Continuum Mechanics And Transport Phenomena

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Lecture – 80
Fluid at Rest : Pressure Distribution

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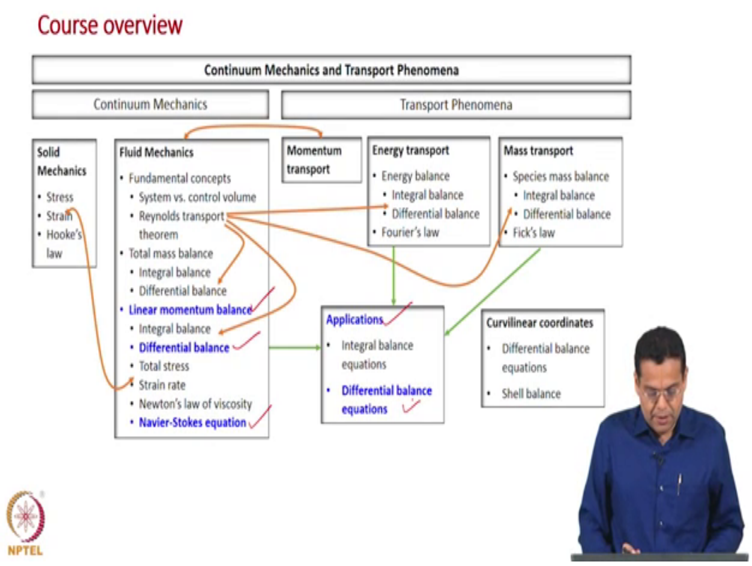

We have derived the Navier-Stokes Equation starting from the differential form of the linear momentum balance. So, now, it is time for us to look at applications of the Navier-Stokes equation and that is what is highlighted here. If you look at the words which are highlighted says linear momentum balance and we are going to look at the application of differential form of that, that is why that is also highlighted.

And then the differential form what you derived is the Navier-Stokes equation and that is highlighted. We are going to look at application so, that is highlighted and of course, the applications of differential balance equations that is the connection between the highlighted words here.

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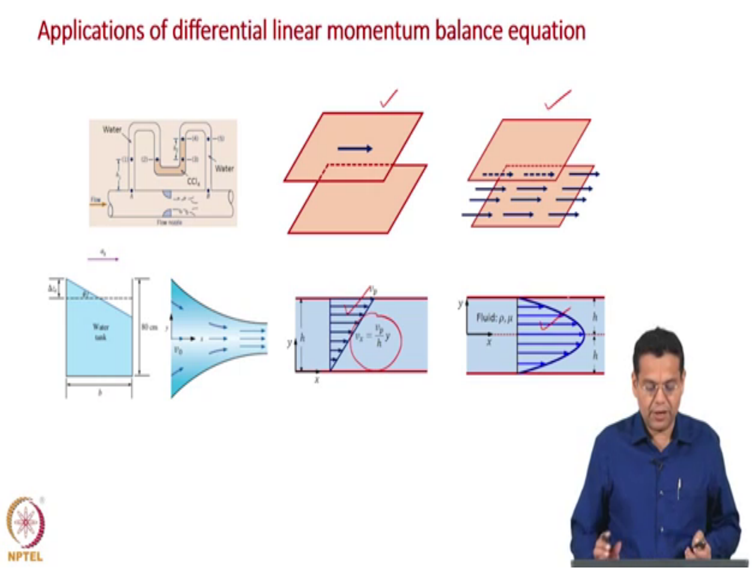

Now, when we began and deriving the differential form of linear momentum balance, we discuss the applications of that and we discussed very detail applications. I would say research level applications both which are old and then new, very traditional applications and then even contemporary applications and recent applications we discussed. But and then we later on discussed this particular slide; so, it is a recall slide. We said though the applications can be to any level of detail, we are going to restrict to simple applications.

So, this slide was discussed earlier. So, right time has now come to really look at this application, solve this applications. So, what are the applications we will be looking at? The first one is application for pressure drop measurement using U tube manometer, second one you have a body of liquid translating; let us say water tanker lorry and what is what how does the surface look what is the pressure distribution and what is the pressure distribution in a converging nozzle.

And then of course, more importantly flow between two parallel plates; one in which one is fixed, the top plate is moving other in which the two plates are fixed, but flow is because of a pump. So, we have looked at this profile several times earlier, we have taken it as given to us you know we always kept saying that towards end of fluid mechanics part we will be deriving them. And, now as part of this applications, we will derive these velocity expressions and we will also plot this profiles both for both the cases. So, that gives rough idea of what are the applications which are in store for us.

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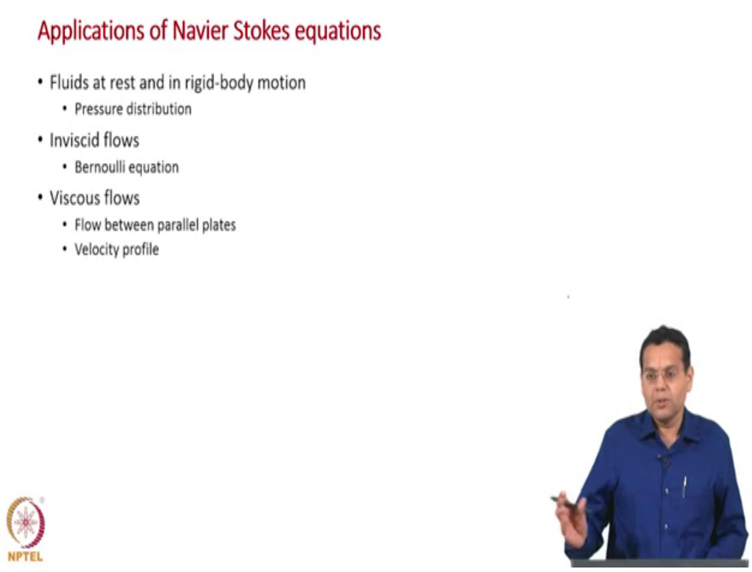

Now, how do we classify these applications? We are going to look at applications hierarchically, what does it mean? We are going to start from simple applications and go to complex applications. In fact, we are going to start the simplest application, what could that be fluids at rest. Not alone fluids at rest, but fluid moving in rigid body motion, what does it mean? If you have a container and then fill with some fluid the entire fluid body motion that is rigid body motion.

We interested in that is pressure distribution; so, we will take up first fluids at rest and in rigid body motion and look at the pressure distribution, then slowly we evolve. We now allow for the fluid to flow, but now consider the case where the viscous forces are not significant. Those are called inviscid flows and where we will discuss the Bernoulli equation. And then once again we evolve further, we allow for fluid to flow and also consider viscous forces.

They are called as viscous flows and that is where we discuss the flow between the parallel plates and what are we interested now in the velocity profile or in the velocity distribution. So, we hierarchically go from the simplest is just fluids at rest to flows with viscous effects.

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So, now let us start with the first level of applications namely fluids at rest and in rigid body motion. So, what is the outline? Fluids at rest first; look at the hydrostatic pressure distribution; we will understand these terms as we go along and then fluids in rigid body motion where the whole fluid body is subjected to motion and look at pressure distribution. As we discuss we will understand these outline much more clearly.

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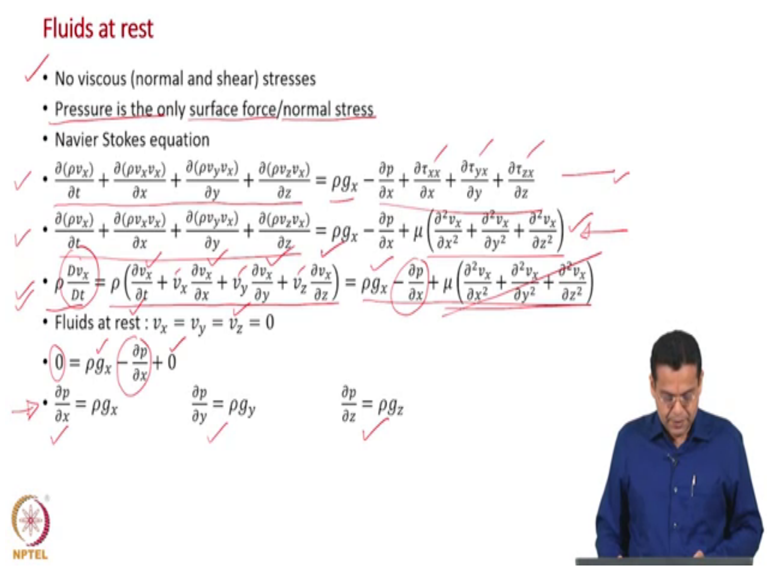

So, let us start with fluids at rest; fluids at rest have a tube or a some container and you have a full of liquid that is all is a fluids at rest. It could be gas, it could be liquid we are going to see both of them. And, now we will have to recall back our discussion on fluids at rest. First, we discussed about stress for solids and then we came to fluids back and discuss about total stress and we said total stresses two components; the hydrostatic contribution and the viscous contribution.

Now, we said fluid under rest has only hydrostatic contribution when it moves, there are additional contribution namely viscous stresses. Now for the present case the fluids are at rest and so, there are no additional contribution namely viscous stresses. So, that is why the first bullet says, there are no viscous stresses both normal stresses and shear stresses.

Now, what is the only stress present in a fluid at rest? It is only the normal stress which is the pressure which is thermodynamic pressure and the that is why it says pressure is a only of course, normal stress and it is a surface force. Pressure is a only surface force to be considered and that is a normal stress. Now let us see, how does the Navier-Stokes equation get simplified for this condition. Now we have discussed the physics of fluids at rest it is a in fact, recall we have already discuss that.

Now, we are going to simplify the Navier-Stokes equation for this particular condition. Now, let us write down the equation

$$\frac{\partial(\rho v_x)}{\partial t} + \frac{\partial(\rho v_x v_x)}{\partial x} + \frac{\partial(\rho v_y v_x)}{\partial y} + \frac{\partial(\rho v_z v_x)}{\partial z} = \rho g_x - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

Now, when we derive the Navier-Stokes equation, we expressed this viscous stresses normal in shear stresses in terms of velocity gradients using Newton's law of viscosity and obtained this Navier-Stokes equation.

$$\frac{\partial(\rho v_x)}{\partial t} + \frac{\partial(\rho v_x v_x)}{\partial x} + \frac{\partial(\rho v_y v_x)}{\partial y} + \frac{\partial(\rho v_z v_x)}{\partial z} = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right)$$

The linear momentum balance substituting the Newton's law of viscosity for the viscous stresses, we obtained the Navier-Stokes equation.

Now, the left hand side, we have seen can be expressed from a material particle view point.

$$\rho\frac{Dv_x}{Dt} = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right)$$

In fact, when we discuss the linear momentum balance we took the left hand side and expressed as the material derivative in and then when we discussed about the Navier-Stokes also. We quickly recalled that the left hand side can be written in terms of the substantial derivative and that is a form we have taken. And remember when we discuss applications almost all the times, we will take the last form of the Navier-Stokes equation.

The left hand side is in terms the substantial derivative of velocity and then these represent the expanded form of the substantial derivative, the local component and then the convective component or local acceleration and convective acceleration right hand side of course, the same terms are present.

Now, for fluids at rest

$$v_x = v_y = v_z = 0$$

So, the equation get simplified to

$$0 = \rho g_x - \frac{\partial p}{\partial x} + 0$$

So, I have the pressure gradient in the x direction and of course, the viscous stresses terms will not be present now because in the fluid is stationary and there are no viscous stresses. We have taken the pressure gradient to the left hand side; the reason is that the variable of interest for a fluid under rest is that is a pressure.

So, we express always the unknown in terms of the known and that is why the pressure term or the pressure gradient term has been taken to the left and gravity on the right hand side.

$$\frac{\partial p}{\partial x} = \rho g_x$$

Similarly we can derive for the y direction and for the z direction.

$$\frac{\partial p}{\partial y} = \rho g_y ; \qquad \frac{\partial p}{\partial z} = \rho g_z$$

So, now, the Navier-Stokes equation which was looked so formidable has become very simple for the case of fluids at rest.

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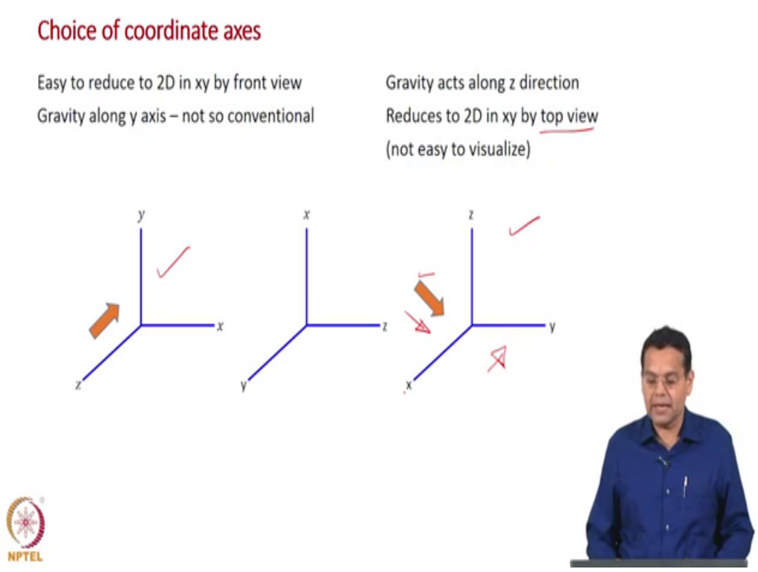

Now, to proceed further, we will have to discuss this slide which you have discuss at least two times earlier which shows the different possibilities of coordinate axis. This is our usual choice of coordinate axis x along horizontal y along vertical and z perpendicular to the slide ok. And, we said that it is easy to reduce these two dimensional case looking at the front view, but gravity now acts along y axis which is not so conventional; always we talk about z as the vertical coordinate.

So, the choice now will be the third coordinate axis where y is along horizontal, z is along vertical, x perpendicular to the slide and now advantage is that gravity acts along z direction which is our usual vertical coordinate. And difficulty we said to reduce to 2D, it is little difficult. Now what is the view we are going to take? We are not going to take the top view we are not going to take that. We are going to take this view so, your z axis is going to be vertical that no doubt about it and x axis going to be the horizontal axis.

So, if you look from this direction, then y is horizontal z is vertical. Now we are going to look at this direction so, x is going to be horizontal axis z is going to be vertical axis. Why is it required? In the previous slide, we have written ρg_x, ρg_y, ρg_z, g_x, g_y, g_z are components of the g vector. To proceed further we should know the orientation of the coordinate axis that is why we are discussing this choice of coordinate axis.

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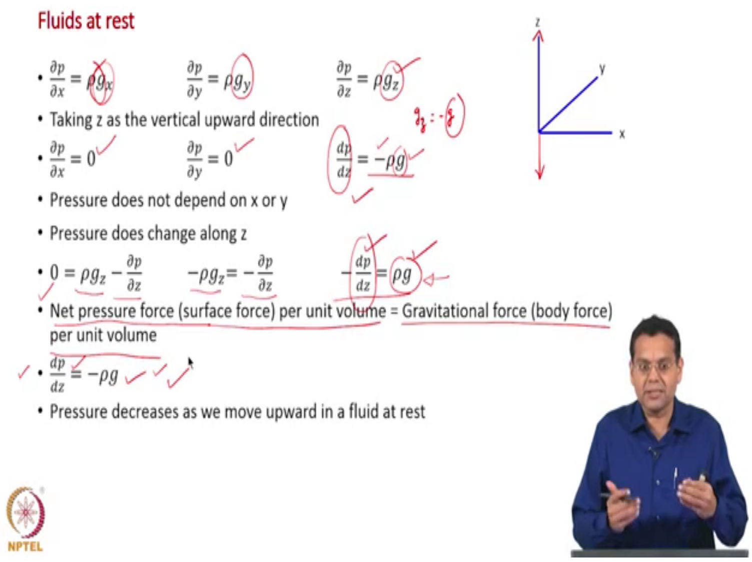

So, now as I told you this is going to be our axis, x is horizontal. z is vertical and y perpendicular to the slide. Now having identified the coordinate axis, let us see how do we proceed.

$$\frac{\partial p}{\partial x} = \rho g_x ; \qquad \frac{\partial p}{\partial y} = \rho g_y ; \qquad \frac{\partial p}{\partial z} = \rho g_z$$

So, these are the equations from the previous to previous slide. Now taking z as the vertical upward direction and once you take that there is no component of gravity along x direction so, $g_x = 0$. There is no component of gravity along y direction. So, $g_y = 0$. Now, the gravity acts towards the negative z axis and so, $g_z =- g$.

$$\frac{\partial p}{\partial x} = 0 ; \qquad \frac{\partial p}{\partial y} = 0 ; \qquad \frac{\partial p}{\partial z} =- \rho g$$

When I write this g has a value of 9.81 magnitude of g and that is what is shown here. Here g_z there is the z component of g vector when I write like this, I have taken care of the minus sign. So, g is just 9.81 or just magnitude of g vector because g acts towards the negative z axis.

How do you physically interpret this? If you have a fluid pressure does not depend on x direction, y direction and of course, pressure does depend on the z direction. In fact, depends only on the z direction.

$$0 = \rho g_z - \frac{\partial p}{\partial z}; \qquad - \rho g_z = \frac{\partial p}{\partial z}; \qquad - \frac{dp}{dz} = \rho g_z$$

Also like to mention we have written as $\frac{dp}{dz}$, the reason is to begin with we said p can depend on x, y, z. But now after taking the vertical direction to be along z and x and y are on horizontal plane and now p depends only on z. So, the partial derivative becomes the total derivative. Now how do you interpret this is equation or in terms of words how do you put? What is $-\frac{dp}{dz}$? Net pressure force per unit volume please recall back our discussion on physical significance of the linear momentum balance.

All the terms in the equation are per unit volume basis. So, right hand side we have $-\frac{dp}{dz}$ which represents net pressure force which is the surface force per unit volume and what is this ρg_z of course, in the momentum balance, it was on the right side again. And what was the significance of that? It is gravitational force per unit volume both are per unit volume. So, one is a surface force other is a body force the surface force is the pressure force and the body force is the gravitational force.

So, for a fluid under rest these two forces balance that is what is shown here. The net pressure force per unit volume is equal to the gravitational force per unit volume that is a physical interpretation of this simple equation. What is a working equation or how do usually write what is shown here that is how you write.

$$\frac{dp}{dz} =- \rho g_z$$

This tell you that pressure decreases as we move upward in a fluid at rest. If you are here if you have still filled with liquid and if you are moving up the liquid there is decrease in pressure.

So, this tells you that z is upward. So, this tells you pressure decreases as we moved upward in a fluid at rest. So, like to mention that this equation would be discussed almost in the second chapter of any fluid mechanics book. Now where we have discussed is almost towards end of fluid mechanics.

What is the reason? You have fluid mechanics book first talks about fundamentals; the first chapter, then slowly they evolve starting from fluids at rest. So, their second chapter is fluids at rest where they consider gravity as a body force and pressure is a surface force and arrive at this relationship. What we have done based on the scope of this course, we started from the law of physics for the system expressed that for control volume using Reynolds transport theorem.

We got the integral balance and then we expressed and started from that got the differential balance. While doing so, we accounted for all the terms I would say we derived all encompassing Navier-Stokes equation. What do we mean by that? We have transient term the convection term; we took all the forces into account the body force both the surface was we are taken into account pressure and viscous stresses.

And now we are simplifying without considering whatever terms which do not play a role now and we have arrived at the same equation. The equation is same the path in which we have arrived with this equation is different. Straightaway this equation is derived a second chapter of fluid mechanics book considering pressure has the only surface force, they will not consider viscous forces because it is too early to discuss about viscous forces in the second chapter.

So, only pressure is considered as a only surface force of course, gravity is the body force. Now we are considered all the terms and then taken out the terms do not contribute and arrive at the same equation. So, this connection should be clearly understood because we have a different approach and we should know we are arriving at same equation from a more general approach to a more specific case. And, that is why I said we are going to start discussing the application starting from the simpler application not even simpler, simplest application. So, in this form does not even look like Navier-Stokes.