Environmental Quality: Monitoring and Analysis
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Lecture-39
Dispersion Model Parameters - Part 1

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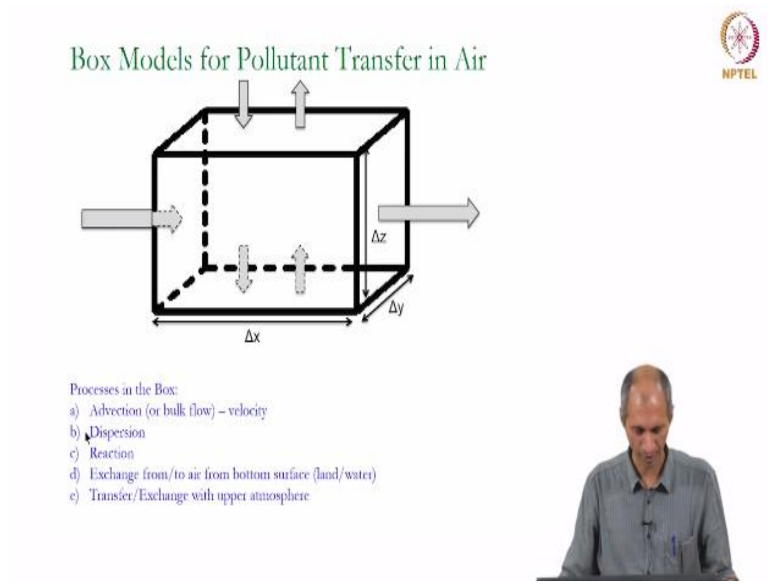

So we just recap just to get a speed we have been away for a while, looking at box models for pollutant transfers in air. So essentially this is generic box model for air we are looking at these are the processes that we are considering in the box. Advection, dispersion, reaction, exchange and all that.

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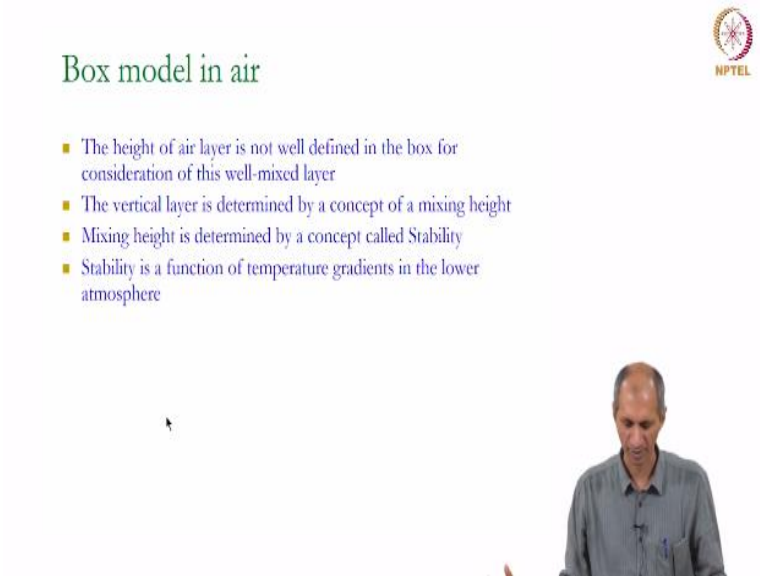

So the specific problem for air is that the height is not very well defined, so we look at what is called as a mixing height and mixing height depends on concept called stability and stability is function of temperature in the lower atmosphere

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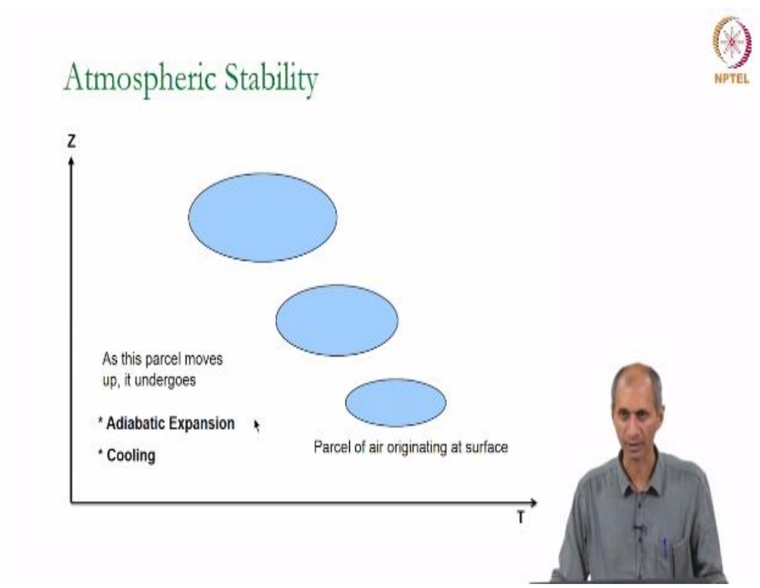

And we looked at stability at this, stability is the behavior of an air parcel when it originates somewhere in the near the earth surface and then it travels upwards and then what happens to it, so the ideal case of that is called an Adiabatic Expansion, cooling as it goes up.

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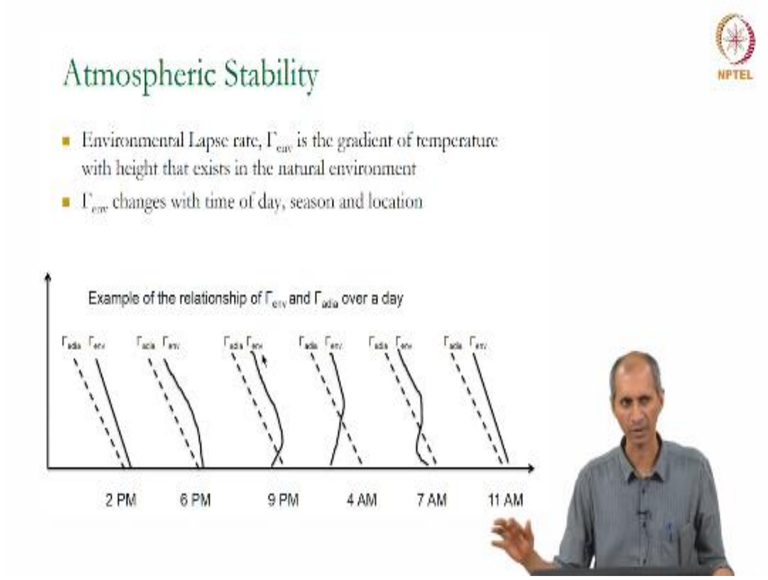

So this is a summary of that. So atmospheric stability is the behavior of a parcel in conjunction with an environment, whatever is environmental lapse rate or the temperature variant in the environment that exists and any point at time. So this figure shows the change of environmental gradient. One example this may not happen all the time depends on the place and time of the season, but adiabatic lapse rate form a particular process stays there.

It does not change, the exhaust temperature of the ground is wherever, whatever it is temperature and of the emission of an exhaust or whatever process that is happening.

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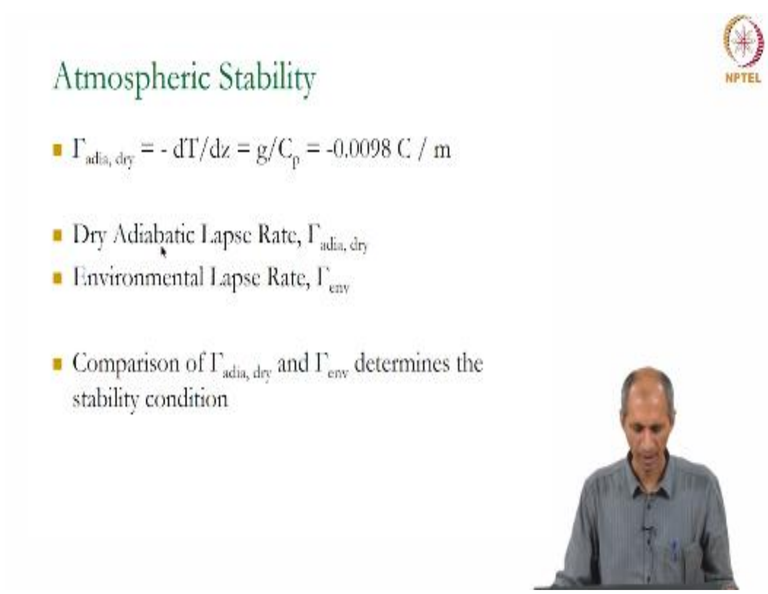

The atmospheric stabilities by lapse rate by Gamma the adiabatic lapse rate is given as -0.0098 centigrade per meter or 9.8 centigrade per kilometer this is a dry adiabatic lapse rate.

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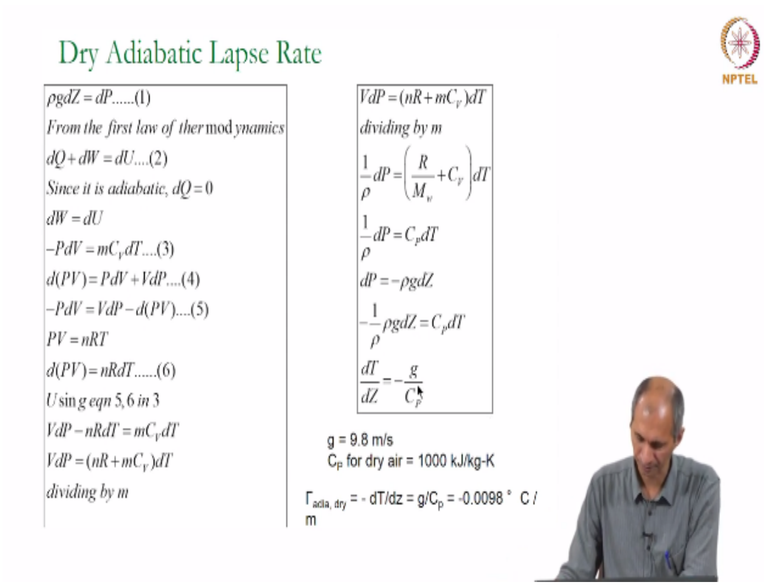

And this is derivation of that its there in many textbooks nomenclature various but we are trying to, we can go through this is basically two things one is the static change in pressure is rho g dz this is static pressure definition and this first law of thermodynamics which means it is an adiabatic process, which is d q is zero d w = d u, so minus of Pd V equals m Cv dT is the ideal, the ideal gas and all that.

We can go to the derivations mainly straight forward and then we use the definitions of cp, cv and all that. We come to this point and then we come down here. You get dT by dZ equals minus g by Cp. Cp is the specific heat of dry air. So if you insert these values here, you get - 0.0098 but the actual and this also has assumptions that when the path is moving up, there is no heat transfer.

It happens very quickly, so there is no let say assumption of adiabatic this thing any way. Then it is insulated, so there is no effect of there is no heat transfer from the surrounding things points. There only bow and see effect that is all. So the density is changing based on the air.

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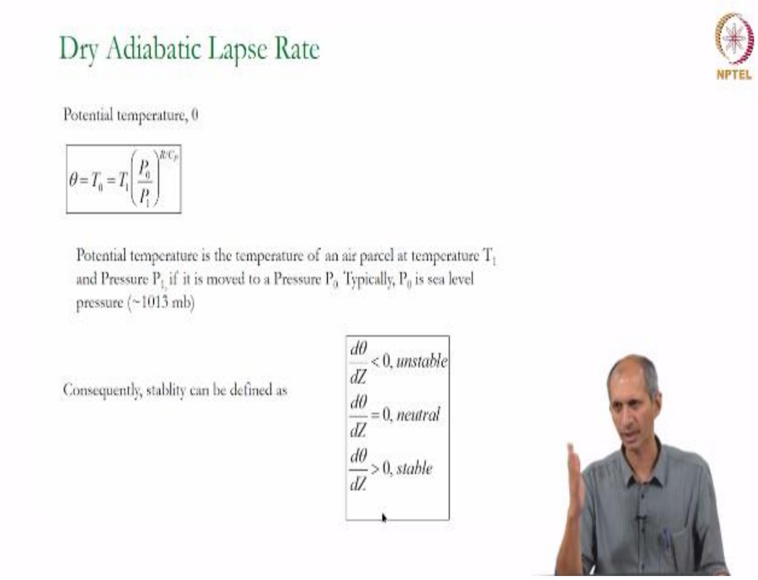

There is another term called potential temperature with defined like this theta equals T0. This is the temperature corrected to particular pressure, pressure with reference to sea level pressure. Temperatures of an air pass that temperature T 1 and pressure P 1, if it is move from to pressure P2. So it is similar, it is corrected temperature. So, just like dt by dZ you can also called theta by dz and the definition are given, so these are the more.

Theta is more normalized way of handling it, if you see many textbook theta are rather than temperature, but for practical considerations and temperature variant is what you will be looking at, when you look at the heat flux. So you look at that way or this way the temperature variant immediately tell you whether it is inversion or not an inversion and what is the value of it?

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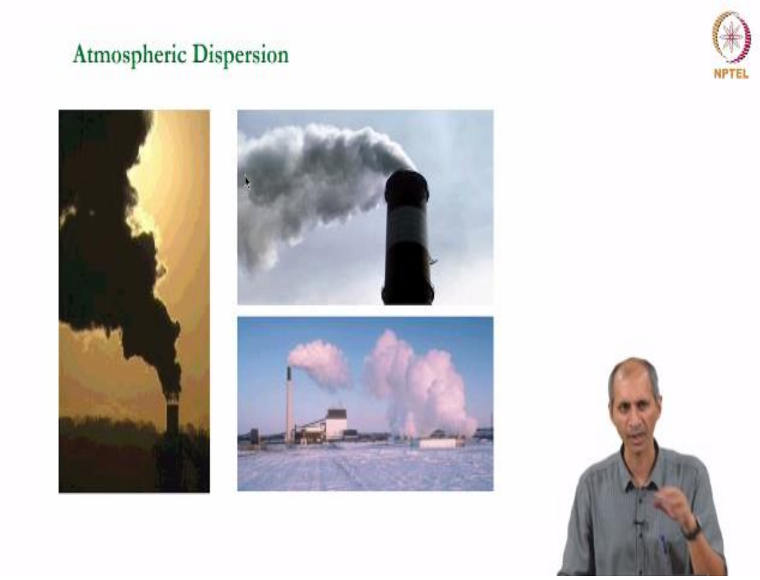

We did all this to define stability. We also looked at these concept of mixing height mixing height as the function of place where the intersection of the environmental airplane and adiabatic happens. As we also defined that this is plume the boundary of the so you can say within this a lot of things happen plume may go in and out but something called as a time average plume.

So you keep looking at it for a long period of time. There is shape at which emission take place and that is called as the plume.

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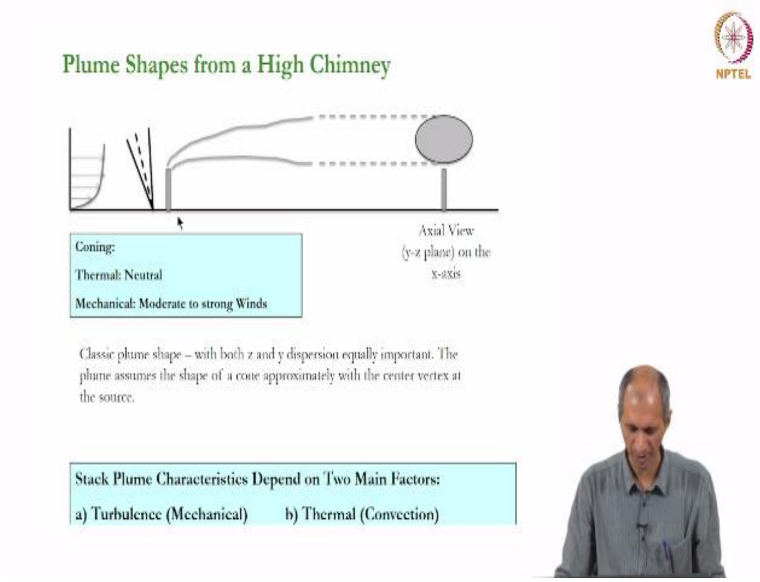

We looked at different shapes. So this is we designed, we drawn again to explain some other things for each of these different types of plume shapes that can. This is a limited set of plumes here, you can have a large number of plume shape based on various combinations of the environmental lapse rate and the plumes and the source high and all that so this is one set of conditions, but if you know how to basic fundamental aspects behind it you can predict what is kind of, what is the plume shape that can expect for a given situation. So there are different kinds of plume shape that you can expect.

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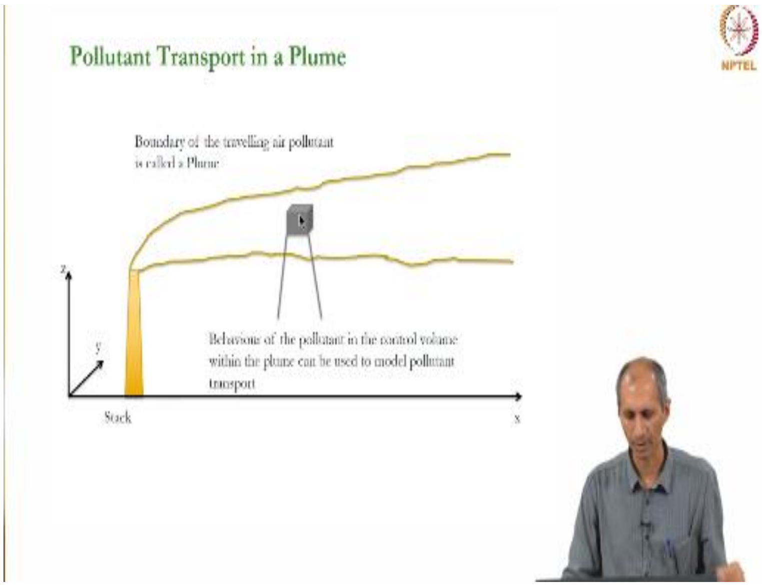

So last time when we were looking at we are looking at the pollutant transport, our goal is to be able to predict concentration are the function of phase and time x, y, z and time. So we look at one control volume within the plume, it is where the pollutant is moving and we try to model it.

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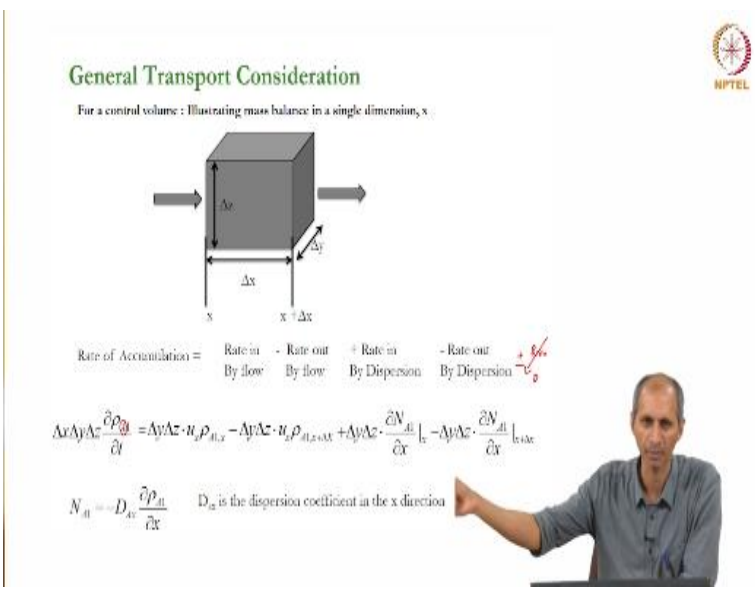

So this is we did this last class we will go over this one again. So take this box just dimension of delta x, delta y, delta z, we have this term here rate of accumulation equals rate of in by flow or rate out by flow, rate in by dispersion, rate out by dispersion. These are the process we are identified before we write the equation we have to figure out we have to determine what are the processes that we are considering in this system.

The transport model can have anything the generalized transport model will also have reactions. We also have adsorption, we will also have dispersion all these things will happen this multi-phase model but we are not doing that here we are looking at only rho A1, so rho A1 is gas vapour phase concentration only we are only looking at rho 3 one which is particulate matter we are looking only rho A1.

So, but this gives you an idea as if you want to do a very complicated model to start here. In this thing you have processes here, so I can add other processes here in this equation and then from here you derive the differential equation that you need to solve. So solution algorithm equation is a different issue that is a mathematics part, make the mathematics solution easier we make more assumptions and make it easier you know.

So that is different sections, from this point of view this class point of view you need to write this equation and then somebody else can solve it. That is not an issue. You need to understand what are these processes? And how they happen whether they are important or not. So rate of accumulation rate by flow, so what we mean by flow is wind is bringing it and taking it out away. Dispersion is the different process.

Dispersion is happening because of beyond sea, because of conviction, wind rewind conviction of anything else so these two are the processes which we have said, so you can add other reactions and all that here, but this case we are going to assume this to be 0. Do you write the balance always for one component? So in this case, we are writing it for this particular A. In the phase that we are interested in, the vapour phase only and this is the phase we are interested in calculating the exposure at some point.

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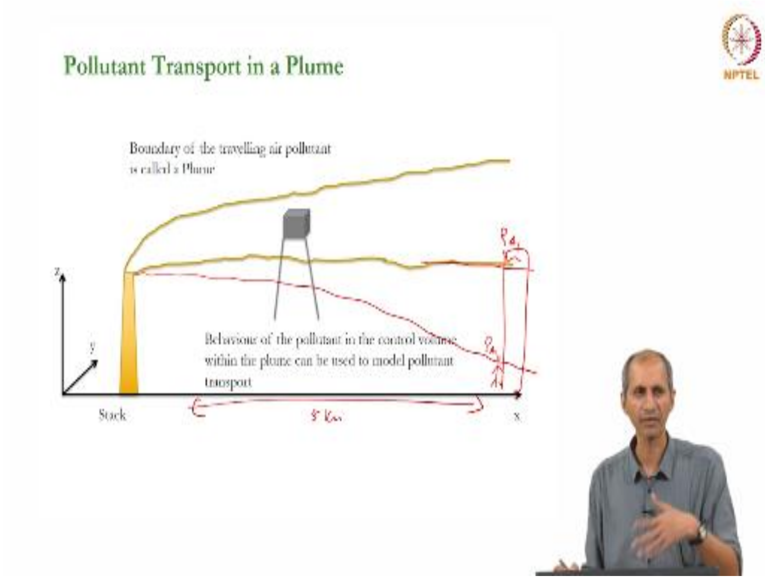

But we are interested in this case is that say, I am interested in this person standing here on the ground and what is the concentration that is being exposed to, so from this point of view I would like to know if this plume is going to travel to this person standing at a distance of 5 kilometers or someplace, some distance or there is a building here and this building somebody is living in this building is there is a plume is going to intersect it what is going to be the concentration at here, so these kind of things is what we are interested in calculating.

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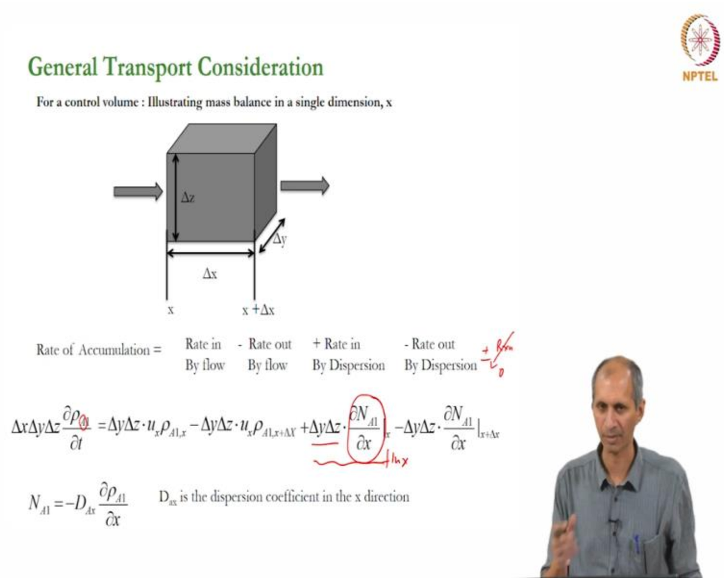

So in this equation, we write this rate of dispersion is this is a flux term, this is flux multiplied by area. So this flux here is given by Na equal – DAx, we are going to draw the A because we are going to consider A in next precise, we are only looking at a you know other component here, so just dx this is a fixed law, this is an equation that is based on fix law. If you are not familiar with fixed law, we will come back to it later after this session.

But it is a very generic law for diffusion it is a very common kind of equation that we see this form of this equation for any flux, so for now take it from me that this is the structure of this particular term we come back to the fundamentals of that later.

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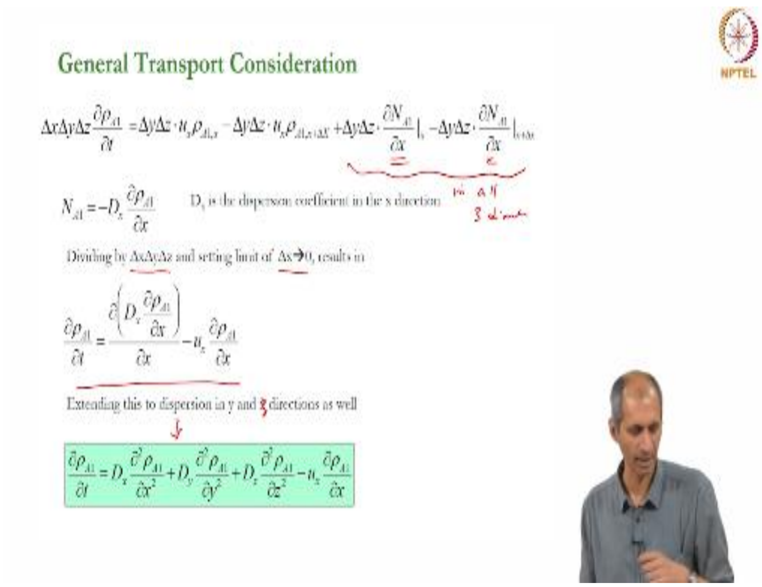

So we take that equation and start adding all this and dividing by delta x, delta y, delta z, setting the limits of all of them to 0, so first thing what we are doing here is we are only doing it for x, we are not done it for any other things. This equation will become two big write for all components for writing for x only I am writing this so I will get this similarly if I extend this to y and z as well. So y and z directions, this is mistake there.

So you only have when we are writing this previous equation, we are only writing the rate in by flow is only in x direction, we know that already there is no flow there is no flow pressure given flow in the y and the z direction, so only x but these two terms can be in all three phases. So that is what we are writing here writing this extended term, so we will write these three terms in all three dimensions.

We will write delta Na by delta y, delta Na by delta z, terms also this case so that will become so there if I only doing for one x, I get this equation. If I do the others I will get this extended equation, we have dispersion in all the three directions. This is where we stop last class.

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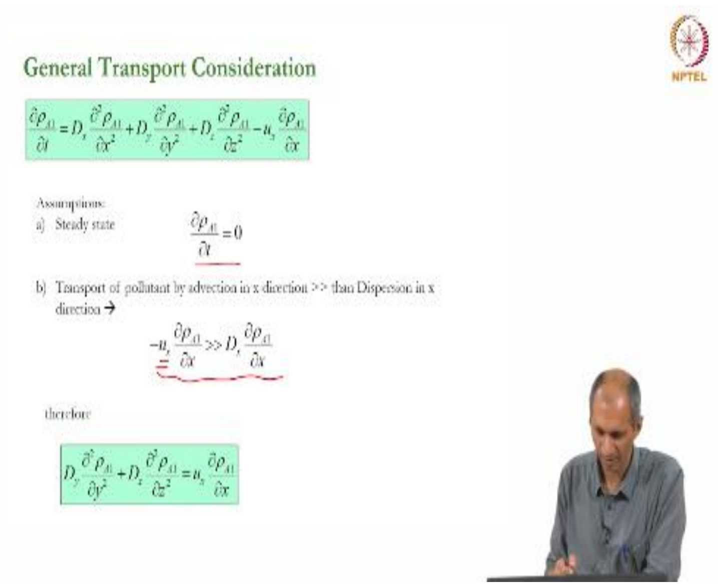

Now here we make true assumptions one is a steady state assumption we do not have to do this but for that this was in dispersion model. Dispersion model that we generally present we use the steady state assumptions where we say rho a to rho a one by rho t equals 0, which means that at any point in time the concentration is the same is you at any place any location you measure it concentration will be will not change with time.

But it will be different with space but it will not change with time. So in a plume if you are looking at it in a plume nothing is going to change so which means for this to be true. Everything else as we true the emission has to be constant; the properties have to be constant. Nothing should change with the time. If something is any of the parameters in this model changes with time, then this is not true.

You cannot use unstated assumptions. So which means it is an assumption it in the environment nothing is constant everything is changing. Which is why we use average values and then we use standard, the variation and then we see if you use average and you use the variation what is going to be the range in which this continuous values are going to be fluctuating from that therefore we make our decisions based on it.

The second thing that we do is since we already have ux bulk flowing in the x direction. We neglect this the x term, so this is entire thing reduces to this simpler equation the equation this is where we stopped last class.

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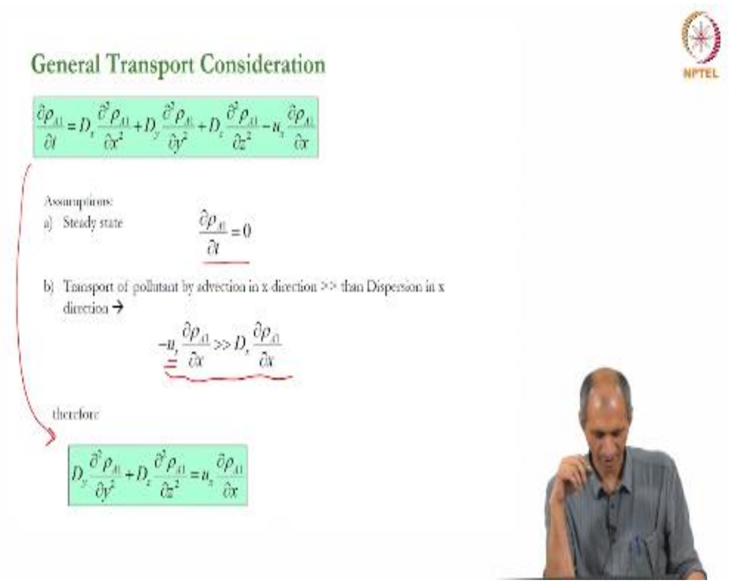

Now, the general solution for this this is an equation there are solutions are already there this form or this form, we do separation of variables and do all that.

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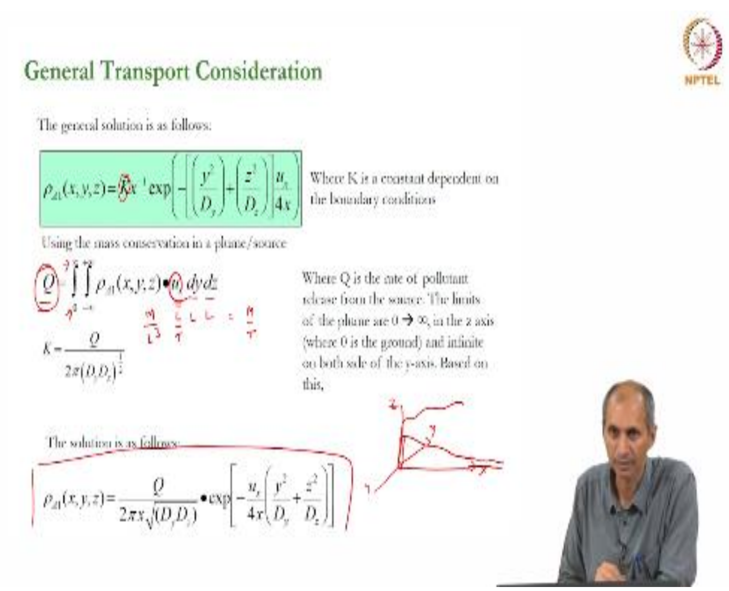

You will get an equation of this form so this there are multiple constants that will come out c1, c 2, c3 because there are three dimensions here x is there, y is there, z is there. There are multiple things so we club all of them into this constant everything comes into one constant and then we use this, people use this single equation as combination of all the boundary conditions. So normally when you say boundary condition will say x equals to x 1 value of rho is this much or something happens to no flux, there is a wall boundary condition and all.

That we fix the values or we give some indication so what it is here we are doing all of that in one shot what we are saying is there using a mask conversation in the plume you take the entire volume of the plume. The entire volume is coming from the source so we are saying q is the rate of pollutant release equals u multiplied by dy and dx, ux into dy into dz will be you look at the dimensions L by T into L into L to M by L cubed equals M by T.

So this Q and so we are integrating y from minus infinity to plus infinity, which means this is the y axis is this, the x axis is, z axis this is the y axis. So there is a plume that is originating let us say here and it is going here. It is expanding in the y axis, it can it is free to expand wherever it wants y axis. In the z axis, it is not free to expand whether it wants there is a limit it will attach to stop at 0 that is a ground cannot go beyond that.

So, that is why the limits are 0 to infinity. Above the ground, it can go however long it wants. So the plume boundary is fixed on the in the y direction y minus infinity plus infinity when x star in the z direction in the x it is anyway moving and there is a set by this term ux. So it is the rate at which it is moving and that the rate at which so the plume boundary is expanding in the x direction and in y direction and the z direction.

So, overall boundaries is now the total amount of mass present in this plume equals Q. If you use this the constant now is determined as the Q divided by 2 pi Dy Dz raised half the substituted back into this equation, this is the general solution. So this you get is the general solution.

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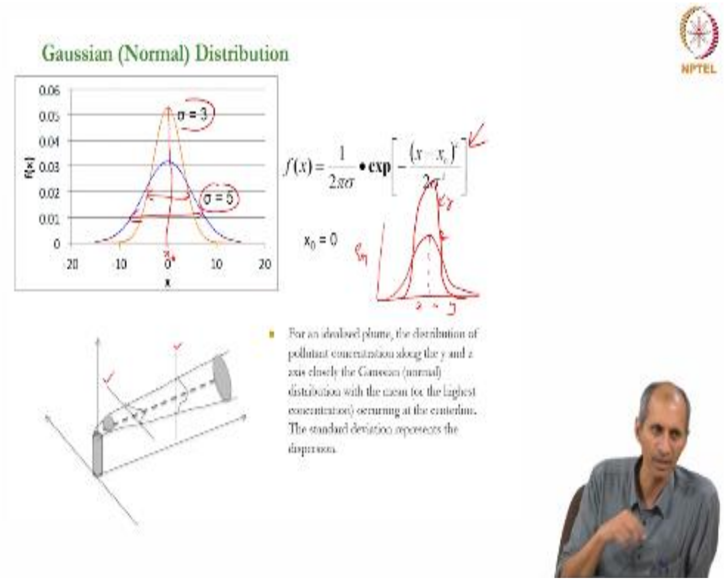

Now here this general solution is you know, you need this term Dy and Dz and all that. So, this is all for you can use this equation as it is. But there is something that people have gone ahead and manipulated this particular equation with the ideal assumption that if you look at this equation, this is very similar to another equation which is called as a Gaussian distribution or a normal distribution.

So where we equation for normal distribution is this f of x y equals 1 over 2 pi sigma into exponential of minus of x minus x0, so this is x0, so in this is a distribution this is x0 means value at this value, what is the value of x0 is the value of f of x is the maximum. So it is a symmetric bell curve and this spread sigma is a spread and the value of the spread and 60 percent some value some percentage of the main f of x.

The magnitude of the sigma indicates how much of spread there is more spread, because of concentration we are assuming that it spreads more. The highest concentration is going to be smaller. If it spreads less that has concentration is going to be higher. Consisting with this thing that you are plume is confined to a smaller volume concentration is likely to be higher plume spread more concentration is the highest concentration is likely to be lower.

So, where do you find the highest concentration? Where will you find it? So for which we look at an ideal plume. So an ideal plume is like this it is nicely going in this cone kind of fashion.

The cross section is looks like an ellipse or even a circle some form of ellipse or a circle. So in this if you look at the distribution of pollutant concentrations, so if I am plotting instead of this if I plot instead of, f of y what I am plotting this rho A1 and the function of z or y either.

I am likely to find that this concentration will look like this, which means that some value the highest concentration occurs at some value of z or y. So let us say this is z and this is y the dispersion occurs like this z dispersion occurs like this depending on the how much of dispersion is occurring in the y and z direction is different. So this curves here show the dispersion is spread of the distribution of the concentration.

So it goes to 0 outside the plume at the plume boundary it stops concentration goes to 0 somewhere in the middle of the plume in the center point it is the highest concentration. This is an idealized curve; you have to understand that this is an ideal curve. This does not usually happen a lot of times this is not like this it is a bit skewed on all that but this is a good starting point for a lot of these kinds of things. So, what people did is to fit this equation into the format that we have.

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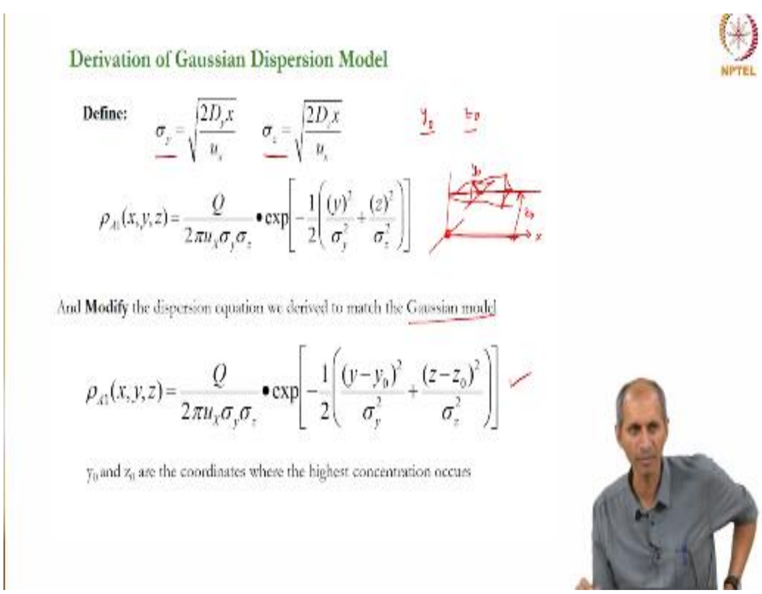

So this equation here does not look it looks almost the same but is not in the same format. So we are fitting this into the same format. So we mean we have to define some new parameters to fit this equation into a permit. So, this is the Gaussian equation. If you look at the bottom equation here the Gaussian dispersion model looks like the other equation, except that there are few other additional terms here.

Now to make it look like this you have to do some transformation so one of the transformation is this one you do sigma y and sigma z to 2 Dyx divided by ux and so on. So you make this transformations and we force that the concentration that at y0 and z0 these are the points at which the highest concentration occurs which means this is a center of the plume wherever so this is the value of y at which the highest concentration occurs this is the value of z that is the highest concentration will occur.

So this depends on the source itself. This depends on the shape of the plume and where is the source and all it so if you have an idealized to this thing, so the plume is going like this. This points, this line in the along the x axis or top of the x axis, we will determine what is z 0, because this is the center point when we are looking at the concentration the distribution of along this will be like this.

So at this high you are going to get the highest concentration. Similarly if you look at this from axis being added from the y at y direction, this is the y axis. So I have but again along the x axis as a and the x axis is called as the center line where the y is there some height and height z equals zero is the highest concentration but also at y equals to 0. The highest concentration occurs here.

So this is y0. So in this particular case, y0 is on top of the x axis. If you look at it by flipped the axis this way if I am looking along the x axis towards the source it is along the x axis is on top of the x axis, which means y is 0 that is center point. It could be anything so it depends on how we are defining the coordinates and all that so we will come to that later y when that will become important.

So, this is not a there is no general reference because you have to take a reference point if you take the source as the origin from there everything else is defined. That is why it is the Lagrangian concept of this model either we are there is no fixed reference point. There is no universal listing it is the reference to some the fluid itself and the stack itself. So this final equation looks like this. So the Gaussian dispersion model in its preliminary form looks like the; this equation here.

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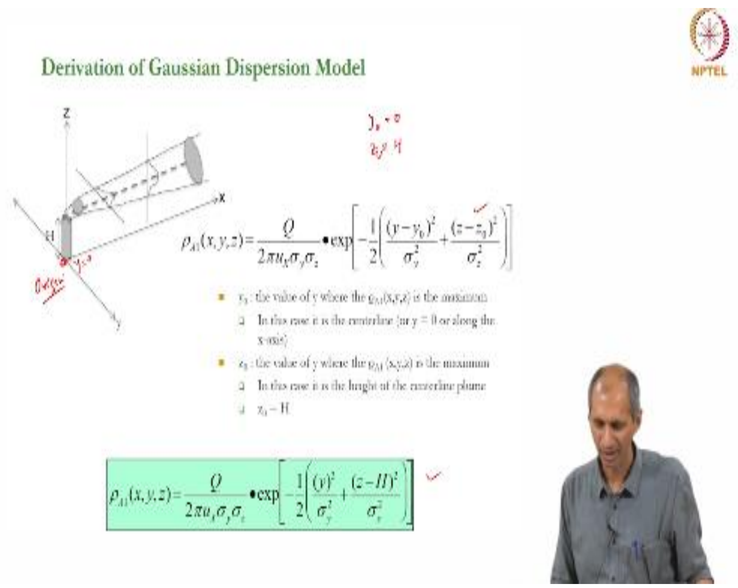

Now, when we put input this y - y0, x - x0. So if the plume the plume if the stack source is that the origin. This is x equals to y x, this is y equals 0 and there is a height of this emission the stack. So let us say there is a chimney and this goes to a certain height and that is where the emission is occurring. So you expect the highest concentration to occur at that point. So therefore here in this case, y0 equals 0 and z0 is the height.

So therefore this equation modifies itself and becomes this that minus h where h is the height of your source. Now, sometimes the source may not be in a chimney it could be on the ground. So that time it should be 0 so it will become then the equation modify further, so you can modify this equation whichever way you want so this is a general equation. Since we have now I would recommend starting here because there are lot of cases in which you will not have a stack you will not even have y you will not be defined as x axis.

But most often y is in central line that is central line so do not worry about that. The second thing that is important here is the x axis itself is defined as the direction of the wind speed x axis can change wherever it wants it can point north it can point east west south rather than the reference for the Gaussian dispersion model x axis is the direction of the wind so the wind speed keeps changing over if you look at Chennai.

For example wind speed is in the morning in one direction the afternoon it is in different direction, it keeps changing the season and all that so you cannot have the wind speed you cannot have fixed frame of reference with the wind. Speed so you have to find out what is the wind speed and then start there and then the rest of the axis is defined y axis is defined based on this so there is no fixed x axis,, this is very important so the first thing you need to start all of this you need to know what is the wind direction.