

**Continuum Mechanics And Transport Phenomena**  
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**Lecture - 84**  
**Flow Regimes: Laminar and Turbulent flow**

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**Inviscid flows - Outline**

- Flow regimes
- Euler equation
- Bernoulli equation



We have been discussing about the applications of Navier Stokes equation. We have been discussing hierarchically from simple examples to complex applications simple applications to complex applications and we have discussed the I would say the simplest class of applications; namely fluids at rest and fluids in rigid body motion.

Now, we are going to go to the next level of application, what do you mean by that there is going to be flow, but the viscous stresses are negligible. That is the level of application we are going to discuss, and the title is inviscid flows for this class of applications. What is the outline? We start with discussing the flow regimes and discuss the Euler equation and derive the Bernoulli's equation. So, as we proceed you will understand these titles better.

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## Flow regimes

- x-momentum balance
- $\rho \frac{Dv_x}{Dt} = \rho \left( \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = \rho g_x - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right)$
- Ratio of inertial forces to viscous forces
- $= \frac{\text{density velocity velocity/length}}{\text{viscosity velocity / length length}}$
- $= \text{velocity length density/viscosity}$
- $= \frac{(LT^{-1})(ML^{-3})}{ML^{-1}T^{-1}} = M^0 L^0 T^0 - \text{Dimensionless number}$
- Reynolds number  $= Re = \frac{\text{Inertial forces}}{\text{Viscous forces}}$
- Flow regimes based on the range of Reynolds number
  - Low - Viscous forces dominate - Laminar flow ✓
  - High - Inertial forces dominate - Turbulent flow ✓



Now, the flow regime tells about characteristics of a flow and different regimes will have different characteristics and they depend on the velocity of the fluid, the properties of the fluid and the geometry as well. Now, we need to discuss about this flow regime now because that will tell the scope of the present discussion namely inviscid flows.

So, how do we quantify flow regimes and that is what we are going to do now. We quantify usually flow regimes in terms of some dimensionless numbers so, let us arrive at that number now. To do that we will start with the x momentum balance or in general the Navier stokes equation, here written for the x direction.

$$\rho \frac{Dv_x}{Dt} = \rho g_x - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right)$$

So, left hand side has been written in terms of the substantial derivative and right hand side we have the body force surface forces due to pressure and viscous stresses, which is the Navier stokes equation in the x direction.

Now, what we will do is consider the viscous forces on the right hand side. The left hand side we have expressed as mass into acceleration per unit volume, density into acceleration of the fluid. So, the last terms represent the inertial forces. They are all representation of inertial forces.

So, what we are going to do now is take ratio of the inertial forces to the viscous forces. What do we mean by that? Take terms on the left hand side when I say terms representative term

will be taken on the left hand side and the right hand side the viscous terms will be taken, and the ratio will be taken. So, let us do that.

$$\frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{\text{density velocity velocity /length}}{\text{viscosity velocity/length length}}$$

So, inertial force if you see you have density and of course, the expression is written in terms of words rather than variables and we have velocity here and then we have gradient of velocity which is expressed as velocity per length.

Now, coming to the viscous forces on the right hand side, we have viscosity and then we have the second derivative of velocity. So, which is because it is a Laplacian, and which is expressed as velocity by the lengths squared. So, what we have done is taken the inertial forces on the left hand side, taken the viscous forces on the right hand side and taken the ratio between these two forces.

Of course, one velocity term cancels and one length term cancels leaving us with velocity, length, density in the numerator and viscosity in the denominator.

$$\frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{\text{velocity length density}}{\text{viscosity}}$$

Now, let us look at the dimensions of these physical quantities.

$$\frac{\text{velocity length density}}{\text{viscosity}} = \frac{(LT^{-1})(L)(ML^{-3})}{ML^{-1}T^{-1}} = M^0 L^0 T^0$$

If you simplify of course, you get a dimensionless number. That is not surprising because these terms are being taken one from the left hand side one from the right hand side of a conservation equation. Dimensions of all the terms on the left hand side and the right hand side in a conservation equation should be same. That is why when you took the ratio of these two terms we ended up in a dimensionless number and this dimensionless number is called Reynolds number named after Osborne Reynolds a scientist and so, the Reynolds number  $Re$  is equal to the inertial forces by the viscous forces.

$$\text{Reynolds number} = Re = \frac{\text{Inertial forces}}{\text{Viscous forces}}$$

Now, how do we classify the regimes based on range of Reynolds number? We said, flow regimes tells about characteristics of a flow different regimes have different flow characteristics and we said we are going to classify flow regimes based on a dimensionless number and that dimensionless number here in this case is Reynolds number.

Now, let us say the Reynolds number is low. What does it mean? The inertial forces are less, viscous forces are large, which means they dominate and we call such a flow as laminar flow. When can it happen? The velocity is less, the length is small. What do you mean by length here that tells about the dimension of the region through which the fluid flows for example, if you have a flow through a pipe then the diameter is used as a length. More formally we call as length scale velocity scale etcetera and velocity could be the average velocity of the fluid through a pipe.

So, when can laminar flow happen? The velocity is less and the fluid flows through a channel of small dimension and the density is less and viscosity is high. For example, it could be a viscous oil; a very viscous oil flowing at low velocity through a channel of small dimension can result in laminar flow. In fact, flow through all the microfluidic devices will be in the laminar flow regime.

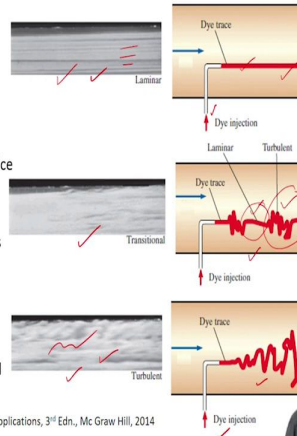
What happens at high Reynolds number? The inertial forces dominate, the numerator the inertial forces dominate and we call the flow to be turbulent. When can it happen? The velocity is high, the channel dimensions, let us say diameter is very high and density is also high and viscosity is less. For example, could be flow of water which has relatively low viscosity compared to oil and so, flow of water at high velocity through a large diameter pipe will result in turbulent flow. Not could even flow of air at very high velocity through a channel of large dimension.

So, at low Reynolds number we have laminar flow, at high Reynolds number we have turbulent flow and viscous forces dominate a low Reynolds number, inertial forces dominated high Reynolds number. Which means in the laminar flow we have viscous force is dominating, turbulent flow inertial forces dominating.

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### Flow regimes

- Laminar flow
  - At low Re
  - Movement of fluid in "layers"
  - Orderly, often slow and steady
  - Rare both in nature and in engineering practice
- Transition
  - Laminar flow undergoes a transition and eventually becomes turbulent as flow speeds increase
- Turbulent flow
  - At high Re
  - Chaotic, disordered, and unsteady
  - Encountered in almost all flows in nature and engineering practice



Cengel, Y. A. and Cimbala, J. M., Fluid Mechanics : Fundamentals and Applications, 3<sup>rd</sup> Edn., Mc Graw Hill, 2014



Now, what are we going to discuss now, what are the characteristics of the flow? We said different flow characteristics prevail in different flow regimes and that is what is shown here the figures are taken from Cengel and Cimbala, very nice representation. So, let us start with laminar flow, as we have discussed, we observed laminar flow at low Reynolds number.

What happens in laminar flow? The word laminae means layers fluid layers. Generally lamina means layer and laminae means fluid layers. So, in this case that is why we have termed this flow to be laminar flow. And in this case movement of fluid takes place in layers and that is what is shown in the first photograph (top) of a laminar flows and you can easily absorb the fluid to flow as in layer by layer.

Very nice flow behaviour and if you have flow between let us say it could be flow between two parallel plates, it could be flow in a pipe etcetera. So, flow happens layer by layer. So, movement of fluid in layer by layer and it is very orderly flow, often slow because Reynolds number is less and steady as well.

Now, remember we discussed about different techniques of flow visualization. We discussed about streamlines, path lines, streak lines and we said experimental if you want to visualize flow we measure the streak lines. How do we measure streak lines by injecting a dye and that is what is shown here a dye is injected and the streak lines are observed. So, in the flow is laminar you get yeah smooth streak line without any disturbance that is what you see there. So, that tells that the flow is laminar. So, if you want to visually see whether a flow is laminar

or not, inject a dye and if you observe defined line smooth line without any disturbance then you can conclude the flow to be laminar.

Now, laminar flow is rare in nature and in engineering practice. So, let us look at the other extreme. We will come back to transition shortly. We look at the other extreme namely the turbulent flow, which happens at high Reynolds number, that is what we have seen. Now, this turbulent flow is exactly almost opposite to the laminar flow behaviour. Why is it? It is chaotic completely chaotic and disordered and unsteady.

Inherently turbulent flow is unsteady, always there will be velocity fluctuations. So, turbulent flow is inherently unsteady and a photograph is shown of a turbulent flow, you can see the difference between this photograph for laminar flow and this photograph for turbulent flow. The flow here is chaos disordered and unsteady and look at the words, almost opposite orderly and then disordered and this is steady and this unsteady and of course, here it is slow here is of course, going to be at high Reynolds number.

Now, how do you visually observe? If you inject a dye look at the streak line there completely zigzag, completely disordered; opposite to what you observed in laminar flow. So, rapid mixing of fluid elements take place resulting in such a streak line and usually you find turbulent flow, both in nature and in engineering practice.

Of course, the in engineering practice whether laminar flow, turbulent flow is in your hand because you can send fluid at low velocity high velocity. But in terms of high better heat transfer characteristics, mass transfer characteristics, those characteristics are better under turbulent flow conditions. So, always prefer turbulent flow conditions in industry practice.

Of course, if you look at applications like microfluidics the flow is laminar. Now, between these two extremes, we have the transition flow regime and look at the photograph here, it is somewhere in between the nice laminar flow behaviour and highly disordered turbulent flow and that is why we call as transition.

So, while laminar flow happens at low Reynolds number, turbulent flow happens at high Reynolds number and in between you have the transition regime. You have a gradual transition from laminar flow to turbulent flow. In terms of streak lines how do you observe and that is what is shown here; part of the streak line will be laminar, part of the streak line is turbulent. This part if you observe relatively smooth, but this is zigzag.

So, part of the streak line laminar, part is turbulent which means that turbulence has already set in and if you increase the velocity it will become completely turbulent. That is why we have used the word flow regimes. Flow regimes tells about characteristics of flow and now we have seen laminar flow regime has certain set of characteristics, turbulent flow has certain set of characteristics. Of course, transition is somewhere in between.