Environmental Quality: Monitoring and Analysis

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Lecture-38
Transport of Pollutants - Gaussian Dispersion Model

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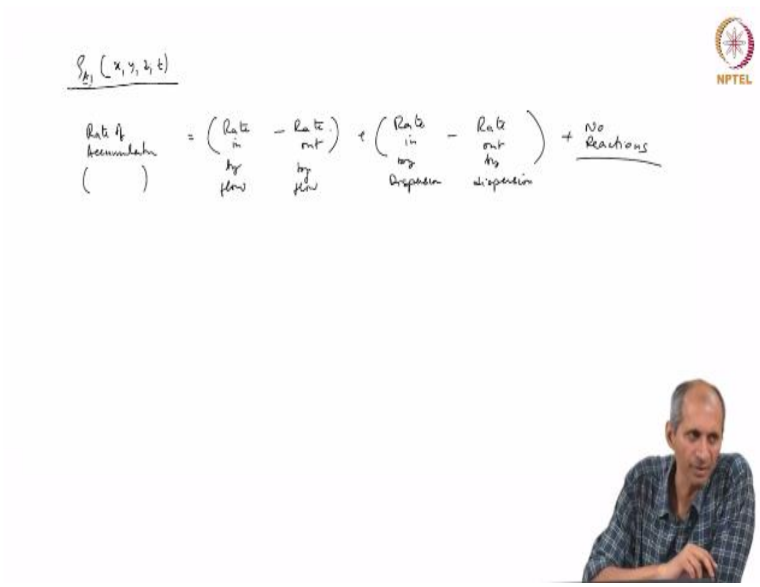

So in order to, our goal is to predict rho A1 as a function of x, y, z and time. This is our general prediction; this is our general goal in what we are trying to do. One of the things; so, our general model will be as follows. If you are trying to do this if you look at, if you invoke the mass balance and try to develop mathematical models for doing this. Now, so here two things may be happening, its rate in rate out.

This rate in by flow, by dispersion nothing else is happening. So rate of accumulation equals rate in dispersion plus there is no reaction, there assuming no reactions here. This is an assumption. If you add reactions, then things will become very different. We are also considering only rho A1. This is vapour phase concentrate. We are not looking at particulate matter and all that. If, you add that, that is a different thing, it will also settle down and get trapped in different this things.

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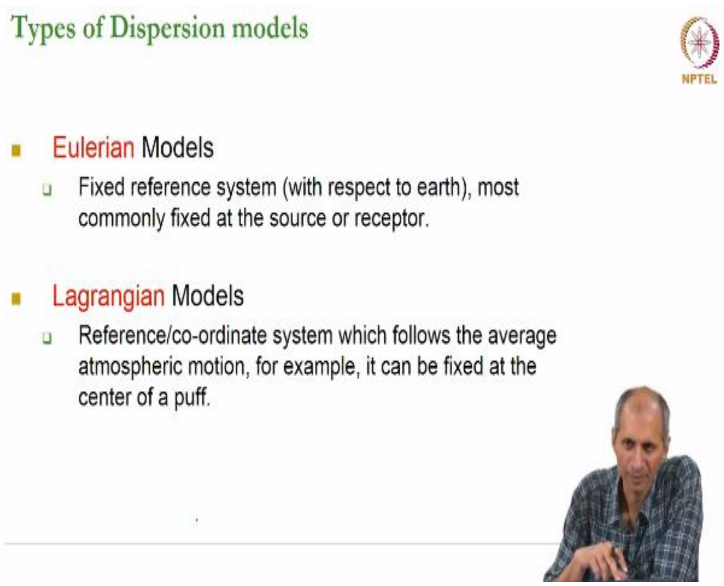

So dispersion models can be of two different kinds. So, one what is called as an Eulerian model, which is fixed reference frame. So what this means is that I am modeling this room here. I have watching from here. So x equals 0 begins at that end goes to this end start from here and here. Lagrangian model on the other hand is that you are moving with the fluid and your frame of reference is that body of fluid.

So here in the way the dispersion model is set up the most commonly used model this thing is what is called the Lagrangian model and the basis for that is that;

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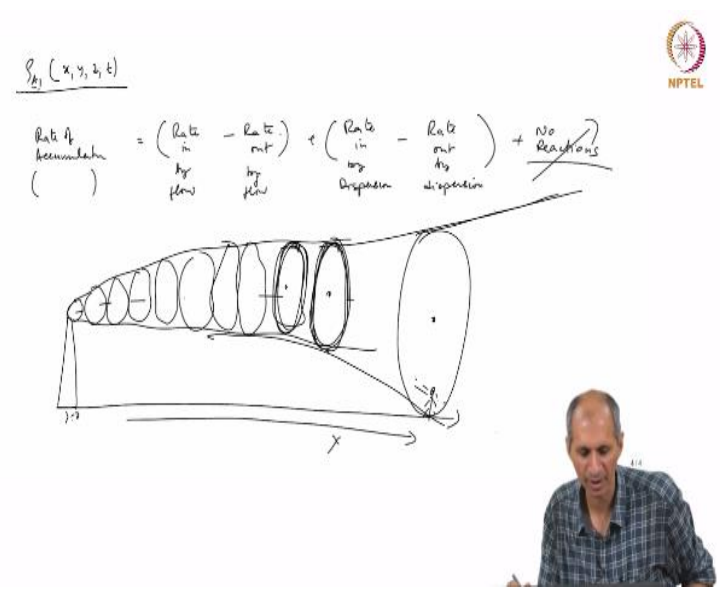

We are now looking at proof, we are looking at the entire system, but we are also seeing that when we are talking about the z and y and all that the dispersion it is the reference to this particular, it is not with the reference to a fixed reference frame this is the fixed reference frame where x equal to 0 but the dispersion itself, spreading itself is happening each of these volumes, so if you imagine one puff that is going out. But this is a series of puffs that is coming out.

Because there is rate at which this is emission is happening. You are burning something there is a rate at every second there is a mass of exhaust that is coming out and this exhaust is going out. So the concentration that you are going to be exposed to is a concentration within this puff. So it is irrelevant, what if you model everything around here is not going to be of much use to you. We are trying to what will be more useful is modeling what is inside this particular puff alone.

So if this puff becomes very large at some point and then there are also issues of this puff you know, spreading very wide, so if the plume spread like this. Which means that this puff will now occupy a big volume and therefore and if there is a receptor standing here, this receptor is exposed to a certain concentration. So this is the goal is to find out what will be the concentration at a particular distance x at a particular height?

But the plume behavior itself the behavior of this thing itself is modeled, in this you will see that when we write down this equation. It is in with reference to this particular plume with this particular puff only it does not refer to anything below or after or before. So to come to that point, you have to write down these equations, so we have no reactions here.

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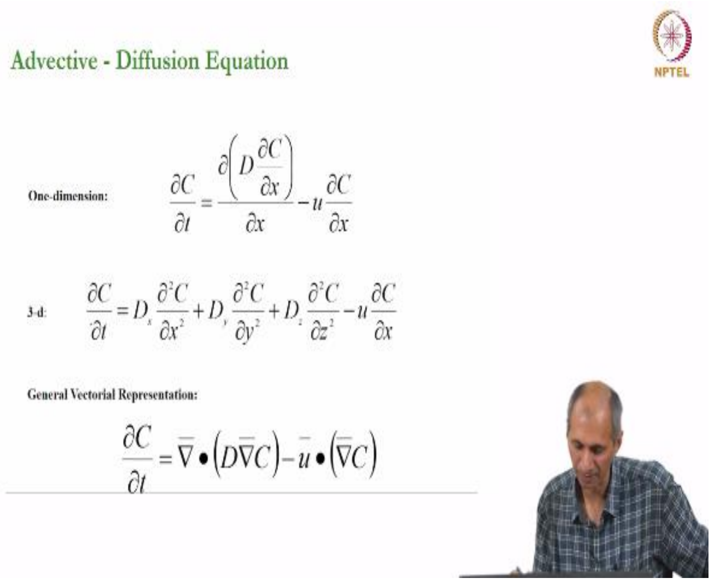

So when we write the general equation we have written down we see here I will change it to derive this equation for you.

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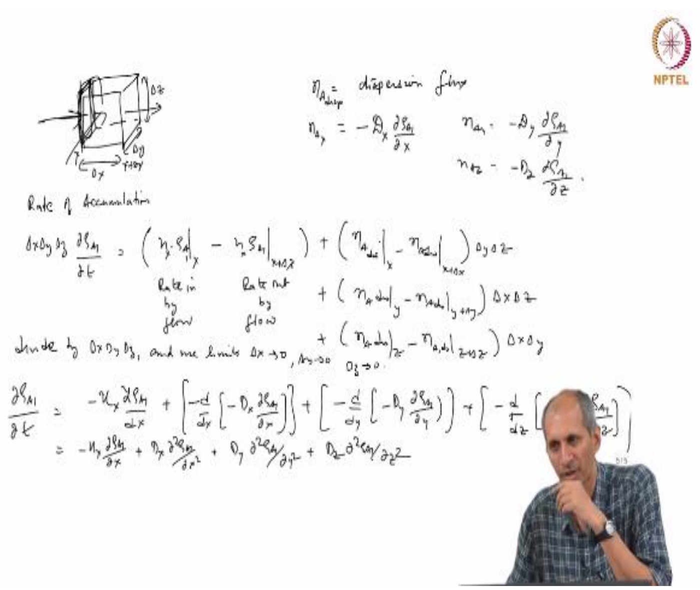

So we have let us say that we have a small volume. We take a three-dimensional volume. This is delta x, this is delta y, this is delta z. This is the volume of something, it is a volume inside one of this the gas pollutant inside the plume, somewhere inside the plume we are trying to find out how it is moving and all that. So the rate of accumulation here, so we are saying, is usually delta x delta y delta z dau t equals.

Now this is the direction in which this is happening flow is happening. There is no flow in the y and z direction. We are looking at average velocities. Now the y, z component y component and their fluctuations all that is now captured as dispersion, that is all captured in dispersion terms that will come separately. So by flow rate in by flow we are talking about x and this x + delta x. So rate in by flow is u at x is multiplied by rho A1.

Sorry this is not ux delta at x, this is ux x + delta x. In all the 3 directions, you can have dispersion. Dispersion can happen so dispersion is again happening this thing. So you have dispersion terms that we use is dispersion we have Dx, we are only taking the main components of Dx in x direction multiplied by the general terms. So we are looking at a flux multiplied by area, so we look at the dispersive flux.

We use the word we used a term in a dispersion flux. This in the x-direction and the area of this is delta y delta z. So the x-direction flux is like this. The area it goes to is this one so that delta y delta z plus you have another term which is y - n dispersion, y + delta y this is in this direction which is delta x delta z + any dispersion z + delta x, delta y. If no reaction here, if I divide everything by divide by delta x delta y delta z and take the limits delta x tends to 0 delta y tends to 0 and delta z tends to 0.

We will get dau rho A1 by dau t equals the first term will become minus of ux d rho A1 by dx. The rest of the terms nA, if you use basis for what we call a fixed law model, this will become a dispersion term multiplied by rho x, this is the format we use so n Ax is this, n Ay will be minus of Dy dau rho A1 by dau y and n Az will be - Dz dau rho A1 by dau z. So if you use all these three terms here the other terms now become, this minus of minus it will become - d by d z of Dx into dau rho A1 of d x.

And dau rho A1 by dau x - d by d y of - d y dau rho A1 by dau y - d by dz - dz of dau rho A1 by dau z. So in this equation plus, plus will cancel out then this equation essentially will become minus of ux is a dau here dau rho A1 by dau x + D x dau square rho A1 by rho x square + D y dau square rho A1 by d y square + Dz rho square row a1 by dz square. In the absence so if you have reaction or something you can add those terms here all add up there.

It will become a big general equation is the three dimensional equation unsteady state including everything, this is the derivation of this full thing.

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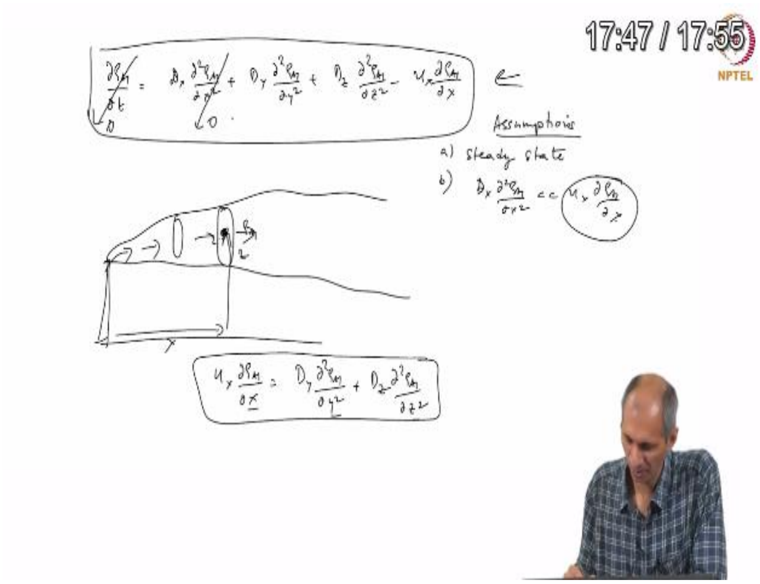

Now so you can write this equation like this dau rho A1 dau t equals Dx dau square rho A1 by dau x square + dau y square + Dz - u of x. So here this is a general equation now you can solve this with appropriate boundary conditions and all that but before we solve it you can do this equation from people do this all the time so what this is applicable to is anything so what we mean by unsteady state is that any point is concentration is going to change.

When will concentration change let us say if I have a plume here, if I am measuring concentration at this point. I would like to find out what is the concentration at this location which has a certain particular z, particular x and y. This point if I want to measure concentration it will only the, it will it be unsteady state. When it will be unsteady state? What are the conditions under which this will be what is the meaning of unsteady state?

Changes with respect to time, when will it change with to respect time? Or let us put the reverse question, when do you expect it not to change the time? There is nothing to equilibrium steady state the equilibrium is a steady state, but steady state need not be equilibrium that there is a difference, steady state may not be equilibrium. So in this case we are not talking about equilibrium.

There is only one phase is just moving we are talking purely about transport when is it when we will when you can when can you make an assumption of steady state here which means that nothing is changing with time, when can that happen? What we essentially are saying is rho A1 is not changing with time at this location, so which we mean at this location rho A1 not changing with the time requires something.

Turbulence, when you say something not changing with time what is else should not change with time? Environmental conditions should not change with time and anything else. Source should not change with time it means you have a constant source or emission and for a given retime period of time, nothing is changing environmental wind speed is all the same, you expect that this will not change.

So now this being a general case we want to use this is a very general case but solution of this is quite complicated which means that you must have parameter you as a function of time everything dispersion has function of time sources function of time and all that will come so the first easier limiting case to this is we assume a steady state. We make various assumptions to simplify this equation.

This is general approach in transport phenomena we take a very general model and then we cut several terms out we make assumptions only assume steady state. So which means that this whatever we are going to do is valid for constant source emission systems, so there are several of them, for example. I can take industries they are routine is known we know that they are going to be producing this amount of waste we know it fixed.

For a given period of time, you know that you can predict where the plume is going because we know for that period of time we might be able to predict the wind speed and everything and we can do that when it is changes we can then do the same convenient for a different set of conditions and we can run it so it is easier to do it that way it is like the box model. Except that we are doing it in time now we are taking it in time and saying this.

So when we say steady state this is a constant flow coming here. So whatever is coming here is leaving here, so at this point the concentration is always be the same it will be different from whatever it is here but it will be the same with reference to time does not change so that is the idea of steady state. Second this term d x is a dispersion in the x direction is much smaller the term.

The Dx dau square rho A1 by dx square is much smaller than the contribution of d rho A 1 by dau x, this is wind moving in that direction the distance the amount of spreading by x it is dispersion is much smaller compared to the wind moving you are going in that direction any way it does not. So this term is negligible so what we do is these two terms this goes to 0 and this goes to 0.

Which essentially the equation becomes now u x dau rho A1 by dau y square equals dy dau square rho A1 by dau y square + Dz dou square rho A1 by dau z square. So this is a steady state equation that we need to solve. So this equation leaves a few boundary conditions now which is boundary conditions are related to which is your x and y and z is three boundary conditions are needed in this case.

Three normal boundary conditional corresponding to x y and z, that all of them are needed in this case. The second order equation so I will stop here we will go from here to the derivation of the what we call as a Gaussian dispersion model from this point requires the bit of work.