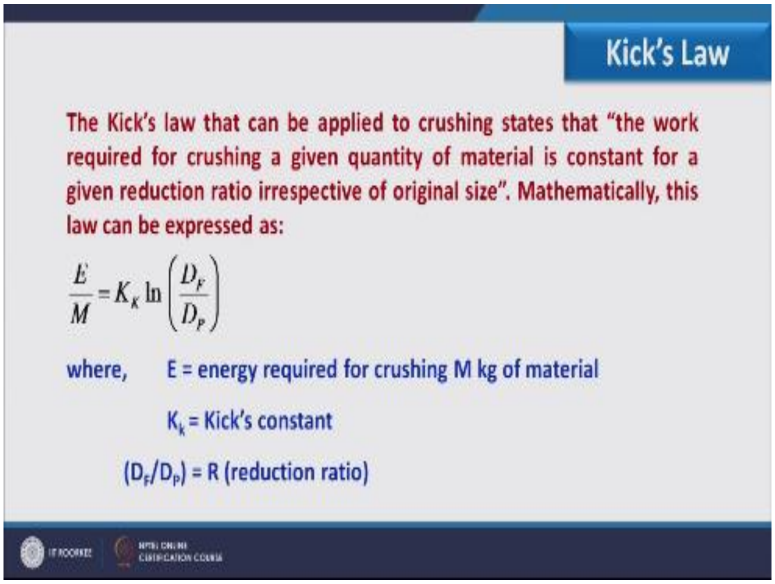

We can write the expression has E/M, that is energy required for crushing M kg of material equal to K_k that is the Kicks constant $\ln(D_F/D_P)$, that is the D_F is the feed size and D_P is the product size, so ratio of these two we have defined as the reduction ratio, so ratio of these two we have defined as the reduction ration therefore energy consumption is proportional to the reduction ratio, and here the proportionality is equated with the constant that is the Kicks constant.

So therefore comminution energy depends only on the reduction ratio and is independent of the original size of the feed, what is the meaning of this? For example.

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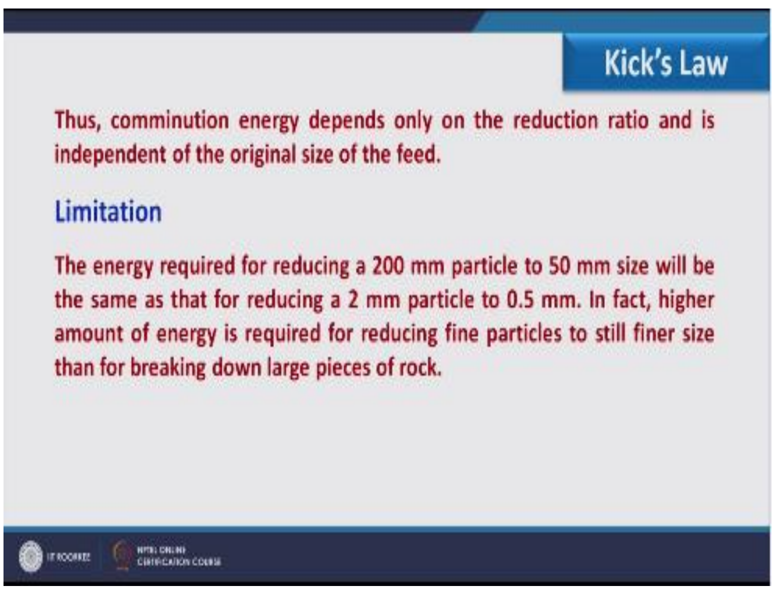

If I say that the particle of 200 mm size is reduced to 50 mm size, for example if particle of 200mm size is reduced to 50 size what is the reduction ratio of this, 200/50 we have 4 so reduction ratio is 4. Now if I consider the particle of 2mm size and we reduce it to the 0.5mm size so 2/0.5 again the reduction ratio is 4. So if we consider the Kicks law the energy consumption while crushing 200 mm material to 50 mm as well as 2 mm material to 0.5 mm will be same because reduction ratio is same.

Now what is wrong with this? If you remember the last lecture that is lecture 5 of week 2 there we have discussed that when particle size are smaller and when we have to convert this to further smaller size it requires more collusion for reduction purpose, so if reduction ratio is same to crush 200 mm particle to 50 mm will require less energy in comparison to if we reduce 2 mm particle to 0.5 mm product. So though reduction ratio is same but obviously energy consumption will we significantly high because when we consider the smaller particle the more collusions are required and many of these collusion will be wasted and they are not participated in size reduction.

So when the statement of Kicks law is concerned that it totally depends on the reduction ratio this statement is not correct when we deal with the fine particles. In fact higher amount of energy is required for reducing fine particles to still finer size than for breaking down large pieces of rock, so Kicks law can be applied to coarse crushing where the.

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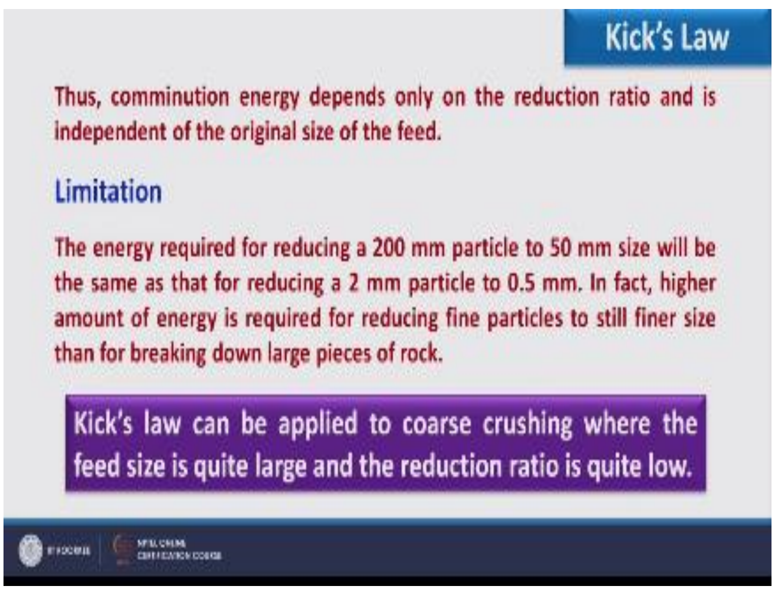

Feed size is quite large and reduction ratio is low, so that is the limitation of Kicks law that when we deal with this smaller particle though it will have same reduction ratio energy consumption cannot be seen in two cases, therefore Kicks law is applicable only for coarse crushing were reduction ratio is small.

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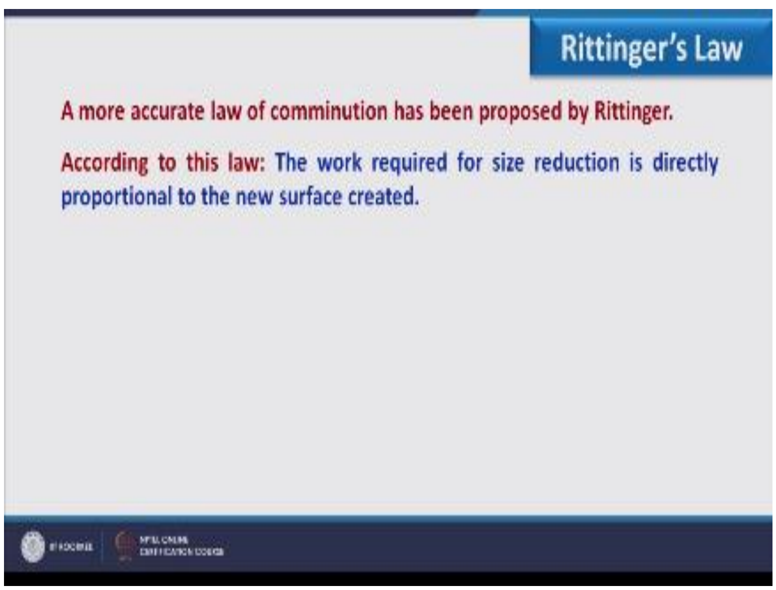

Second law we have is the Rittingers law which is more accurate as far as size reduction or comminution is concerned. Now what it says according to this law.

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The work required for size reduction is directly proportional to new surface created. How we show it mathematically? This is the expression where we have.

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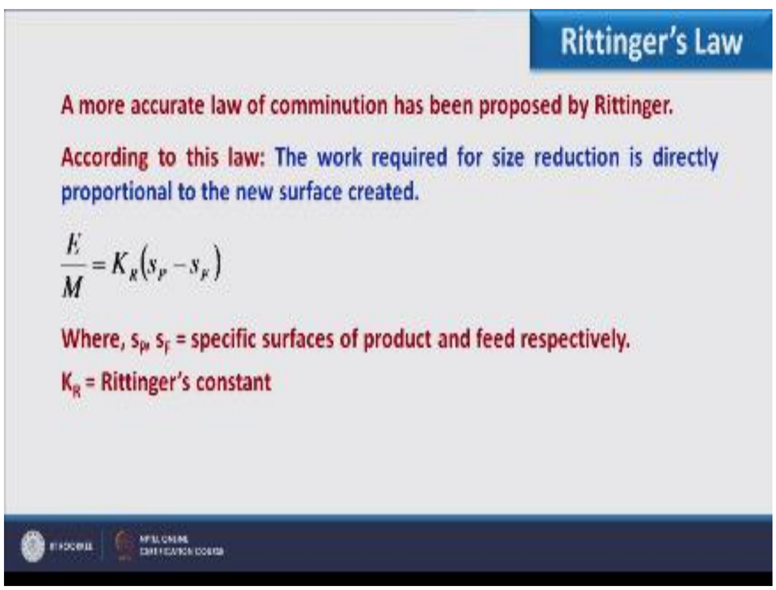

E/M that is energy consumption to handle M kg of material, it is equal to K_R (S_P - S_F). Now what is SP and what is SF? These are the specific surfaces of product and feed, when we crush the feed obviously feed will have a large size in comparison to the product so surface area for product is larger than that of the feed, so S_P – S_F will give the new surface created and.

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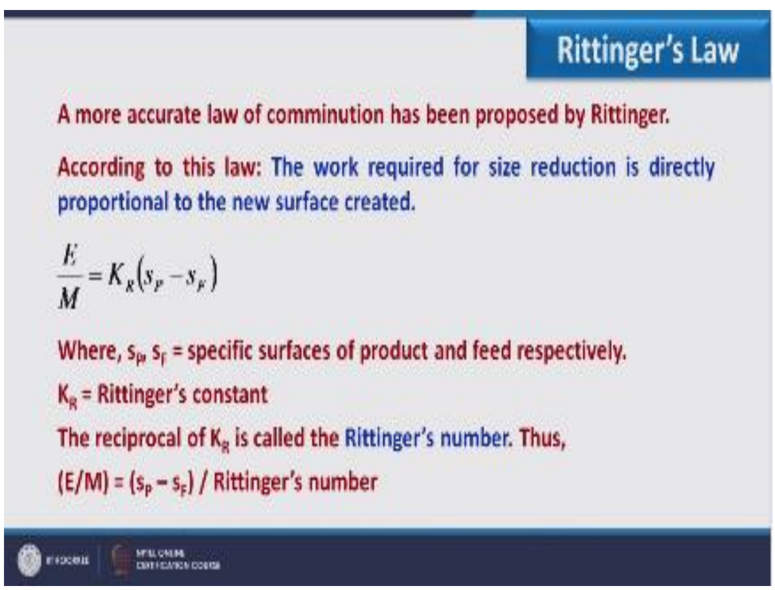

K_R we call as the Rittingers constant, the reciprocal of K_R is called as Rittingers number therefore the above expression is represented as (E/M) = (S_P – S_F) / Rittingers number. Now what is Rittingers number?

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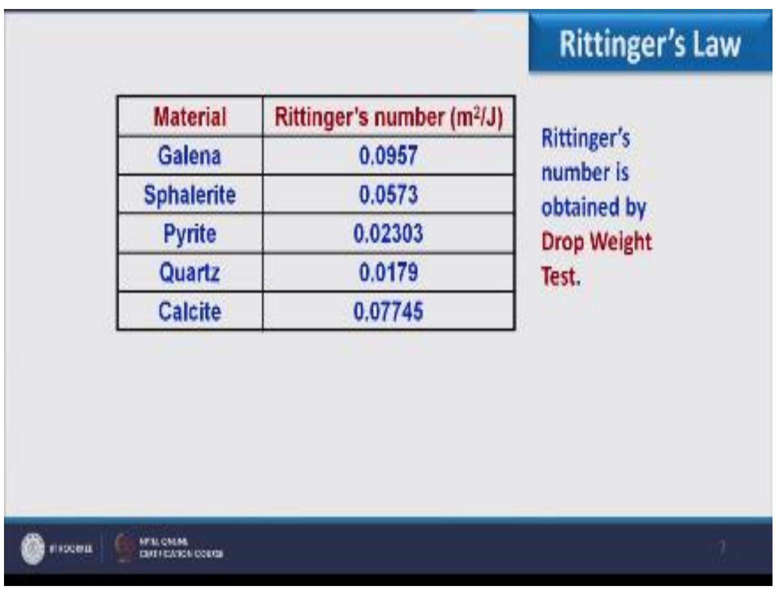

If you see this table for different material the Rittingers number is shown which is the as far as unit is concerned it is (m^2/J) so depending upon the material the Rittingers number vary, but how we find this Rittingers number? Rittingers number can be obtained by drop weight test, now what is this drop bit test? For example if I am having this feed and we put the feed over here.

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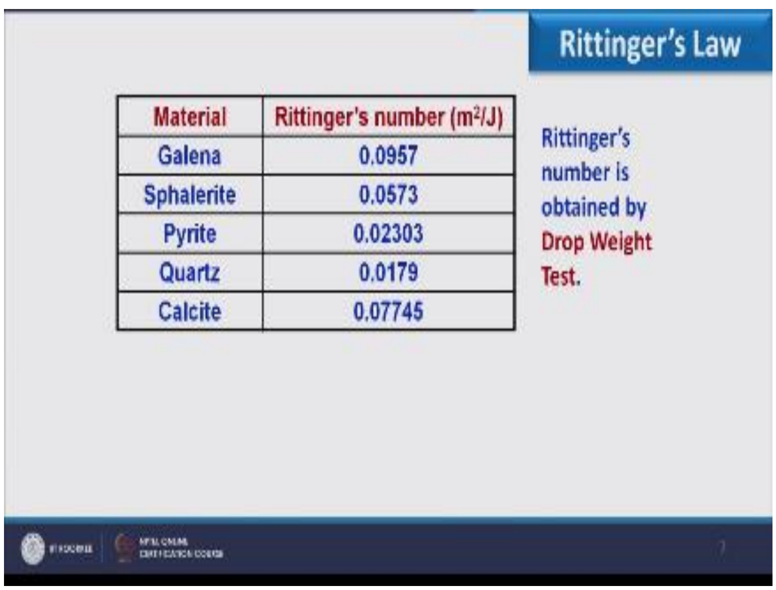

Material	Rittinger's number (m^2/J)
Galena	0.0957
Sphalerite	0.0573
Pyrite	0.02303
Quartz	0.0179
Calcite	0.07745

And we take the weight of known quantity let us say M kg and we drop this weight up to a given height, so from this height if M amount of weight is dropped over here so in one drop it will consume MGH energy, so when we keep on dropping this, this from the same height the energy consumption would be MGH into number of drops. So when we carry out this experiment, so if you consider the Rittingers expression that is.

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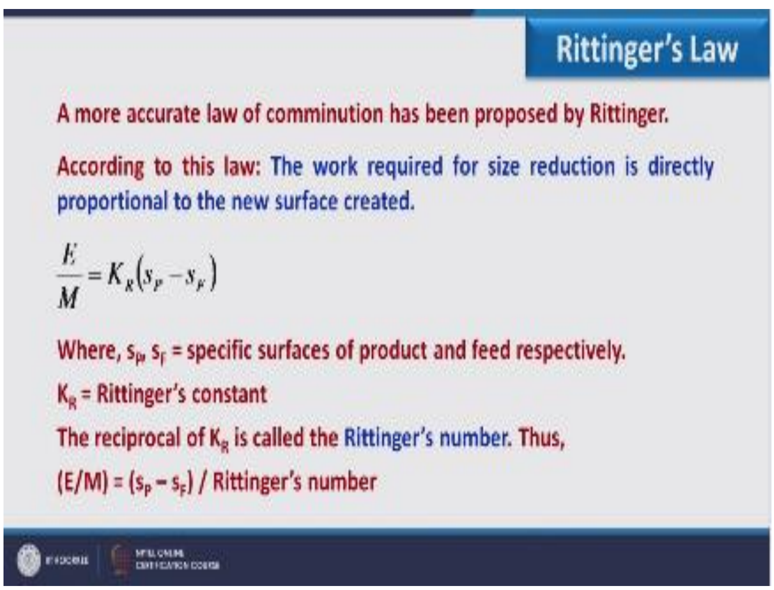

(E/M) = (S_P – S_F) Rittingers/ Rittingers number so using this expression we can calculate the Rittingers number because when we have the continuous drop of a known weight from a given height we already know the energy consumption and we know the surface which is available for feed as well as surface, for surface acquired by the product so that we can calculate by a screen analysis. First we carry out a screen analysis for feed and then we carry out screen analysis for product.

So using these screen analysis data of feed and product we can calculate $S_P - S_F$, that is the new surface created and energy consumption we can calculate as.

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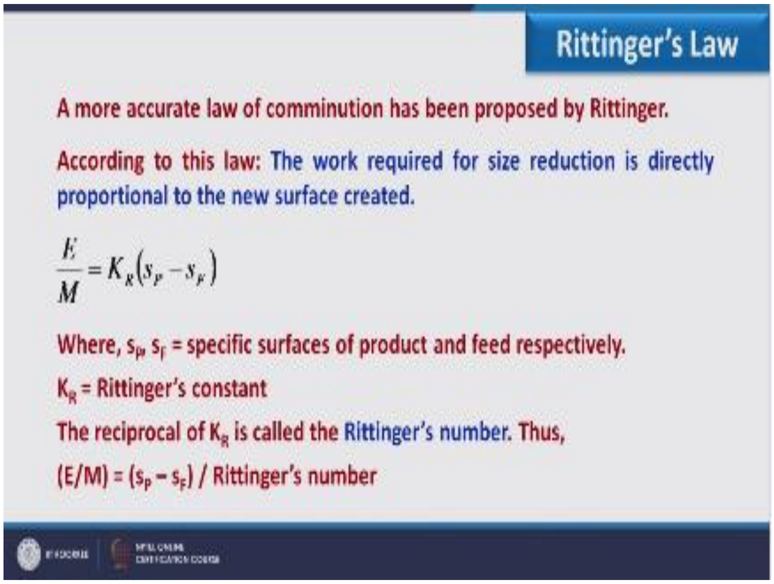

MGH x number of drops so division of these two will give the Rittingers number, therefore the unit is the meter square that is new surface created.

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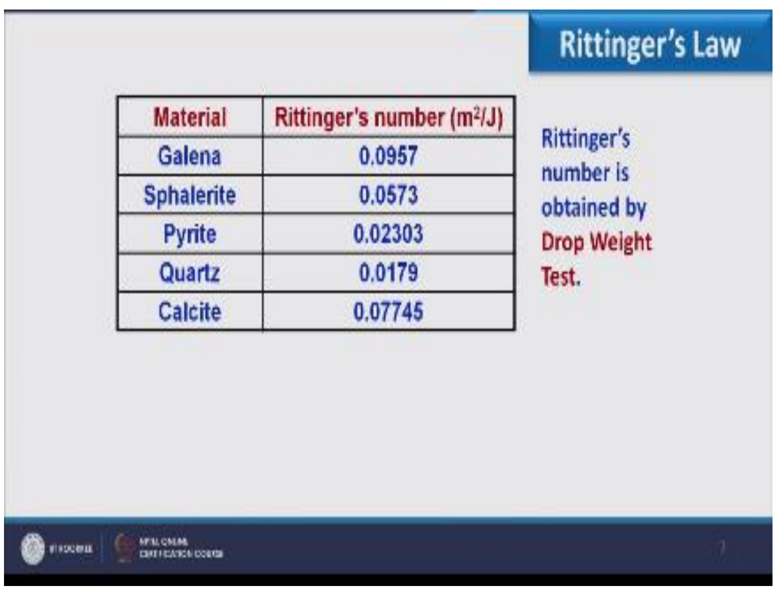

Material	Rittinger's number (m^2/J)
Galena	0.0957
Sphalerite	0.0573
Pyrite	0.02303
Quartz	0.0179
Calcite	0.07745

Divided by energy consumed in Jul. So Rittingers number designates the new surface created per unit mechanical energy absorbed by material being crushed. Large value of Rittingers number of material indicates that it is easier to grind. Now why it is so, because Rittingers number speaks about the new surface created divided by energy consumption so if Rittingers number is more it will consume lesser energy, therefore the grind ability would be easier for that material.

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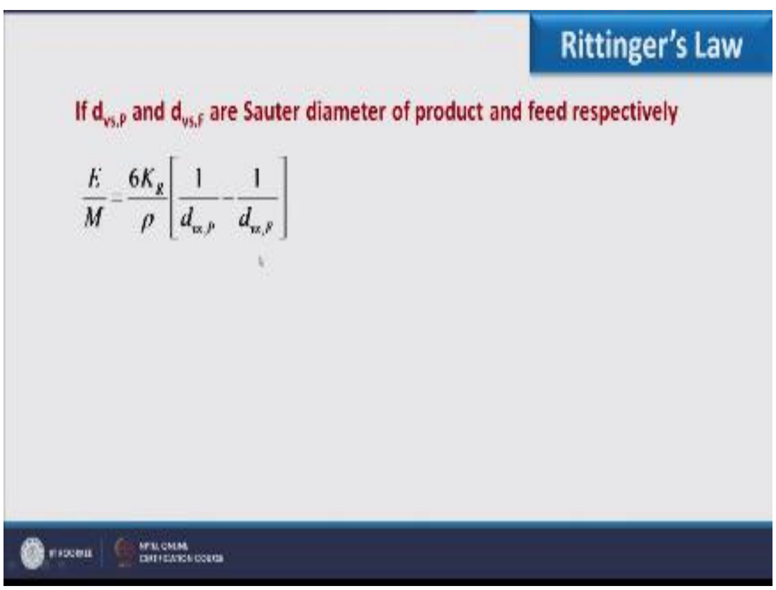

So if I am having d_{VSP} and d_{VSF} as Sauter diameters of product and feed then we can write the Rittingers expression as $E/M = 6KR/\rho\ 1/d_{VSP} - 1/d_{VSF},$ now how we can write this because.

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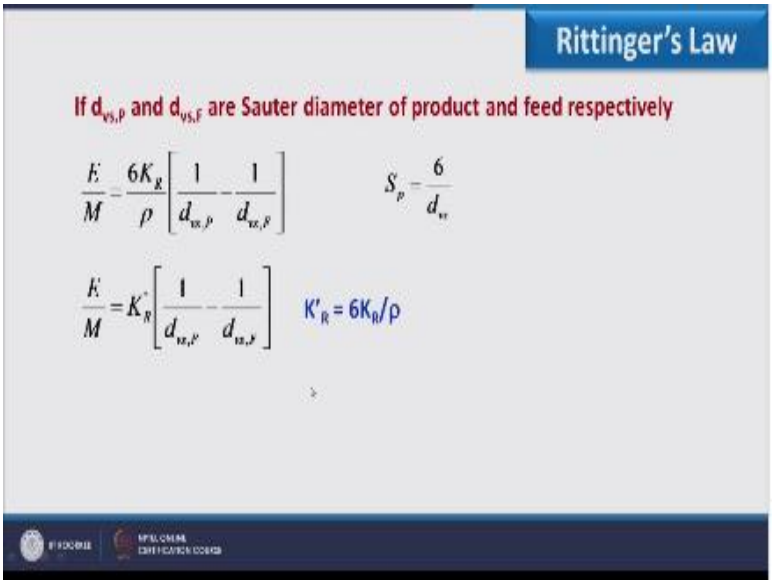

6 over d_{Vs} is the specific surface, specific surface means surface area per unit volume because d_{VS} is the Sauter diameter which is defined as volume surface mean diameter, so 6 over d_{VS} is nothing but the S_P and this S_P is the surface area per unit volume. Now if we consider the

Rittingers law then specific surface is defined as surface area per unit mass, therefore this S_P we have divided by this row so that is the 6 d_{VS} in two row. 6 and row is constant so we have taken it out and 1ƿ d_{VSP} - 1ƿ d_{VSF} will speak about the new surface created.

So when we consider d_{VSP} and d_{VSF} we can write this expression finally where we have replaced 6 K_R/ƿ as K_R – and these are the Sauter diameter of product as well as feed, so in terms of Sauter diameter we can calculate the energy consumption using Rittingers law. Now if you consider the Rittingers law what is the limitation of this law, that the mechanical losses.

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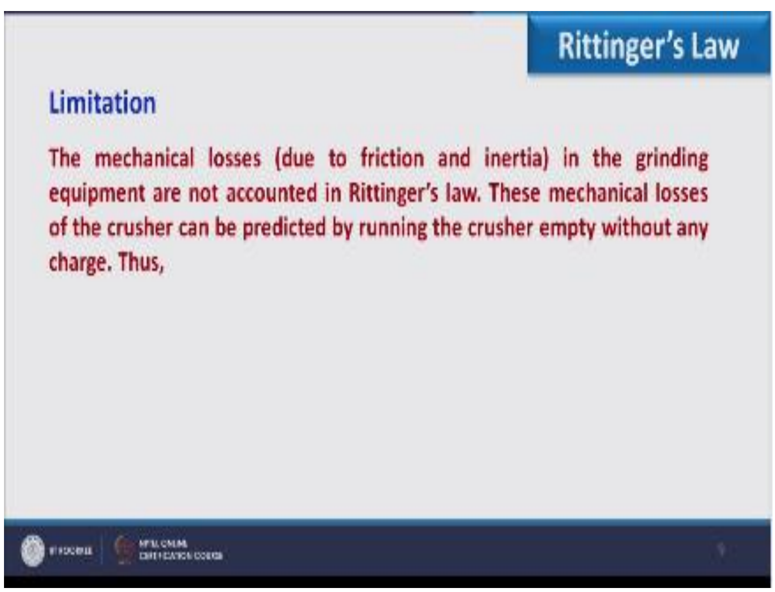

Due to friction and inertia in grinding equipment are not accounted in Rittingers law. Why it is so, because if you consider the Rittingers law it only speaks about new surface created however some of the energy is also required.

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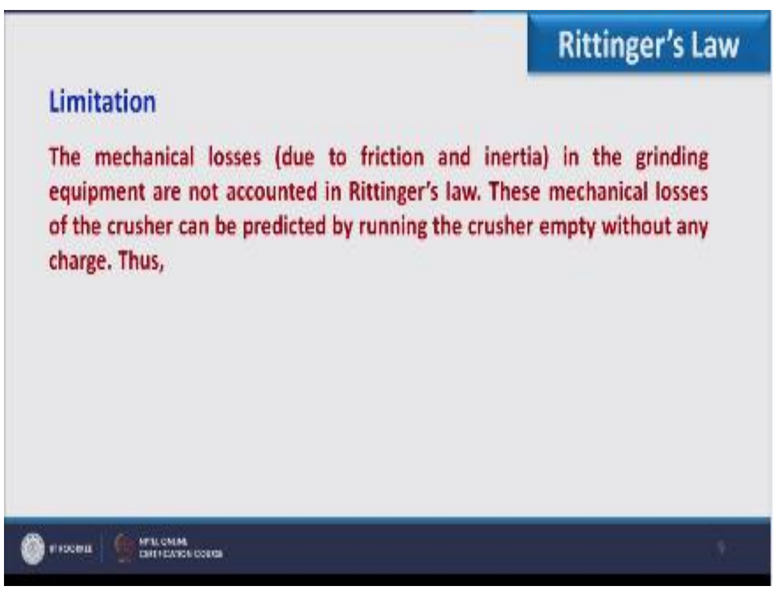

To run the crusher without any material so this we call the mechanical losses, so Rittingers law does not account the mechanical losses and that is the limitation of this law. And here we can define the overall energy efficiency of the crusher and this can be defined as energy required to create new surface divided by total energy supplied. Now what is this total energy supplied, is the energy consumption to run the machine empty plus energy consumption to create new surface, so that is the total energy supplied.

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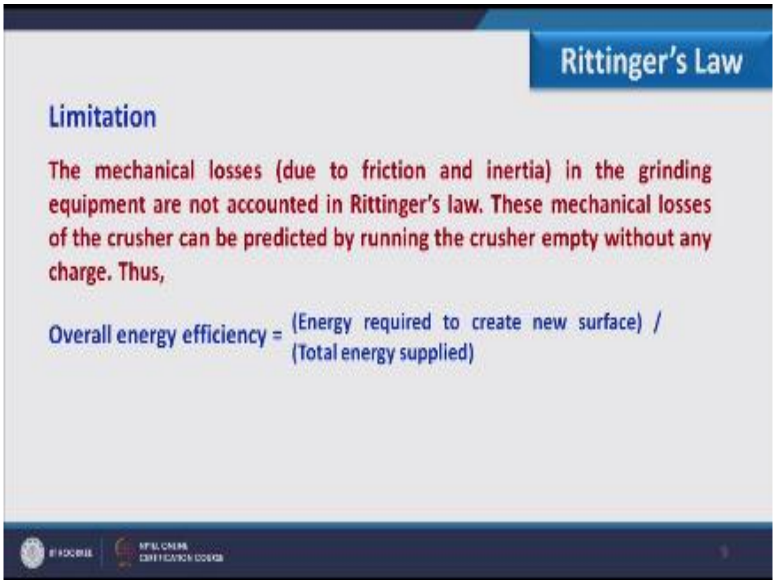

Considering this we can calculate overall efficiency of the crusher, apart from this we have another efficiency and we call it theoretical effectiveness.

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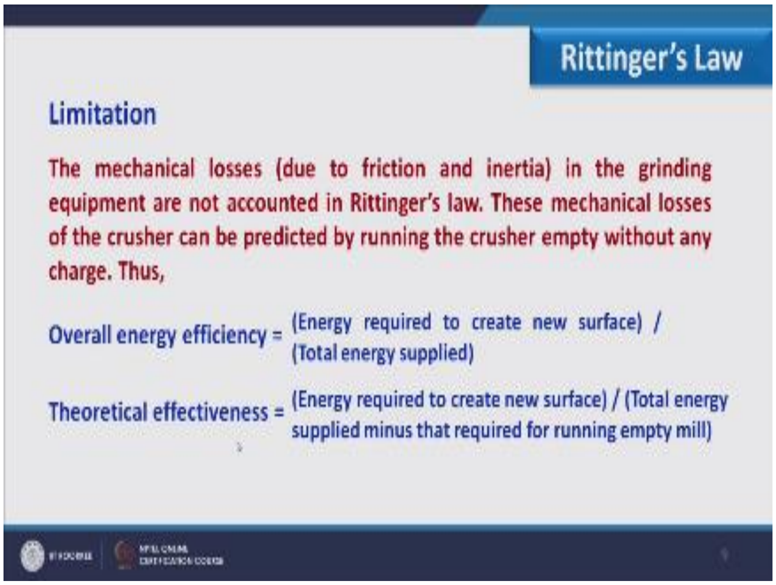

Or theoretical efficiency of crusher, and this we can define as energy required to create new surface divided by total energy supplied minus that required for running the empty mill. So here you see total energy supplied minus that required for running empty mill, it means we have considered only that part of energy.

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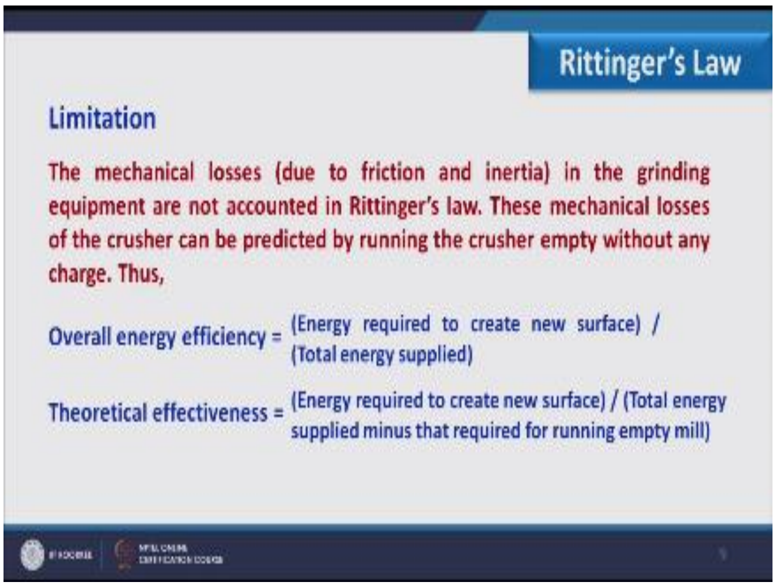

Which is utilized for crushing only, so this is the theoretical efficiency as well as overall efficiency.

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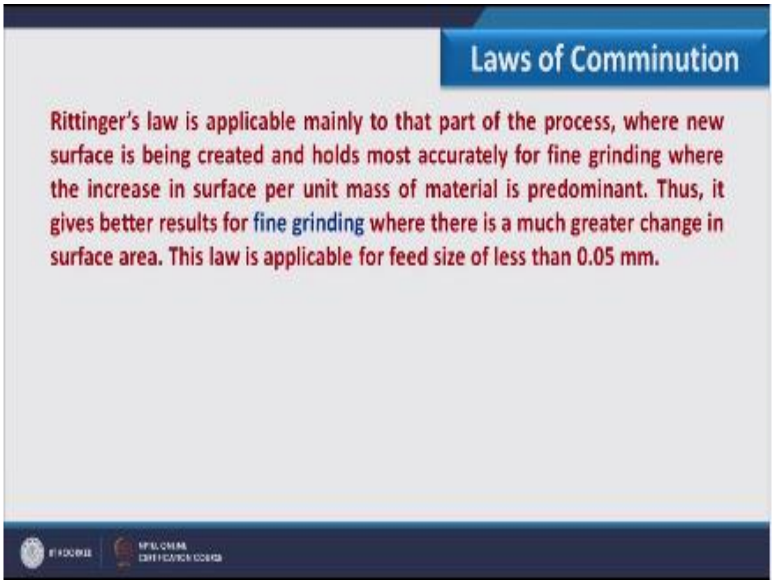

Now Rittingers law is applicable mainly to that part of process where new surface is being created and holds most accurately for fine grinding where the increase in surface per unit mass of the material is predominant. Thus, it gives better result for fine grinding where there is much greater change in surface area. Therefore, Rittingers law is applicable when we deal with the fine material because it has more chances to create new surfaces.

So Rittingers law can calculate energy in that region more accurately. So this law is applicable for feed size of less than 0.05 mm. Now both Kicks as well as Rittingers law have been shown to apply.

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Over limited ranges of particle size, like if I consider Kick's law it can be applicable accurately for coarse crushing where reduction ratio is small, and if we consider the Rittingers law it handles the fine particles to be converted into more fine particles.

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So for this Kick's as well as Rittingers law K_K that is the Kicks constant, and K_R that is Rittingers constant are determined experimentally.

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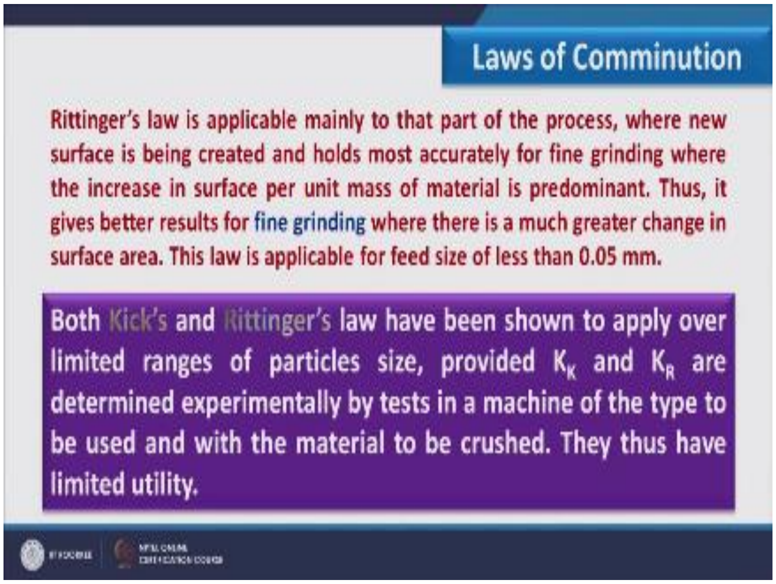

By tests in a machine of the type to be used and with the material to be crushed, they thus have limited utility. An intermediate and more realistic method for predicting power consumption in crushing as well as grinding was proposed by Bond in 1952 and we call it Bonds law.

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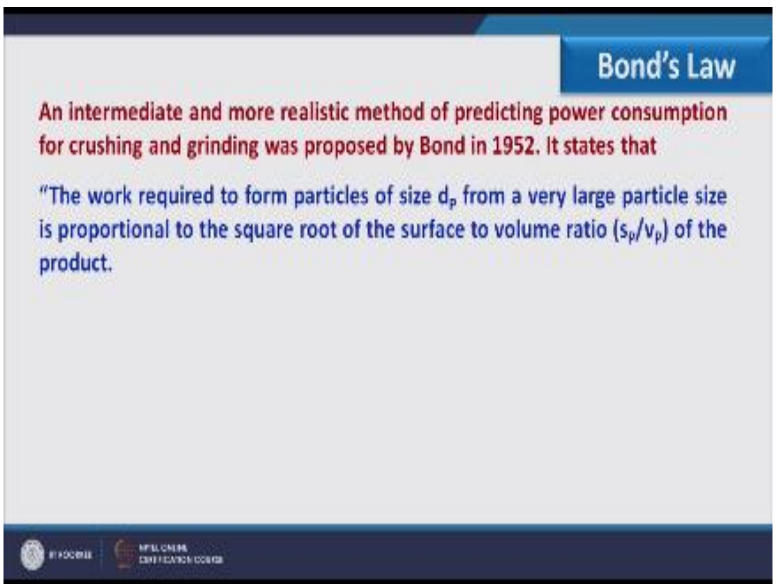

And what is the Bonds law, it says that work required to form particles of size d_p from very large particle size is proportional to square root of surface to volume ratio of the product. So here you see we have surface as well as volume ratio and work required for crushing the feed of large size to the product of size d_p will proportional to square root of surface, that is s_p to volume that is v_p ratio of the product. Now how we can define this, how we can show this mathematically it is, if we consider s_p over v_p.

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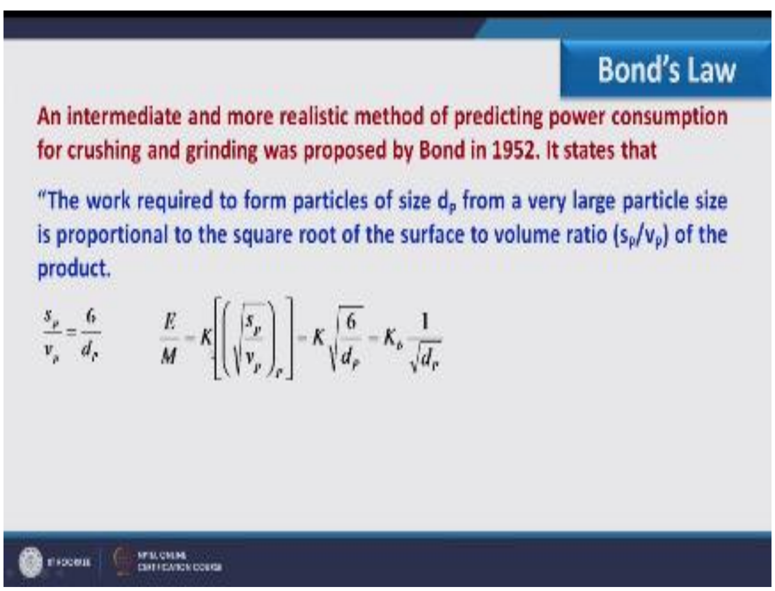

That would be $6/d_p$, now this surface area over volume ratio how we can convert this into $6/d_p$, here if we consider surface as well as volume then we can consider as Sauter diameters so if d_p we consider as Sauter diameter so that here we can have the ratio of surface area to volume and that we can equate with the $6/d_p$. And Bonds law mathematically we can represent that E/M total energy consumption for crushing M unit of material is equal to K and $\sqrt{s_p/v_p}$ of product. So E/M is proportional to $\sqrt{s_p/v_p}$ and what is this K, that we will see later on, further if we resolve this expression that.

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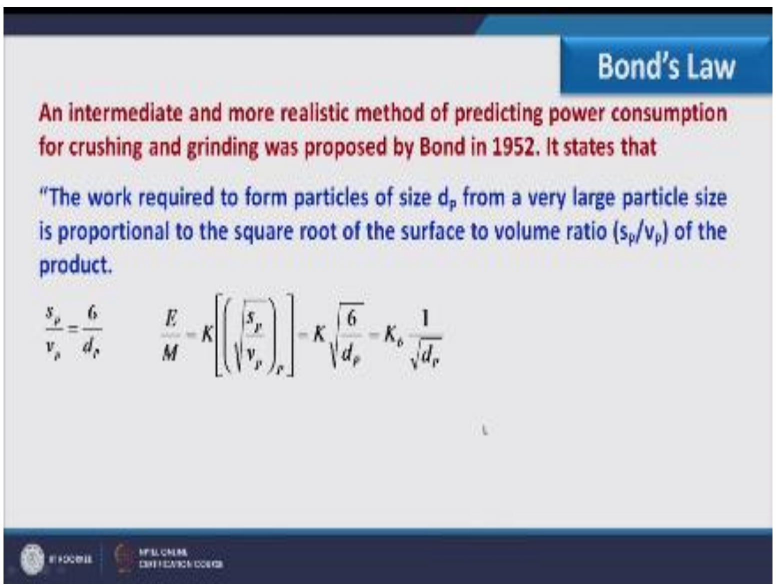

If we write the expression of s_p/v_p that is K and $\sqrt{6}/d_p$ and this 6 we will take out from the square root so we have $K_b/\sqrt{d_p}$. So this is basically the Bonds law where K_b we call as Bonds constant, more precisely we can write the equation as $E/M=K_b\,[1/\sqrt{d_p} - 1/\sqrt{d_f}\,]$.

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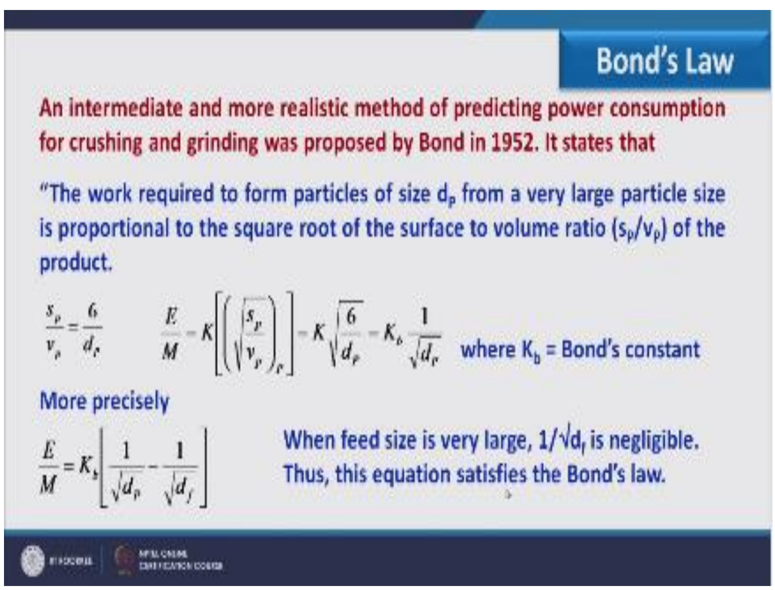

Now how we can include this d_f, if you read the definition carefully it says that feed of very large size, it means when we consider very large size feed $1/\sqrt{d_f}$ is negligible and thus the equation satisfies the Bonds law. So when we deal with very large size of feed we can write the equation of Bonds law in this form considering size of product as well as of feed.

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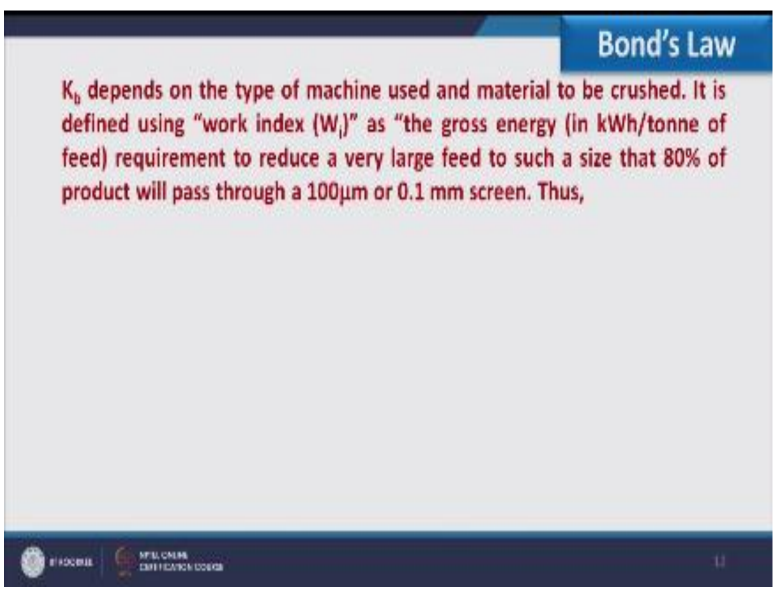

So K_b depends on type of machine used and material to be crushed, it is defined using work index, we represent the work index by W_i and it is defined as, the work index is basically the gross energy requirement to reduce very large feed to such a size that 80% of product will pass through 100 µm or 0.1mm screen. And this gross energy we can define in terms of kilowatt hour per tonne of feed. So what is this gross energy, when we have very large feed size we have to crush the feed size in such a way so that 80% of product will pass through 0.1 mm screen, it means from large to very fine size we can.

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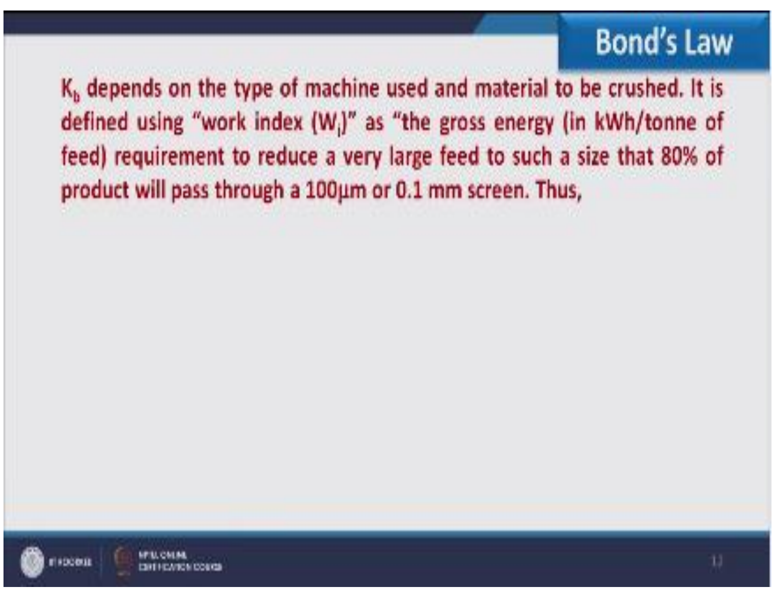

Fine in grinder or in crusher and accordingly we can compute energy consumption using Bonds law. So if I am having very large feed size.

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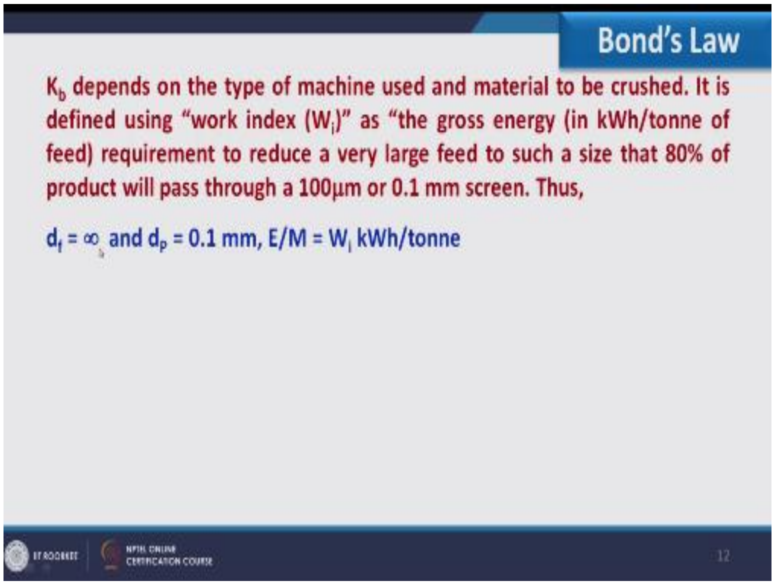

We can consider $d_f = \infty$ and $d_p = 0.1$mm because 80% product should pass through 0.1mm screen. In while doing so whatever energy consumption we are having that would be w_i which we call as work index. If this is the expression here instead of d_p we can right 0.1, however this expression or this factor will become negligible so $w_i = k_b / \sqrt{0.1}$, if we resolve it further then k_b would be 0.3162 w_i. So putting the value of k_b in the expression, the final expression of Bonds law is $E/M = 0.3162\ w_i\ \sqrt{/1/d_{pb-}}\ 1/d_{fb.}$ Now what is this d_{pb} and $d_{fb,}$ this we have defined considering 80% feed particle pass through d_{fb} mm screen and 80% of product pass through d_{pb} mm screen.

So you see in this particular law we have to use diameter in mm only, not in µm or centimeter

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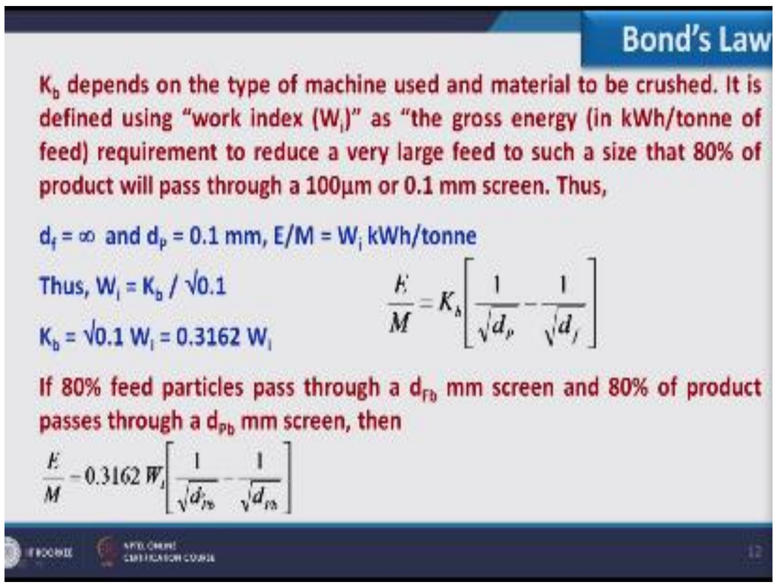

Therefore here I am having some of the values of work index which depend on the material only

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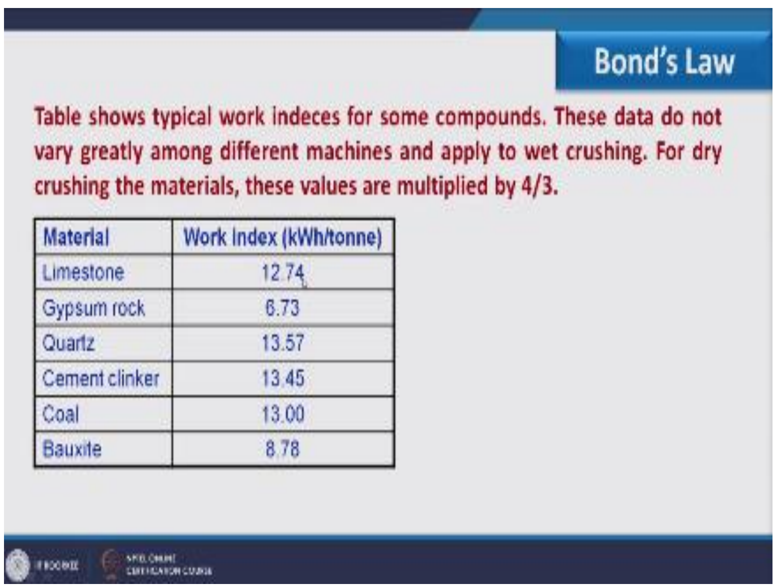

For limestone it is 12.74, for coal it is 13.00 and many other work indexes of material is shown in this table. So these data do not vary greatly among different machines and apply to wet crushing, if we have to handle the dry crushing or if we have to handle the dry material then these value would be multiplied by 4/3. So what happens when we have considered work index work index has been defined with respect to gross energy, gross energy means total energy which is consumed while crushing the process.

So obviously it will include the mechanical energy, therefore Bonds law predicts the gross power consumption in size reduction or comminution

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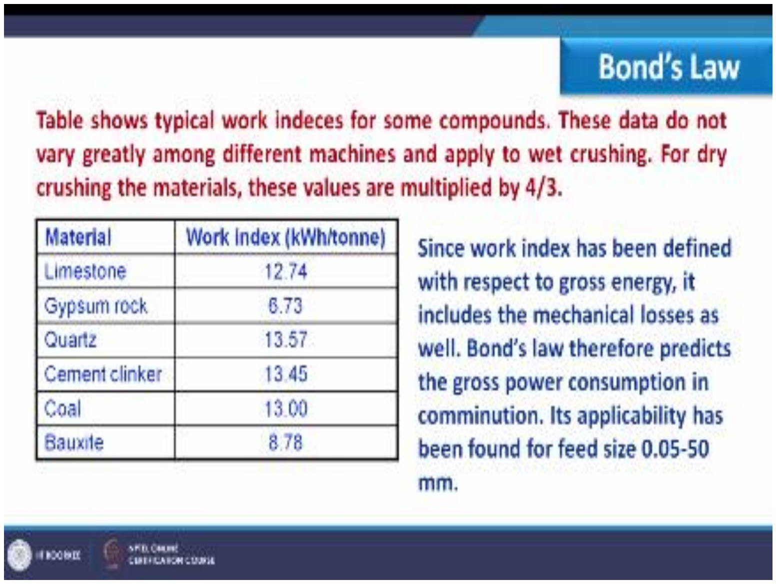

Its applicability has been found for feed size 0.05 – 50 mm. Now here we can represent all laws of comminution using a single equation, if I am having this equation in differential form that is $DE = c\ dd/d^n$

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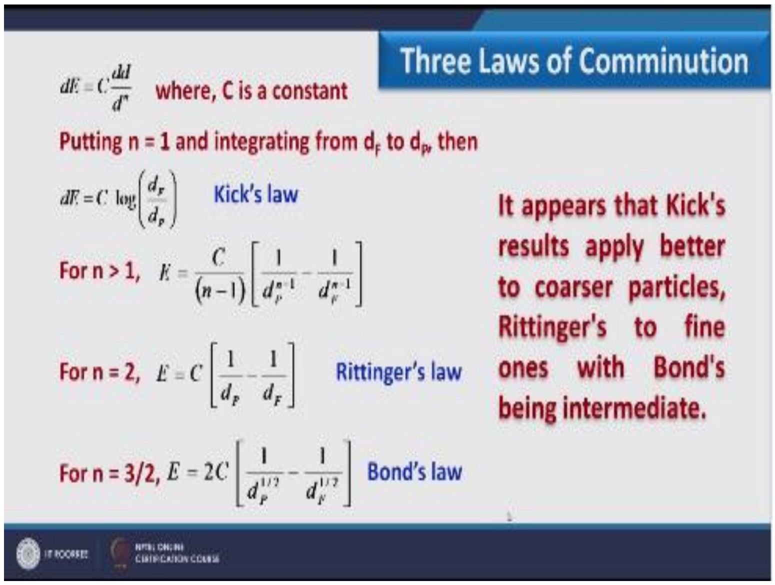

Now this is, this the first d is basically representing the differentiation and the first d is representing the differentiation however second d is the diameter, and here we have this diameter over n where C is the constant. Now when we consider n=1 and integrate the above equation from d_F to d_P we can write the equation in terms of dE or instated of dE I should write over E that is the mistake in this, here I should write E only equal to C log(d_F/d_P) so when we consider n=1 and integrate it from d_F to $d_{P.}$

We can have the Kicks law expression if value of n is greater than 1 and we integrate this particular expression we can have E=C/(n-1) [1/d_P^{n-1} – 1/d_F^{n-1}]. If I use n=2 in this particular expression then finally we have equal to C[1/ d_p^{n-1} -1/ d_f which is nothing but the Rittingers law and when we consider n=3/ 2 in this expression then we can get the Bonds law, so you see using the single differential equation we can represent all three laws of comminution. It appears that Kicks law results apply better to coarse particle, Rittingers to fine particle and Bonds to intermediate, so we have three different law to calculate energy consumption in three different region. As clearly shown by this graph.

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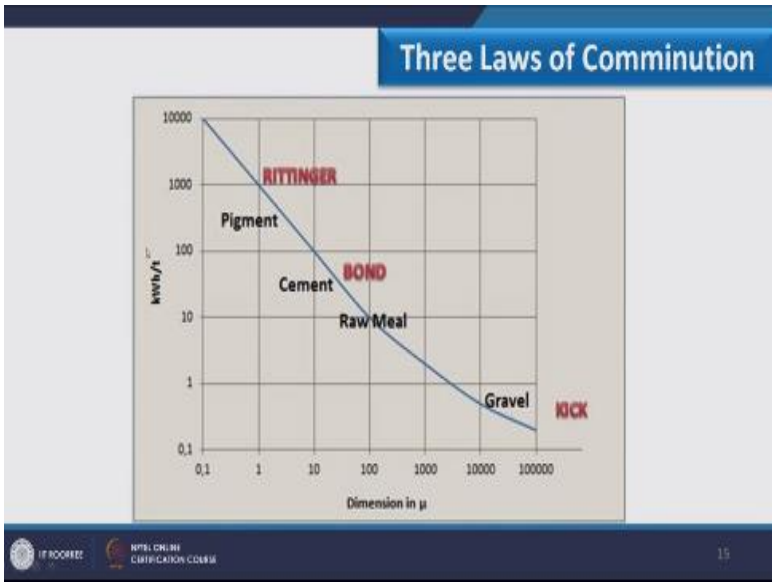

Here we have the energy consumption kwh/t and here we have the size so if you see the coarser size like gravels etcetera we can calculate energy consumption in this by Kicks law. If we deal with the cement or raw meal we can use Bonds law because it is applicable at intermediate size and if we are dealing with pigments or very fine particle we can use Rittingers laws, so these are the zones of applicability of three laws of comminution.

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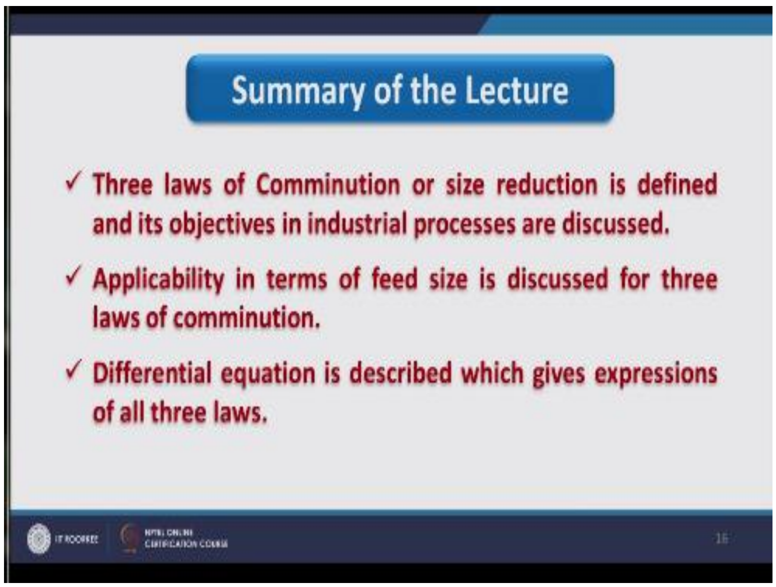

And here we have the summary of this lecture and in this particular lecture we have discussed three laws of comminution or size reduction along with its objective in industrial processes. Second, secondly we have discussed the applicability in terms of feed size for three laws of comminution, that is Kicks laws is applicable for coarse crushing, Rittingers law is applicable for fine grinding and Bonds law is applicable for intermediate feed size, and finally we have described all these three laws using a single differential equation while changing the parameter while changing the n value of this differential equation we can get all three laws.

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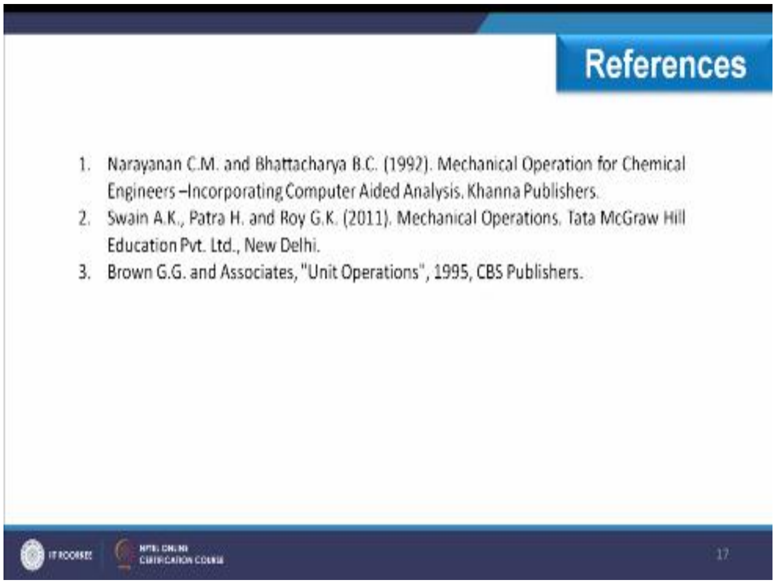

So these are the references for this particular lecture and that is all for now, thank you.

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