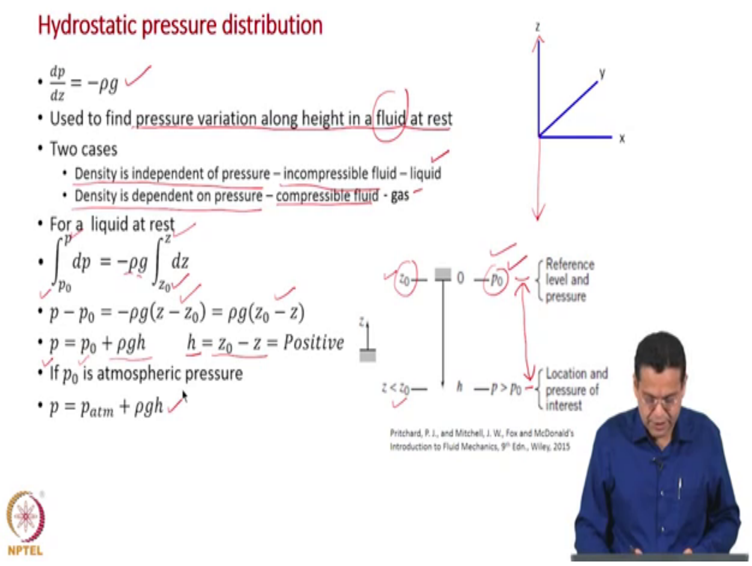
Continuum Mechanics And Transport Phenomena
Prof. T. Renganathan
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture – 81
Hydrostatic Pressure Distribution in Liquid

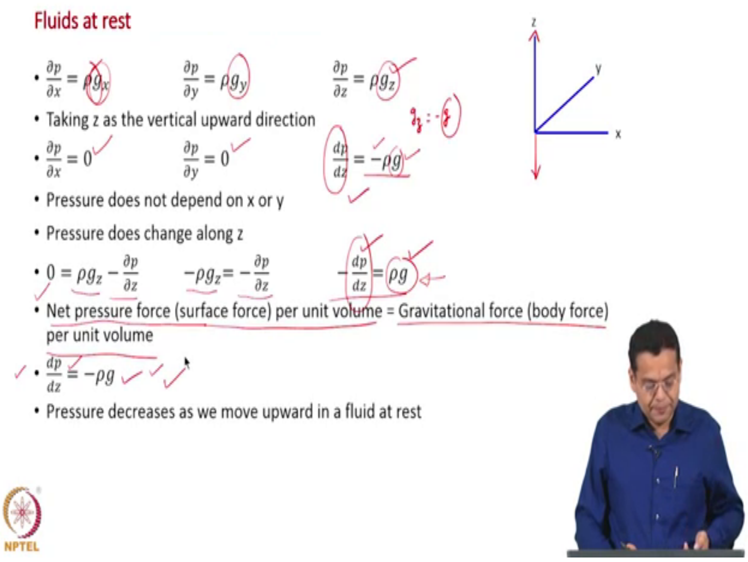
(Refer Slide Time: 00:13)



How are we going to apply this equation? the main application of this equation is to find out the pressure distribution in a fluid, more precisely the word is Hydrostatic Pressure Distribution. Why is hydrostatic? The pressure distribution in a fluid under static condition is call hydrostatic pressure distribution, that is why we said for a fluid under rest there is only normal stress because of pressure.

So, that pressure distribution is call hydrostatic, also like to mention which I will discuss later also. When we discuss for a fluid under rigid body motion, there again we will discuss pressure distribution, but we will not call as hydrostatic pressure distribution, we will call as pressure distribution. Only when the fluid is under rest alone that distribution as hydrostatic pressure distribution.

(Refer Slide Time: 01:12)



So, now we are going to use this equation to get simple expressions for the variation of pressure in a liquid and gas also

$$\frac{dp}{dz} = -\rho g$$

Now, that is the starting equation, we are going to used to find pressure variation along height. When I say pressure distribution, it means that we are going to find out pressure variation along height in a fluid at rest; remember still we are using the word fluid.

Now, two cases. What are the two cases? First case where we take for example, pressure distribution in water in a liquid which is incompressible fluid. What does it mean? Density is independent of pressure that is a first case simpler case and very practical case also. What is the second case? We will consider the case of pressure distribution in a gas for example, air. What is the example? Pressure variation in the atmosphere, we have some let us a atmospheric pressure at the ground level and let us say you are travelling uphill, how does a pressure vary and that is example we are going to take in fact, as a numerical example.

We will see what is the pressure variation from some level to some other level. In this case what happens? Our fluid is air that is why we have here fluid, now we are disguising very clearly liquid and then gas. Gas is the compressible fluid for which density is dependent on pressure. So, that has to be taken into account, when you derive the expression for the hydrostatic pressure distribution. So, we are going to consider two cases: liquid which is incompressible and gas which is compressible fluid. One case density is independent of pressure, other case density depends on pressure.

So now, let us take the case of liquid at rest, for a liquid at rest you are interested in finding out p as a function of z. So, just integrate this equation,

$$\int_{p_0}^{p} dp = -\rho g \int_{z_0}^{z} dz$$

So, let us keep pressure on the left hand side, z on the right hand side and let us integrate. Now what are the limits? We will follow this figure from Fox and McDonald good representation. At some datum level z_0 the pressure is p_0, that is what is one of the limits at z_0 we have p_0 and at some other z we have the pressure p.

So, remember when I integrate of course, g is constant I have taken ρ also the outside because ρ is a constant here for the case of liquids. Now, let us do the integration very simple integration

$$p - p_0 = -\rho g(z - z_0)$$

Now, according to that nomenclature $z - z_0$ is negative. So, we will write as $z_0 - z$ which we will call as h which is the depth. So, $z_0 - z$ is positive which we will call as h which is the depth. So, this equation gets simplified as

$$p = p_0 + \rho g h$$

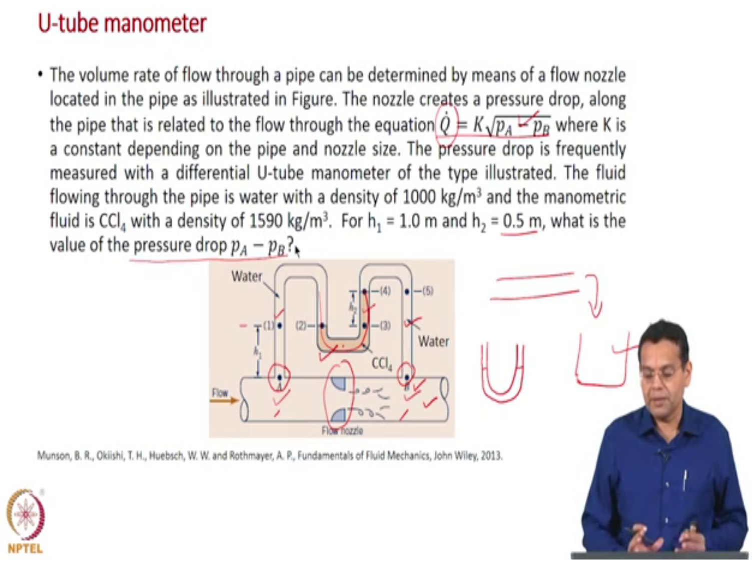
So, we can call it as h as depth. Now, if p_0 is atmospheric pressure, what does it mean? Of course, here you have a closed box, assume it the box is open and you have the surface here opened atmosphere and how does the pressure vary along the depth. So,

$$p = p_{atm} + \rho g h$$

Of course, it is also to be known to us from our earlier physics classes, that pressure increases along the depth and that is what is given by this equation.

This equation is known to you, the concepts also may be known to you. We are doing it more formally starting from the all encompassing linear momentum balance or Navier-Stokes equation, that is a difference I would say.

Example: (Refer Slide Time: 06:20)



So, let us take a example I would say a very practical example. In fact, you will also use this equation several times, you many you may also have a experiment based on this in your fluid mechanics lab as well. What is the application? We have water flowing through a pipe and we like to know the flow rate. One; one way of measuring flow rate is you have the pipe, let us say you allow the water to collect in a let us say a vessel. And, find the time required for a certain amount of volume to be collected, that is not always convenient because water may be continuously flowing in a pipe.

So, in a lab it is possible, but in a industry or in other case, this method of measurement where you take a vessel allow water to collect for sometime divide volume by time to get the flow rate is not always practical. So, what we do is, we have some kind of restriction in the pipe, different ways are possible; you will come across different kind of restrictions in your next fluid mechanics course or in lab as well. What we have is a flow nozzle, in principle what happens is that there is a flow restriction.

So, because of a flow restriction there is some pressure on the we call upstream side, upstream meaning before the flow nozzle. After the flow nozzle the downstream side there is a decrease in pressure because, of flow through a constriction through a restriction, there is a drop in pressure. So now, this drop in pressure depends on the flow rate, if you measure the pressure drop then you can relate that to the flow rate that is objective. Now, let us read part of the question. The volume rate of flow through a pipe can be determined by means of a flow nozzle located in the pipe as illustrated in figure.

You will come across orifice meters, venturi meters several examples, principle begin all of them is that you cause a restriction in the flow. The nozzle creates a pressure drop and that is what you have discussed, along the pipe that is related to the flow through the equation. As I told you, if we measure the pressure drop we can calculate the flow rate. Of course,

$$\dot{Q} = K\sqrt{p_A - p_B}$$

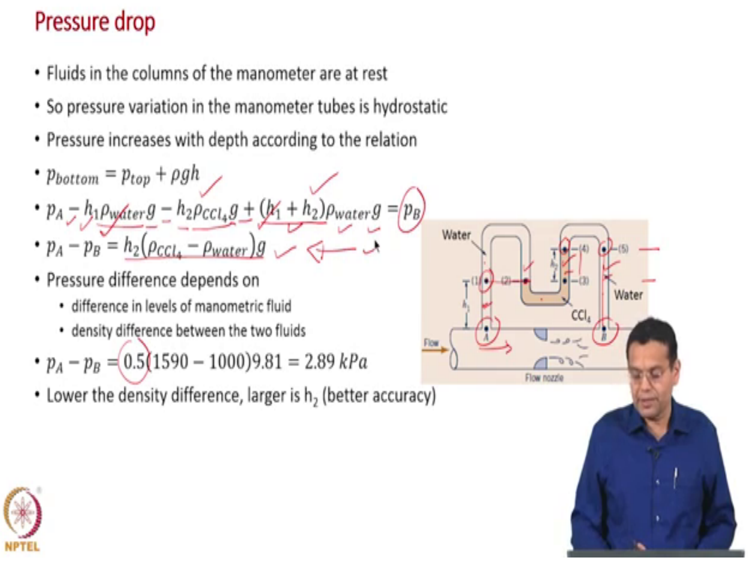
Where K is a constant depending on the pipe and nozzle size. These example we are not going to calculate flow rate that is simple, if K is given you can multiply that with the root of pressure drop and get the flow rate. What we are going to see is how are we going to measure the pressure drop.

Now, how do you measure pressure drop? The pressure drop is frequently measured with the differential U-tube manometer and that is what is shown here. This is the U-tube manometer and you have the connecting tubes here. Also you have tapings on the pipe, we have water column here and you have here is the manometric fluid. The manometric fluid chosen here is carbon tetrachloride and usually you color it and you can also choose a mercury as well ok. These are usual a manometric fluids, if the pressure drop is less you choose CCl_4, the pressure drop is high you choose mercury.

The fluid flowing through the pipe is water, with the density of 1000 kgm^3, the manometric fluid is CCl_4 for the density of 1590 kg/m^3;. Then what is given to us is the height h_1. Now, if the pressures at A and B were same then the level of manometric fluid in both the limbs; we call them a limbs of the U-tube manometer will be same. But, in this case the pressure at B is less, because we say there is a pressure drop.

And, hence the liquid level here, when I say liquid the manometric liquid level in the right limb is higher than that in the left limb and that difference is given as h_2. And h_1 = 1 meter, the difference in the level of the manometric fluid the two limbs, h_2 = 0.5 meter or 50 centimeter. We are ask to find out the value of pressure drop. It may be a very simple example and simple application of a very simple equation also, but very practical as well both in lab, industry everywhere.

(Refer Slide Time: 11:06)



So, let us start this application, few steps only; the we have discussed the pressure distribution in a fluid under rest. So, the fluid in the columns are under rest, but what is the fluid that depends, here you have water, here CCl_4, here again water. So, pressure variation in the manometer tubes is hydrostatic that is a point, we have discussed hydrostatic pressure distribution.

So, we can apply only for the case where the fluids are under static condition, though there is flow here there is no flow in these columns. They are under static condition and we have seen the relationship, their pressure increases with depth according to the relation that is a equation we have derived.

$$p_{bottom} = p_{top} + \rho g h$$

Now, the way in which you do usual you will come across several such problems in fluid mechanics book. The procedure is same for most of them, the way in which we start is we started one end and reach the other end that is what we are going to do now; going through the different columns. So, let us start doing that,

$$p_A - h_1\rho_{water}g - h_1\rho_{CCl_4}g + (h_1 + h_2)\rho_{water}g = p_B$$

We start at A, p_A; now we will go to point 1, now we are going up the column of fluid, we are going up the column of water column. So, there is a decrease in pressure. so, that is why a minus sign in second term. The height is h_1 and the fluid is water. So, ρ_{water} into g, remember g just 9.81.

Now, these two points 1 and 2 are the same level in the same fluid that is water. So, pressures at these locations are same, now once again pressures are 2 and 3 are same because we are in the same fluid, but now CCl_4. So, pressure at 3 and pressure at 2 are same, now we will once again move up the column. Now, what is the fluid? CCl_4. So, once again there is a decrease in pressure. What is the decrease in pressure? $h_1\rho_{CCl_4}g$, because the fluid is CCl_4.

Now, pressure at 4 and 5 are same because, we are in the same fluid that is water; now we will have to reach to position B. So now, we have to come down the column and the column of fluid is water now, now pressure increases. So, $(h_1 + h_2)\rho_{water}g$ is equal to p_B. So, this steps what you have followed can be applied for solving several problems in fluid mechanics book based on manometer, different arrangements can be given.

Sometimes the manometric fluid could be inclined or you could use a manometric fluid is density is lower than water let us say oil, different combinations are possible, but general principle is this ok. Now, let us simplify

$$p_A - p_B = h_2\left(\rho_{CCl_4} - \rho_{water}\right)g$$

So, the pressure drop depends on the difference in the levels of the manometric fluid and depends on the density difference between CCl_4 and water of course, gravity. Pressure difference depends on difference in levels of manometric fluid and density difference between the two fluids ok. Let us calculate

$$p_A - p_B = 0.5(1590 - 1000)9.81 = 2.89\ kPa$$

And, what is the other use of this equation? Looks like a very simple equation but helps us to select a suitable U-tube manometer. What does it mean? Suppose the pressure drop is less and let say you choose mercury, what happens? You will get a very small difference in the levels of the manometric fluid and you will lose it on the accuracy. You like to have a reasonably large difference in level, not very large not very small. If it is very small what happens? You lose out on the accuracy, if it is very large you will have design a very long U-tube manometer.

So, this equation also helps in the selection or design of a U-tube manometer. What is the manometric fluid you should use or what could be the height of the limbs you should use? So, you can use this simple equation to select a manometer as well. So, lower the density difference larger is the h_2 which means better accuracy is there. And, that is why sometimes you use to improve the accuracy, if the pressure drop is very less we use fluid which is lighter than water for example, oil which will have density like around 900.

So, this density difference is roughly about 600 and if you use oil which has density about 900, the density difference is 100. So, you can 6 times increase the level difference between the levels of manometric fluid in the two limbs. So, that is the use of this equation.