Environmental Quality: Monitoring and Analysis
Prof. Ravi Krishna
Department of Chemical Engineering
Indian Institute of Technology – Madras

Lecture – 49
Overall Mass Transfer Coefficient

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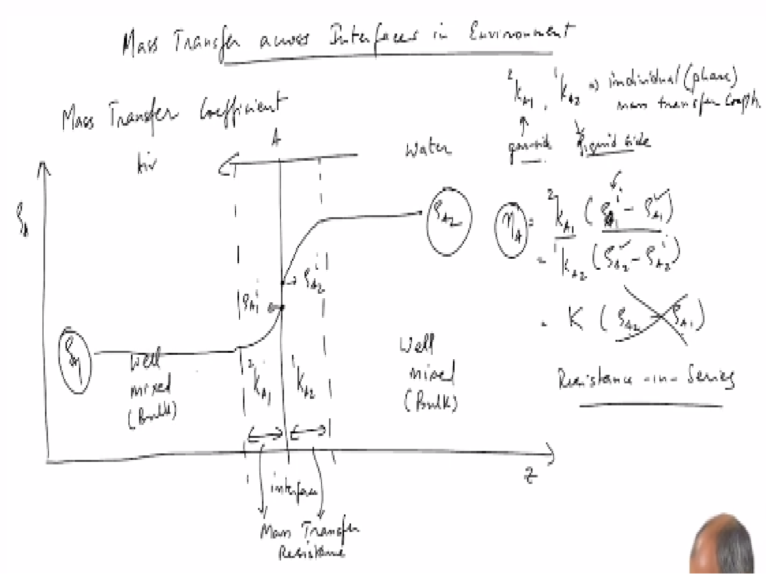

So, we were discussing mass transfer across interfaces. We were discussing the general principles of mass transfer coefficient. So we defined something called as mass transfer coefficient and its dependency on the flow, the properties of the fluid and properties of the solute itself to some extent. So we stopped at a point where we were looking at transfer across an interface. So we will just recap that. So let us say there is an interface between two phases, let us say this is water and this is air.

If mass transfer is happening from water to air, the transfer of A is going from water to air, which means there is a gradient from water to air. So, we also discussed that close to the fluid interface, it is convenient for us to assume that there is a region of mass transfer resistance. So the thickness of this region depends on and so the rest of the region is considered as well mixed or we call it as a bulk and this is also well mixed or we will call it as bulk. So by definition, when we say well mixed, the concentration here is rho A2.

It is straight, it does not change. So you are drawing the scale of concentration on this and on the x axis is some kind of a length scale. It is notional, it is not an accurate scale. We are not going to give any numbers to it at this point, and then there is a gradient that applies within this region and so this point, we call it as rho A2 interface and then corresponding to this rho A2i, there is another number here on the air side and there is a gradient that goes in to this region. So, what this suggest is all the mass transfer resistance is in this phase.

So, if there is mass transfer resistance, there is also a mass transfer coefficient kA1 and then there is a mass transfer coefficient kA21. This is the definition of the mass transfer coefficients and these are called as kA12 and kA21 or individual phase. It is individual phase mass transfer coefficient or individual mass transfer coefficient. So this is the gas side and this is the liquid side individual mass transfer coefficients. So we also defined that we said the flux can be defined as kA12 into rho A1i - rho A1 which is also equal to kA21 into rho A2 - rho A2i okay.

So, this is a quantity of our interest. So, if we want to predict what is the flux, we need to know this and this and the problem sometimes is we cannot, we do not know what this is interface, not sometimes, always interfaces. When there are 2 liquid phases or 2 gas and liquid phase, this interface concentration is very difficult to estimate or unreliable. So, we need something. What we can measure though are these two numbers. So, What we would like to do is something like this, some proportionality constant.

Equivalent mass transfer coefficient for this minus this, but this is an illegal representation because transport, when we are talking about these kind of things, there is a continuum and there is a discontinuity in here because there is a phase in between, there is a phase boundary in between, that is the discontinuity. So, you cannot write these equations without invoking the equilibrium in between. So, I cannot simply write rho A2 – rho A1, as long as I can do this only if it is in one phase, there is a continuum. Because there is no continuum, there is a break there.

So therefore, we cannot do this. So how do we work around this one? So we invoke what is called as a resistance in series approach to do this, okay. So the resistance in series, what it says is I am trying to relate this to this through an equation in the system.

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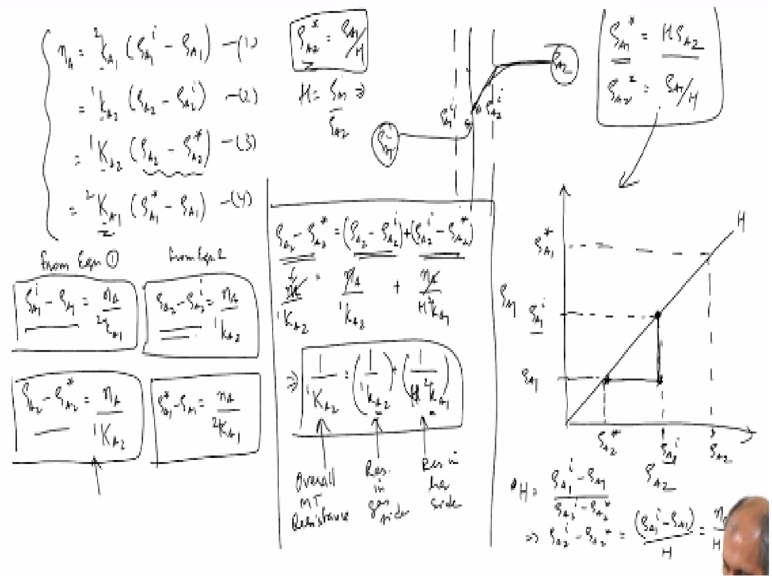

So I am going to write the terms here. So, what we are saying is that if we cannot write the two terms directly, what we will write instead is something like this we will write. A driving force, which is essentially rho A2 minus an equivalent concentration liquid state concentration which represents the gas state concentration and similarly I can write rho A1 star minus rho A1 and I can write another term here. So, notice that this is small k, this is individual mass transfer coefficient.

So, here what is rho A2 star? Rho A2 star see here we are what we are saying is the interface this is rho A2 and this is rho A1, sorry this one. We know that these 2 are in equilibrium. Yesterday's class we said that rho A2i and rho A1i are in equilibrium with each that is the assumption that we can make. If we can make extended assumption to this one, so what we are saying is that representation of this in terms of this can be through the equilibrium. So one relationship that we have is say the Henry's constant okay.

Henry's constant equals rho A1 by rho A2 which are in equilibrium with each other, right. So, this implies that rho A2 star is rho A1 divided by H by invoking the equilibrium relationship. So what this says is there is a number, an imaginary number rho A2 star, which is this rho A1 whatever is in equilibrium with rho A1 okay. So what we are saying is there is some number somewhere, we do not know where, we cannot mark it on this plot because it does not exist really in reality, but this number represents the gas state concentration.

Similarly, other side we are talking about rho A1 star. It is H into rho A2, should be looking at the other side, okay. So, there is a equivalent concentration on the gas side that is in equilibrium with whatever is there in the bulk liquid side. So, these are the bulk liquid concentrations. So, what we are doing is we are circumventing the interface concentrations by attempting to do this okay. So here we have rho A2 star equals rho A1 by H, these 2 are the relationships, we will keep it aside for the time being.

So what this means graphically is the following. So we will draw this here. I am going to draw this here. Can you see this, legible enough, yeah. The rho A2 and rho A1 and this is my equilibrium. This is my Henry's constant curve, this is equilibrium relationship, Henry’s constant is we are assuming is constant this one temperature, this is linear relationship. So, we mark 3 concentrations here. We will mark rho A2, which is the bulk phase concentration here and then we also mark rho A1, which is a very low concentration on the gas side.

We also mark rho A2i and correspondingly if you mark rho A2i we mark on this graph, we mark rho A1 A because we have already established that rho A1i and rho A2i are in equilibrium. So, the intersection, this point lies on the equilibrium curve. What we are now saying is that corresponding to rho A2, there is an equivalent concentration here which is called as rho A1 star and corresponding to rho A1 there is an equivalent concentration which is called as rho A2 star.

This is what it means, what we have said here this rho A2 star, rho A1 star are expressed on this curve here, okay. So now we can use these relationships. So based on this, see we have equation 1, 2, 3 and 4, four equations here. So, from equation 1, we get two and from equation 2, we get rho A2 - rho A2i = nA by kA21. Then from equation 3, we get rho A2 - rho A2 star = nA by capital KA21 and likewise we get the last equation here, rho A1 star - rho A1 = nA by capital KA12 okay. So we will pick one of these equations.

Let us pick the left one, first one here. So we say nA by rho A2 - rho A2 star, yeah. So, we need a term to represent the right hand side here. So, the derivation of this is like this, rho A2, rho A2 star i, we subtract and add rho A2i on this equation okay. So, this term here rho A2 - rho A2i is here okay is by kA21. On the left hand side, this term here is this here nA by capital KA21. This term here from this graph, this term is rho A2i rho A2 star, which is this, this gap here.

So, this gap here based on this Henry's constant equals the slope of this segment here is rho A1i - rho A1 divided by rho A2i - rho A2 star, which means rho A2i - rho A2 star = rho A1i - rho A1 divided by H. This number rho A1i - rho A1 is this term here, which is nA by H kA12. Now you cross out all the n's, now we get 12. What this gives you, you can see that this term on the left hand side, this is called us the overall mass transfer resistance is the sum of this resistance which is in the liquid phase plus this resistance in the gas phase.

This is the individual resistance in the liquid phase and this is individual resistance in the gas phase, but there is an equilibrium term sitting here, so this takes care of that okay. Correspondingly, you can also write an equation for kA12, you can derive it in the same manner and you will get a similar equation okay. This is a very useful representation. So, this resistance is a series, this resistances in the gas side, resistances in the liquid side okay.

Because whenever you have 2 different mediums or 2 different, for whatever reasons, medium for different properties, you can add up the resistance in order to get the mass transfer flux. So, it is like in heat transfer, we have different slabs with different conductivities, you have different transport rates. So, it is a composite resistance in series, but you have to understand that all of this is a steady state system okay, which means that is the reason why this is true. Because it is steady state, they are all equal.

What it means is that the overall rate of transport is a combination of these 2 individual resistances. So if one of them is smaller, that controls the rate at which the overall process is moving.

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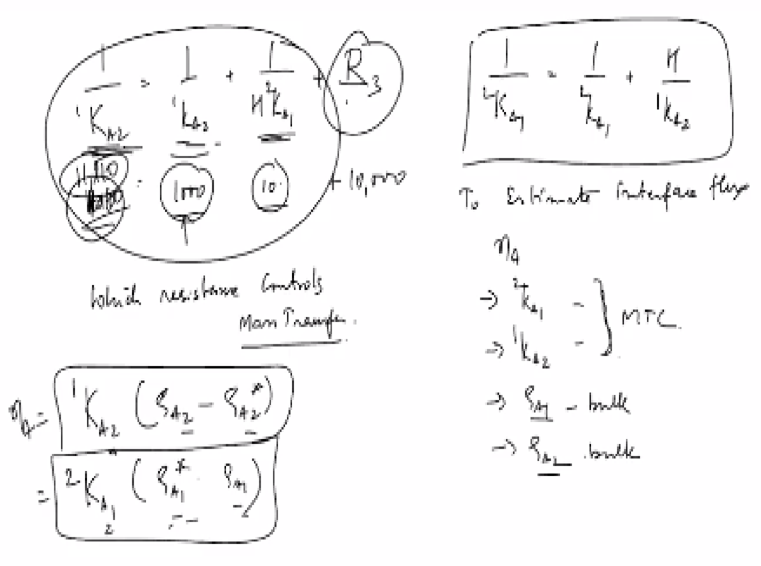

For example, in this case if I calculate 1 over KA21 = 1 over kA21 + 1 over H kA12, it turns out that this resistance is 1000 and the resistance is 10. The overall resistance is 1010. So it is predominantly dependent on this, it does not matter, what this means is that in the gas side, there is no resistance, resistance is very small, but it does not still matter because this is controlling how much is going across from one end to the other end, okay, yeah. So there are different analogies that you can think of and I would like you to think of different physical analogies that this is true and vice versa and so.

Sometimes it is possible that the resistances are almost equal. So, what this means is this analysis is very useful because now you can determine which is the controlling, which resistance controls mass transfer because that will tell you if you need to engineer the situation somehow, you need to reduce the mass transfer flux or increase. In our case in the environmental problems, we are always interested in reducing mass transfer, we do not want chemicals to get across the interface from one or the other.

In your chemical engineering application, sometimes we want to enhance mass transfer. So, you would like to do something to increase this resistance or decrease the resistance. So, here we would like to do is increase the resistance. Once I have increase in resistance, I will add one more term here. I can add another term, which means that there is an addition, another term with another resistance, I will call it R3. Let us say the R3 resistance is 10,000, it is more than these two phases, which means that this now controls the overall mass transfer.

This now becomes 10,000, 1111 whatever 10 okay. So this is useful in this case. **"Professor – student conversation starts."** What is this term capital KA2 to 1kA2 and 2kA1. No. no, you are talking about this term and this term. No, other. This term. This term and this term. No sir, capital K. Capital K21 and small k21, yeah. No sir, capital 1KA2, 2KA1. Yeah, so this is see the kA1, you are talking about the capital, you are talking about the difference between, it is the same difference, this capital KA1 and capital KA2, if you are using capital KA21, the flux you will write it as rho A2 - rho A2 star.

If you are using this, you will write it as rho A1 star - rho A1. Depending on this number, the concentrations that you are using in the driving force changes. Yeah, we will do some examples. **"Professor – student conversation ends."** This is a representation of, you can use either of these equations, you can use any one of these depending on what is convenient. I mean, both of them should be the same. Now, now clear. It is just the way we have defined it, depending on which number we can take, if you want to take the equivalent of the liquid phase or the gas side.

So this is also true for other systems okay. Any questions in this? So, the other equation is KA12, if you derive it in the same manner in which we did, it will be kA12 + H by 21, it will turn out that way. So, this is the equation for. So, this is the preferred method when you have an interface, because then all you need to estimate interface flux which is nA, you need the calculate small kA2, these two, rho A1 and rho A2. These are the bulk phase concentrations and estimate of the mass transfer coefficients, both the mass transfer coefficients.

So, the information that you will need is whatever is needed to calculate the mass transfer coefficient. Now, how do we estimate the mass transfer coefficient? So, this is the problem and if we want to calculate n, we need these 4 numbers. These numbers may be given to you in the problem saying that you already know by measurement that you know say there is a lake which is polluted with some chemical, you measure it and you know the concentration and then you measure the air, ambient air, and you know what is the concentration of this chemical there?

So, you know both these numbers okay. Then you need to estimate the mass transfer coefficients, then you can get whatever is the flux across in this scenario. How do you get the mass transfer coefficient? So, the mass transfer coefficient as we have seen already is a proportionality constant, so there is no priori except for one or two different systems. There is not any derived equation for mass transfer coefficients.