**Continuum Mechanics And Transport Phenomena**
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**Lecture – 91**
**Planar Couette Flow – Velocity and Pressure Distribution**

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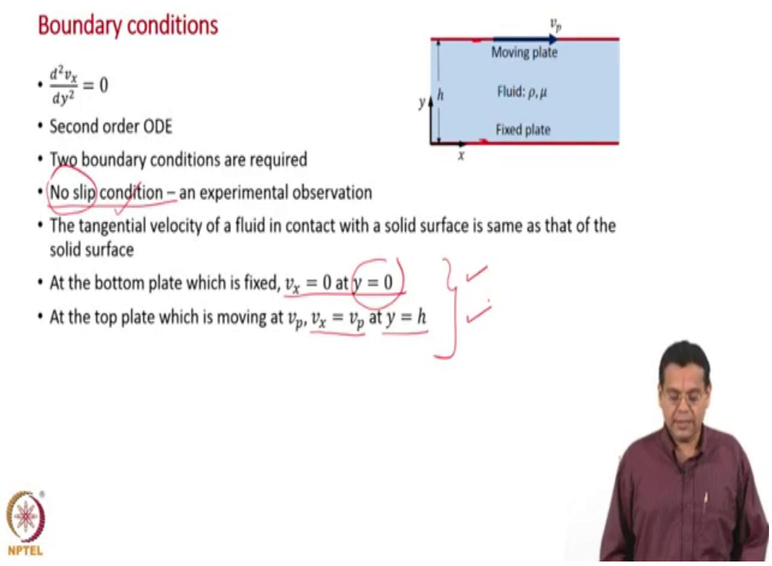


What we will do is discuss the boundary conditions. We have a second order ordinary differential equation and then to solve that we require two boundary conditions and that is what we will discuss now.

$$\frac{d^2v_x}{dy^2} = 0$$

Second order ODE, two boundary conditions are required. Now, so for we have been telling that whenever we came across the example that the fluid clings to the bottom plate also clings to the top plate and fluid near bottom plate does not move and the fluid near top plate moves at the same velocity as the top plate.

So far purposely I have not used a very formal terminology, we used a very informal technology saying that it clings. What is the formal terminology? It is called as no slip condition. What is the formal way of expressing a no slip condition? Let us read that and then we will discuss the tangential velocity of a fluid in contact with the solid surface is same as that of the solid surface.

So, that is what we have been telling that the fluid clings which means that whenever a fluid is in contact with the solid the velocity of the tangential velocity can a normal velocity also, now we are discussing about the tangential velocity the tangential velocity the fluid is same as that of the solid. If we now apply, the velocity of the fluid let us say the fluid layer just in contact the bottom surface is 0.

Now, what about the fluid layer if the just in contact with the top plate? It gets the same velocity as that of the solid now which is moving at the $v_p$ that is why we said the here also the fluid clings to the solid and get the velocity of the fluid is same as there are the velocity of the solid. Now, why do we say it as no slip condition; slip means there is some difference between the velocity of the fluid and the solid when there is no difference in velocity there is no slip or slippage between them that is why we call them as the no slip condition.

So, to summarize this boundary condition the no slip condition means that there is no difference or no slip between the velocity of the fluid adjacent to a solid layer. It gets the same velocity the fluid acquires a same velocity as that of the solid itself. If it is stationary fluid has no velocity; if it is moving fluid also moves along with the same velocity.

So, now one this is clear then we can write down the condition for $v_x$

- At the bottom plate which is fixed, $v_x = 0$; at $y = 0$ and
- At the top plate which is moving at $v_p$, $v_x = v_p$; at $y = h$

So, these two are the boundary conditions done based on no slip condition.

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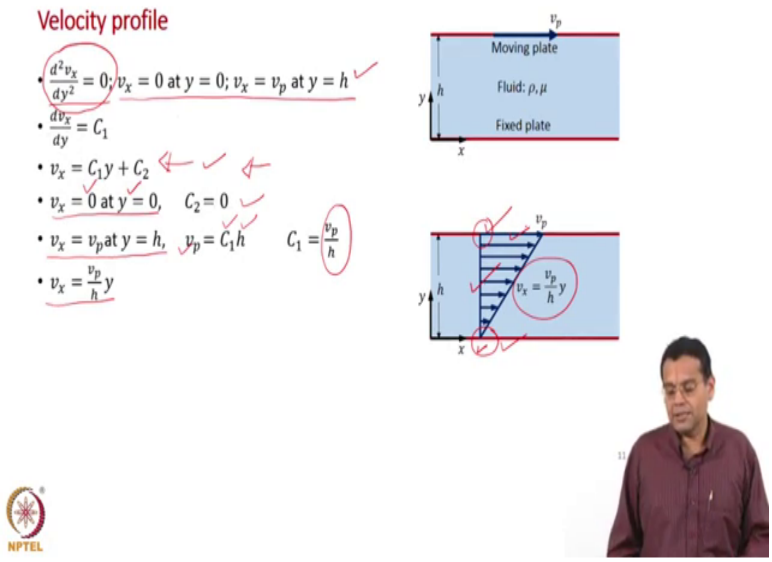


Now, let us summarize the differential equation along with the boundary conditions now like to mention that physics stops here, and  maths starts here.

$$\frac{d^2v_x}{dy^2} = 0$$

$$v_x = 0 \ at \ y = 0; \quad v_x = v_p \ at \ y = h$$

So far we have arrived at the equation based on the Navier-Stokes equation which represents conservation of linear momentum and then based on the no slip condition we wrote this boundary conditions.

Also like to mention that the no slip condition is a experimental observation. We can prove or derive the no slip condition experimentally it is observed that the velocity of the fluid in contact with the solid is same as the velocity of the solid. So, it is experimentally observed that the velocity of the fluid is same as that of the velocity of solid. So, it is experimental observation.

Let us say if you keep measuring velocity closer and closer to the wall you observed that the velocity of the fluid and that the wall becomes same. So, this equation has a physics of the linear momentum balance behind it and based on that equation represents I would say one simplified form of the linear momentum balance. The boundary conditions are also written based on physics which is no slip boundary condition.

Now, if you recall back you would have done courses on mathematics on calculus differential equations. In those courses you would have been given a differential equation with the boundary conditions. Now, this situation now where you have formulated the differential equation yourself along the boundary condition; there some equations was given you would solve as a mathematical problem. But, now all the differential equation represent some form of conservation equations.

Now, because you know the conservation equations yourself of arrived or derived the differential equation and stated the boundary conditions yourself. So, putting in other way now you have formulated the question yourself. So, that is why it said from now onwards what we are going to simple maths just what you would do in a typical first year engineering calculus course; given a linear first order differential equation along with boundary conditions how do you solve that is all we are going to do.

$$\frac{d^2 v_x}{dy^2} = 0$$

So, let us do that this is very much known to you. So, integrate once we get

$$\frac{dv_x}{dy} = C_1$$

Let us integrate once again you get

$$v_x = C_1 y + C_2$$

Now, we will have to evaluate the constants $C_1$ and $C_2$ for which use the boundary conditions. So, we use first boundary condition $v_x = 0$ at $y = 0$. So, if you substitute in this equation you will get

$$C_2 = 0$$

This of course, certainly would have done several times in your calculus course. Now, let us use other boundary condition $v_x = v_p$ at $y = h$ and so, if substitute in this equation of course,

$$v_p = C_1 h; \qquad C_1 = \frac{v_p}{h}$$

So, now, when you substitute in this equation you get the velocity profile

$$v_x = \frac{v_p}{h} y$$

That is the velocity profile which we have been seeing so long. Almost from the beginning of the course we have been looking at this velocity profile we said that we will derive towards the end of fluid mechanics path and that is what we have done now.

And, if you want to represent this is the pictorial representation velocity varying from 0 to $v_p$ at the other end and remember they are all shown as vectors. It is a velocity vector directed along the x direction. From that way it is a kind of I would say milestone been looking at the several times and today's class we have derived it.

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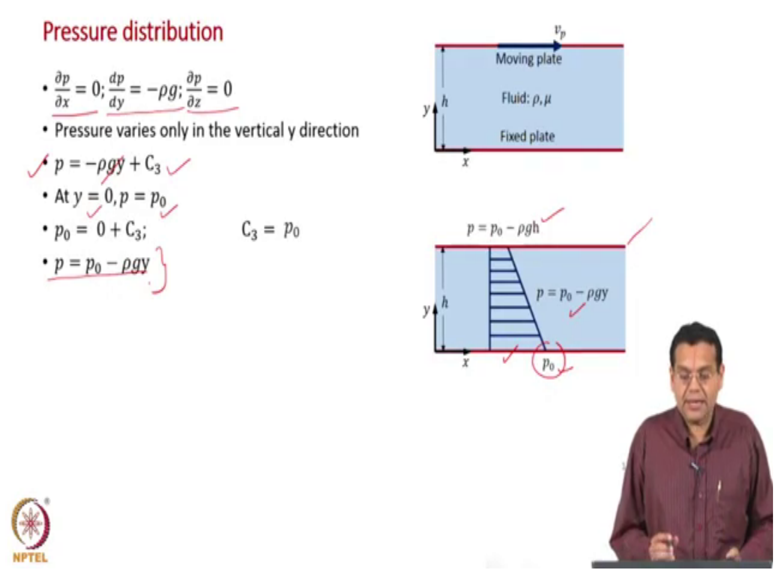


Having derived the velocity profile we can and derive the equation for the pressure profile of the pressure distribution and

$$\frac{\partial p}{\partial x} = 0; \quad \frac{\partial p}{\partial y} = -\rho g; \quad \frac{\partial p}{\partial z} = 0$$

We have seen there is no pressure gradient along the flow direction and then there is no pressure gradient along the z direction also; and there is pressure gradient only in the vertical direction which is the same as the condition of fluids at rest, that is the hydrostatic pressure distribution. So, pressure varies only in the vertical direction.

$$\frac{\partial p}{\partial y} = -\rho g$$

So, let us integrate that. So, if we integrate you get

$$p = -\rho g y + C_3$$

And at y = 0, we specified a pressure $p = p_0$. And so, if you substitute in this equation evaluate $C_3$

$$p_0 = 0 + C_3; \qquad C_3 = p_0$$

So, let us substitute that and you get the equation for pressure distribution

$$p = p_0 - \rho g y$$

This just tells you that the pressure varies linearly with y, it increases if you go down or decreases if you go up and that is what is shown here so, at y = h

$$p = p_0 - \rho g h$$

So, the pressure distribution here is hydrostatic. And, we can also see that because the flow is incompressible, absolute pressure as no meaning we always talk in terms of pressure difference. So, we always find pressure to an arbitrary constant. What does it mean. That is why we said at some position y = 0 we are given the pressure $p_0$.

So, this you get an expression which tells the difference in pressure $p - p_0 = -\rho g h$. Always you find an incompressible flow up to an arbitrary constant that is a more formal way of saying and that is what we are saying here as well that is applicable for all the incompressible flow. So, if it is compressible the absolute pressure can be determined and that is a meaning as well.