Continuum Mechanics And Transport Phenomena
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Lecture – 97
Viscous stress vs. Molecular momentum flux
Part 1

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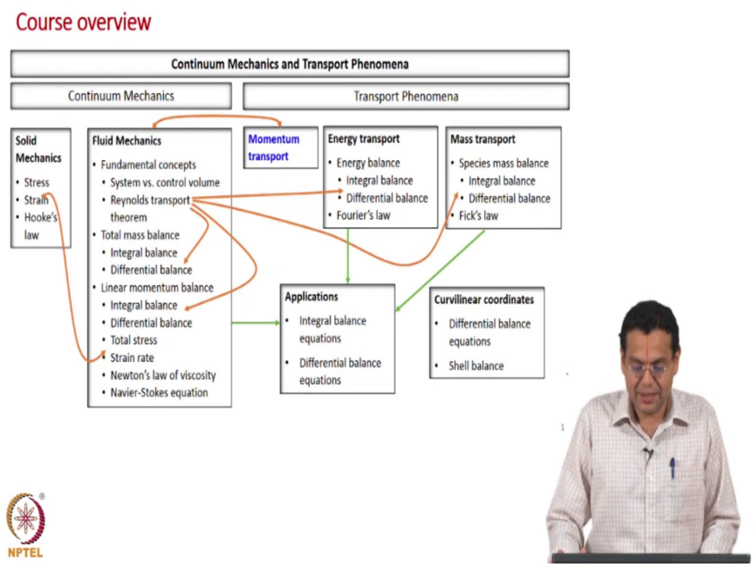

We will start with the second part of the course namely Transport Phenomena which in itself is split into three parts namely momentum transport, energy transport and mass transport and what we are going to discuss now is momentum transport. Fluid flow can be analyzed whether through fluid mechanics or momentum transport and what we are going to discuss in this lecture is the equivalence between these two approaches and what a subtle difference between them. So, to be more specific we should say we are going discuss this double headed arrow which says both are equivalent.

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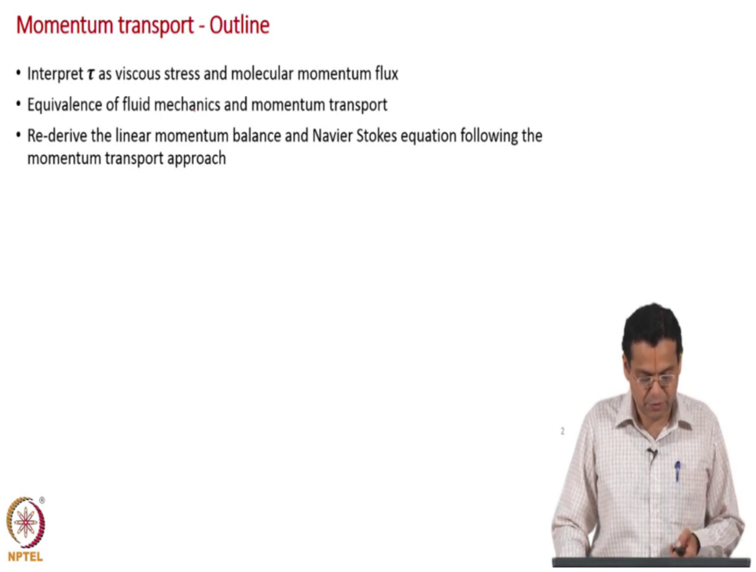

Let us look at the outline. So, as usual you understand this outline better at the end of the lecture. So, to begin with, we are going to see is a two different interpretation of τ. So far we are looking at a viscous stress, now we are going to view in a different view point namely as a molecular momentum flux something like duality of τ just like a duality of light.

Then we are going to look at equivalence of fluid mechanics and momentum transport and of course, discuss the subtle differences between them as well. And, then we are going re derive the linear momentum balance and the Navier-Stokes equation following the momentum transport approach. That is a overall broad outline let us proceed then you will understand this in clay with more clarity.

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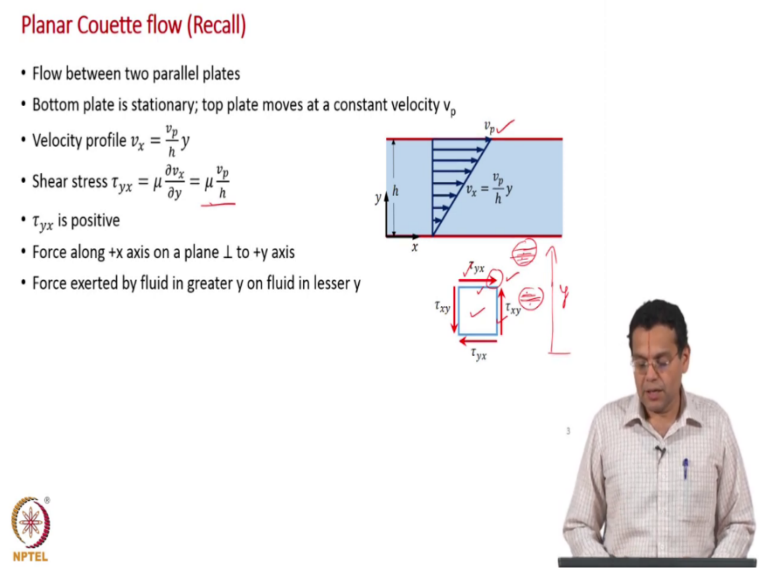

Let us start with our well known example which just a recall we will start with our planar couette flow between two parallel plates in the top plate is set in motion. So, let us quickly recall flow between two parallel plates; bottom plate is stationary, top plate moves at a constant velocity v_p. We have derived the velocity profile use the Navier-Stokes equation now we need not assume this profile. We derive the velocity profile to be linear given by

$$v_x = \frac{v_p}{h} y$$

Now, the shear stress

$$\tau_{yx} = \mu \frac{\partial v_x}{\partial x} = \mu \frac{v_p}{h}$$

Also, we can take it as one dimensional Newton’s law of viscosity. Now, that is a positive quantity and then according to our sign convention this is the stress element because it is positive; on a positive phase, the direction of force is along the positive axis. In this case it is a positive y phase and force along positive x axis and here it is positive x phase and force along positive y direction.

So, because we are considering τ_{yx}, force along positive x axis on a plane perpendicular to positive y axis that is what you discussed just now. Your plane is the perpendicular to the plane is along positive y axis, force acts along positive x axis. Now, we will also interpret in this way which we have not discussed so far, but it intuitively we can understand that. If you are considering this force we are considering positive force, then that force is exerted by fluid in greater y on fluid in lesser y.

What does it mean? We have fluid near the top plate and we have fluid near the bottom plate and this force exerted by top fluid on bottom fluid so, top liquid is in greater y and bottom is in lesser y. So, this force exerted by fluid in greater y on fluid in lesser y.

Of course, we have seen this several times, but we can look this in this way also and why is that the fluid here flows at a higher velocity compared to fluid at a lower y and hence the fluid in the top region exerts force in the positive direction on the fluid in the lower region.

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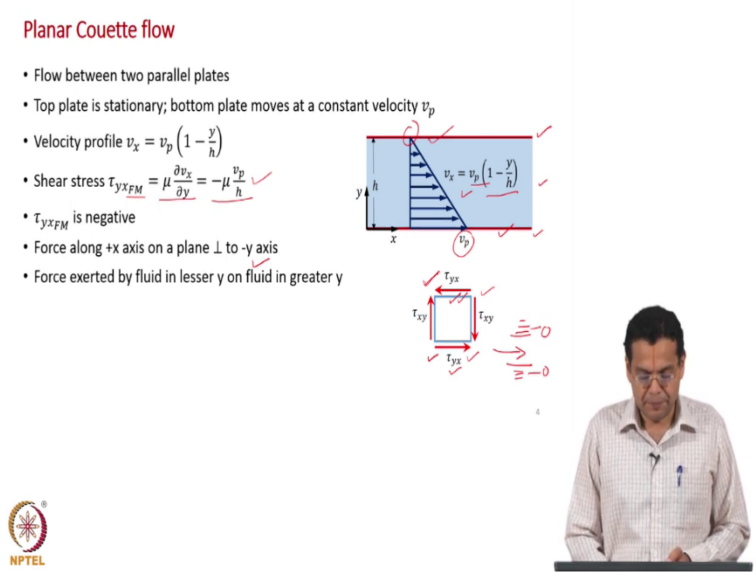

Now, what we will consider is once again a planar couette flow, but now with the top plate fixed and the bottom plate keep moving at a constant velocity v_p. This is the example which we are going to discuss to relate fluid mechanics and momentum transport we will understand why we have come from that geometry to this geometry as we go along.

Let us look at that. So, flow between two parallel plates; top plate is stationary, the bottom plate moves at a constant velocity v_p. Now, what is the velocity profile? If you go through the derivation if you go through the same procedure as we have followed earlier for deriving the velocity profile for planar couette flow for this case what is that the velocity is 0 at top and the velocity is v_p at bottom and solve the Navier-Stokes equation you will get

$$v_x = v_p\left(1 - \frac{y}{h}\right)$$

But, we can quickly check whether it is right at y = 0 that is at the bottom plate, the velocity is v_p and y = h, the velocity at the top plate is 0.

Now, if you find the shear stress $\tau_{yx,\,FM}$ I use the subscript FM meaning fluid mechanics we understand that as we go along what the shear stress expression it is

$$\tau_{yx,\,FM} = \mu\frac{\partial v_x}{\partial x} =- \ \mu\frac{v_p}{h}$$

Now, according to our sign convention if you want to represent shear stress component which is negative then on a positive plane the force should be along the negative axis. So, in this case is a positive y plane and force is along the negative x axis.

And, if you look at the bottom phase the normal to this phase is along the negative y axis so, the force is along the positive x axis ok now. So, τ_{yx} is negative the shear stress is negative. Now, once again we want to consider force along positive axis. So, let us take this force and. So, now, in this case the positive force will act on a plane whose perpendicular is towards the negative y axis that is what we are seen here. Now, what is the equivalents of the last sentence to the previous slide for this case, now if you consider this force, this exerted by fluid in lesser y on fluid in greater y. Why is it so?

This fluid and the lower y or lesser y has a higher velocity and fluid at a greater y has a lower velocity. So, the fluid in the lower y region tries to pull fluid in the greater y region hence this force exerted by fluid in lesser y on fluid in greater y almost everything is opposite to the previous case this is the case which we are going to discuss or use as configuration to compare fluid mechanics and momentum transport.

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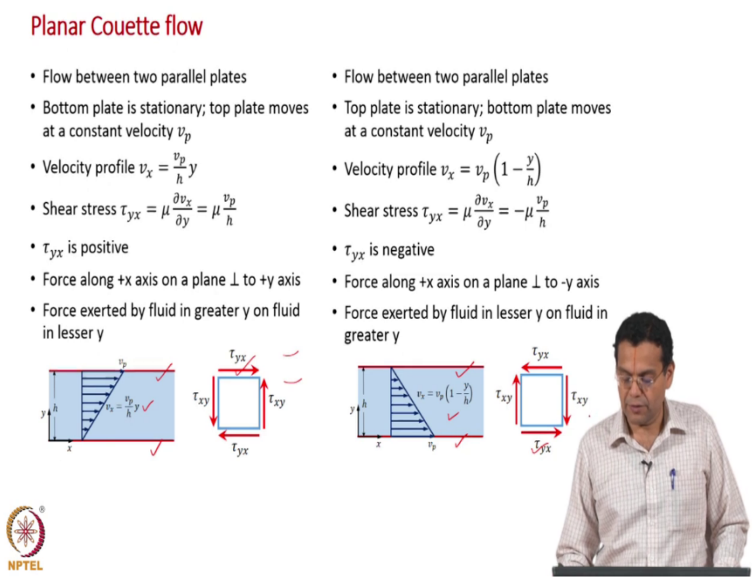

So, let us proceed further. Let us compare both the planar couette flow case. Both of them are flow between parallel plates.

- The first case which I have been discussing so far, the bottom plate is stationary; top plate moves at a constant velocity v_p. In the new planar couette flow top plate is stationary, bottom plate moves at a constant velocity v_p.
- What is the velocity profile? It is $\frac{v_p}{h}y$ in the first case and now the velocity profile is $\frac{v_p}{h}\left(1 - \frac{y}{h}\right)$.
- What about shear stress? $\tau_{yx} = \mu\frac{\partial v_x}{\partial x} = \mu\frac{v_p}{h}$ for the previous case and the present case $\tau_{yx, FM} = \mu\frac{\partial v_x}{\partial x} =- \mu\frac{v_p}{h}$ and
- The first case τ_{yx} is positive second case τ_{yx} is negative and
- If you consider a positive force along x axis that acts on a plane whose perpendicular is towards positive y axis. The second case the new case if you consider a force along positive x axis that will act on a plane was perpendicular is towards negative y axis.
- In the first case force exerted by fluid in greater y on fluid in lesser y and the present case force exerted by fluid in lesser y on fluid in greater y and we are going to focus on the case on the right hand side.

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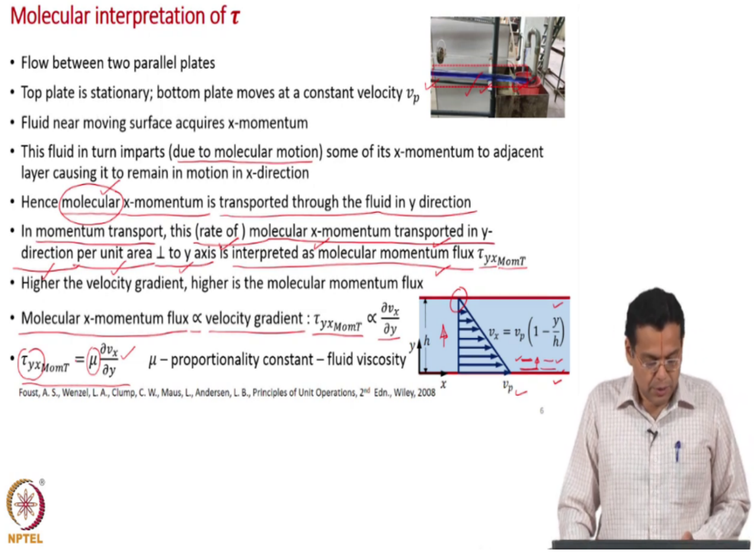

Now, what is it we are going to do now is give a molecular interpretation for τ. So, as I told you we are going to take the case where the bottom plate is moving, top plate is fixed; flow between two parallel plates the top plate is stationary bottom plate most at a constant velocity v_p. Now, the bottom plate has some momentum and the fluid moving near this plate acquires the momentum from this plate and that imparts to the fluid above that. So, fluid near the plate acquires a momentum and then imparts the fluid which is above that.

So, this fluid which is near the plate imparts some of its x momentum, we are talking about x momentum here, to adjacent layer causing it to remain in motion in x direction. At the molecular level what happens is that generally molecules move in a random motion, but now the molecules will have a higher x velocity let us say near the moving plate, the molecules in the above plane will have a lower x velocity.

Overall the molecules have two components one is the random component, other is the x component the molecules, so in the layer near moving plate we will have higher x velocity hence higher x momentum and molecules in the above layer will have lower x velocity and hence lower x momentum. And, now let us say molecules move from the lower layer to the above layer they carry with them the higher momentum and then when they reach the top layer they transfer their higher momentum to the molecules in that region. That is why here we use the word, due to molecular motion. Let us read that sentence again.

So, when you say fluid think in terms of molecules in that region, the fluid near moving plate intern imparts some of it is x momentum to adjacent layer causing to remind motion in x direction. So, how to interpret this? So, molecular x momentum is transported through the fluid in y direction look at the word here very specifically we have use molecular x momentum and that is x momentum, but it is transported through the fluid in the y direction more specifically in the positive y direction.

So, plate has some momentum and the fluid moving near the plate acquire some x momentum and that imparts x momentum the next layer of fluid and in terms of molecular picture, molecules have a higher x velocity at bottom and higher x momentum and when they move to this layer they carry with them a x momentum which means that they transport x momentum in the y direction. Now like to distinguish this molecular x momentum with the our usual momentum which you have been discussing so far our usual momentum when we say momentum in and momentum out this picture has to be kept in mind that we have discussed several times (top picture in the above slide image).

So, if you take a control volume is a momentum entering, momentum leaving etcetera, this is by bulk flow which we call as convective x momentum. In this present case in the y direction remember there is no convection, there is no bulk flow in the y direction we have a bulk flow only in the x direction. So, x momentum gets transferred in the y direction because of molecular motion and hence this momentum is called as molecular x momentum in contrast to our usual convicting momentum which is because of bull flow. So, this has to be kept in mind as we go along.

So far we have been using just x momentum meaning it is convective x momentum, but right now we are two different forms in which momentum is transported, two different forms of momentum being transported – one is molecular x momentum other is convector x momentum. Now, how do we interpret τ how do we relate this τ to this momentum transported. Now, in momentum transport what do I mean by that in the momentum transport approach of analyzing fluid flow; one is the fluid mechanics way of analyzing fluid flow, now the momentum transport way of analyzing fluid flow.

We have talked about this transport of x momentum in the y direction the rate of molecular x momentum transport in a y direction per unit area. What is area? In this case there are two plates so, the area through which this transport takes place and of course, perpendicular to y axis is interpreted as molecular momentum flux. So, this is interpreted as the molecular momentum flux. What is the nomenclature $\tau_{yx,MomT}$ momentum transport is that, that is why previously we used τ_{yx} subscript fluid mechanics. Both are τ_{yx} so, just to distinguish earlier we subscript fluid mechanics, now we are using subscript momentum transfer.

Let us read it again. What is that we are doing now? We the few minutes back we discussed about x momentum being transported in a y direction now we are trying to quantify it or express that as a flux. So, rate at which that molecular x momentum it is transported in y direction and per unit area perpendicular to y axis means, several terms are there here one is rate molecular x momentum and then in y direction and per unit area perpendicular to y axis and that is interpreted as the molecular momentum flux $\tau_{yx,MomT}$.

And, now it is clear that if the velocity gradient is higher, what does it mean? In this case and top velocity is 0 let us say this velocity is higher and higher then the velocity gradient is higher and the molecular flux will be more higher will be the molecular flux. So, the molecular x-momentum flux is proportional to the velocity gradient, this is based on intuition that the x momentum being transferred in a y direction will be more if the velocity gradient is more.

$$\tau_{yx,MomT} \propto \frac{\partial v_x}{\partial y}$$

So, molecular x momentum flux is proportional to the velocity gradient. The proportionality constant is the viscosity and so, we write

$$\tau_{yx,MomT} = \mu \frac{\partial v_x}{\partial y}$$

The τ_{yx} in momentum transport as molecular momentum flux proportional to the velocity gradient with the proportionality constant as the fluid viscosity.

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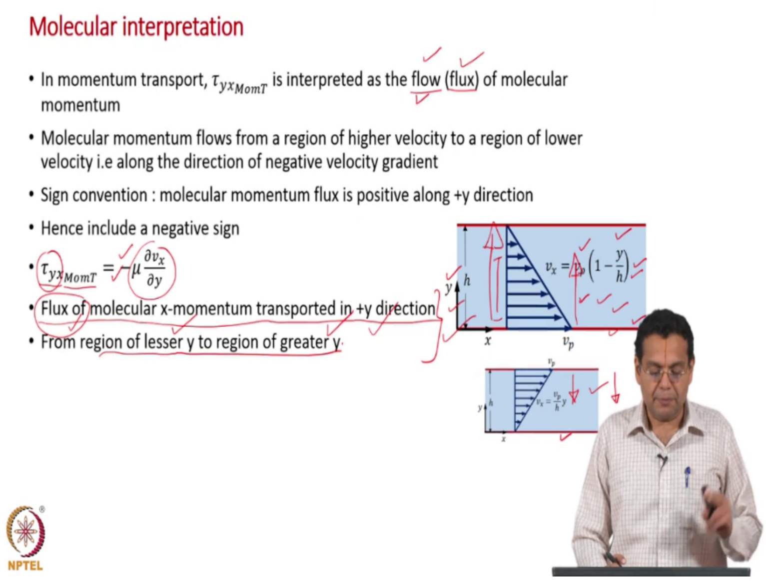

Now, let us proceed further we have seen this just now in momentum transport $\tau_{yx,MomT}$ is interpreted as the flux of molecular momentum, I have written the word flow because tau y x is considered something flowing what is it something flowing that is x momentum being transported. So, flux is a more formal word, but in terms of understanding, in terms of imagination it is a flow of molecular momentum. So, flux is more formal representation of $\tau_{yx,MomT}$, when we say flow it says gives a physical meaning that molecular momentum flows ok and that is what is represented by $\tau_{yx,MomT}$.

Now, based on the discussion what we had this molecular momentum flows from a region of higher velocity to a region of lower velocity that is very obvious based on our discussion earlier. So, molecular x momentum flows from a region of higher velocity to a region of lower velocity which means that it is along the direction of negative velocity gradient. The velocity decreases in this direction and the momentum flux is also in that direction that direction which x momentum is transport it is also along the direction which velocity decreases.

Now, we will adopt a sign convention that is the key in fact. We will adopt a sign convention; what is sign convention? I want my molecular momentum flux to be positive along the positive y direction. So, I want this flux to be positive, remember this flux is along the positive y axis, I want this flux to be positive along the positive y axis. How do we do that?

In the previous slide, we have written

$$\tau_{yx,MomT} = -\ \mu \frac{\partial v_x}{\partial y}$$

Suppose, if I stop at this what will happen for the present case $\frac{\partial v_x}{\partial y}$ is negative and hence I will have a case where I have a negative momentum flux flowing along a positive y axis which is little counterintuitive because whenever we have a flow we want that to be positive along a positive axis. For example, let us say we have velocity; when I say positive velocity I want to float be along positive x axis I do not want to be against through I want I do not want the flow to be towards negative x axis.

Similarly, here because we are interpreted $\tau_{yx,MomT}$ as something as a momentum flowing that is why this flow is written here. Whenever you imagine $\tau_{yx,MomT}$ from momentum transport imagine as if something is flowing what is that x momentum is being transported in the y direction I want that flow that flux to be positive along positive y axis. How do you take care of that? By including a negative sign. Now, what happens for this case $\frac{\partial v_x}{\partial y}$ is negative and you have another negative sign. So, now, my molecular momentum flux becomes positive along positive y axis. That is why we include a negative sign, so that the flux is positive along positive y axis.

Now, this is the reason why this example was considered rather than our usual example. What would have happened in this case I would have had a molecular momentum flux towards the negative y axis, I will have a negative molecular flux towards negative y axis which is little inconvenient discuss. That is why we took this case where there is positive momentum flux towards positive y axis; both result in same sign convention negative momentum flux towards negative y axis, right now we have positive momentum flux was positive y axis.

So, want to take a case where we deal with only positive y axis and positive momentum flux that is why this example was taken, this couette flow was taken rather than this couette flow. So, what is τ_{yx} now we can understand the subscript also τ_{yx} is flux of molecular x momentum transported in positive y direction, that is a formal nomenclature for τ_{yx}. Of course, we can also say molecular flux of x momentum. What does it tell you?

We have a molecule x momentum transported in y direction the flux of that is the tau y x and now just like we had in fluid mechanics force exerted by a fluid in lesser region on a fluid in higher region similar to that what is the direction of transport from a region of lesser y to a region of greater y. We have a region of lesser y to a region of greater y we are; remember you talking about the positive τ_{yx} momentum transport value.

So, when this is positive these two are for a positive value of τ_{yx} momentum transport if it is positive then the flux of molecular x momentum is along the positive y direction and then it flows from a region of lower y to a region of higher y that is what is happening from a region of lesser y to a region of greater y.

Just to summarize this slide the only addition in this slide compare to the previous slide is the previous slide we interrupted τ_{yx} from a molecular momentum transport point of view, in the present side what we have done is including of negative sign that is a only addition in this slide.

Why did we do that? Based on the sign convention that the molecular momentum flux should be positive along positive y direction, based on that sign convention we have included this negative sign and we also given a formal we have also interpreted what a positive τ_{yx} transport means.