Continuum Mechanics And Transport Phenomena
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Lecture – 77
Newton’s Law of Viscosity: 1 D Form

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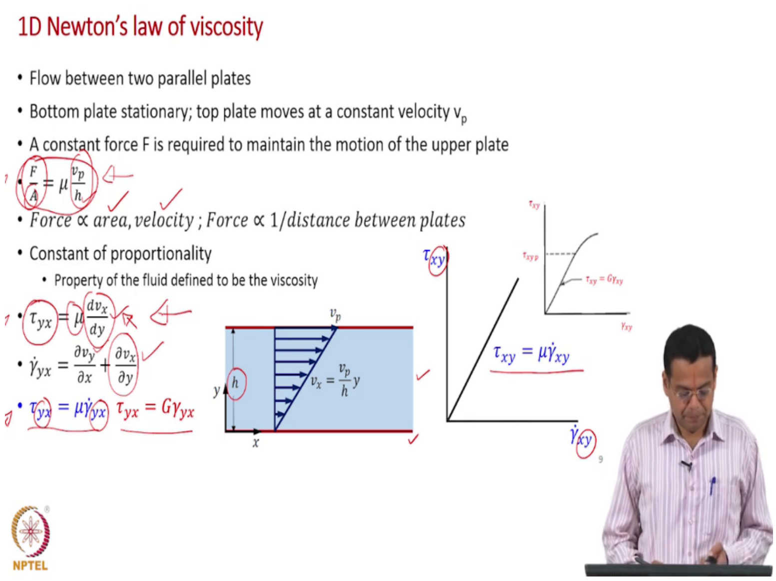

Now, before writing down the 3 D form of Newton’s law of viscosity; we will discuss the 1 dimensional form of Newton’s law of viscosity. So, that you can easily understand the 3 D form. We will consider for the 1 D form of Newton’s law of viscosity is the flow between two parallel plates which we have come across several times; the bottom plate is fixed, the top plate is set in motion at a constant velocity.

And now we require a constant force, you are keeping the bottom plate and then top plate set in motion; which means, some force is required to keep this plate in motion.

$$\frac{F}{A} = \mu \frac{v_p}{h}$$

Now, this force required to keep the top plate in motion is experimentally found to be proportional to the area; obviously, larger the area more force is required.

$$Force \propto area,\ velocity; \quad Force \propto \frac{1}{distance\ between\ plates}$$

That is what is expressed by the equation; force proportional to area, so area has been brought to the left hand side, force per area is proportional to the velocity. So, v_p is on the numerator inversely proportional to the distance between the plates, the distance between the plate is h and that is in the denominator. And what is the proportionality constant is viscosity; it is a material property of the fluid.

The constant of proportionality is a property of the fluid defined to be the viscosity. So, how do you write little more formally, this $\frac{F}{A}$ is the stress; because it is tangential force, it is shear stress. Because the force acts along the x direction, on a plane whose normal is y axis so it is τ_{yx} so,

$$\tau_{yx} = \mu \frac{dv_x}{dy}$$

How do you write little more generally, the shear strain rate is given by the expression which we have seen already

$$\dot{\gamma}_{xy} = \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}$$

So, what we have is; one of the terms, because there is no y component of velocity, so,

$$\dot{\gamma}_{xy} = \frac{\partial v_x}{\partial y}$$

So, if you want to write this more generally

$$\tau_{yx} = \mu \dot{\gamma}_{xy}$$

So, what we have seen is in fact a 1 dimensional form of Newton's law of viscosity. We derived three 1 dimensional form of Newton's law of viscosity, but the second one is very much specific for this case; the most frequently written form is this one. This is the form, moment you come across 1 dimensional Newton's law of viscosity you will come across this form of the equation, $\tau_{yx} = \mu \frac{dv_x}{dy}$.

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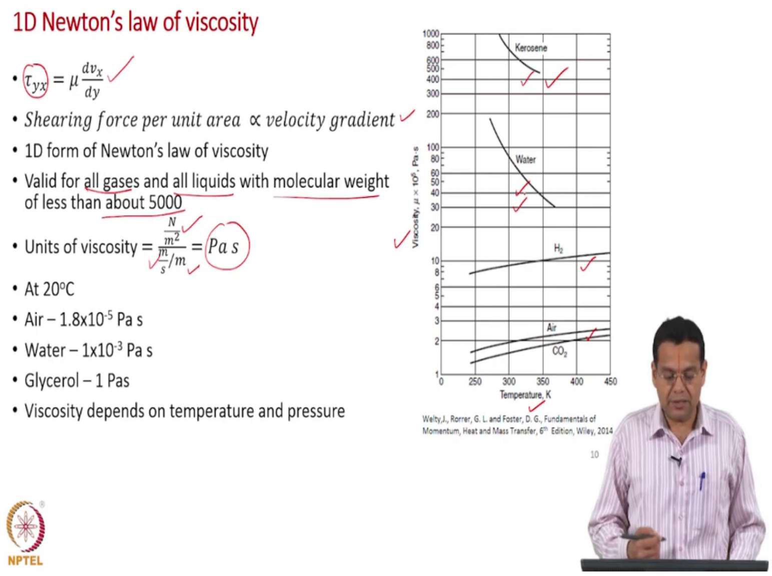


$$\tau_{yx} = \mu \frac{dv_x}{dy}$$

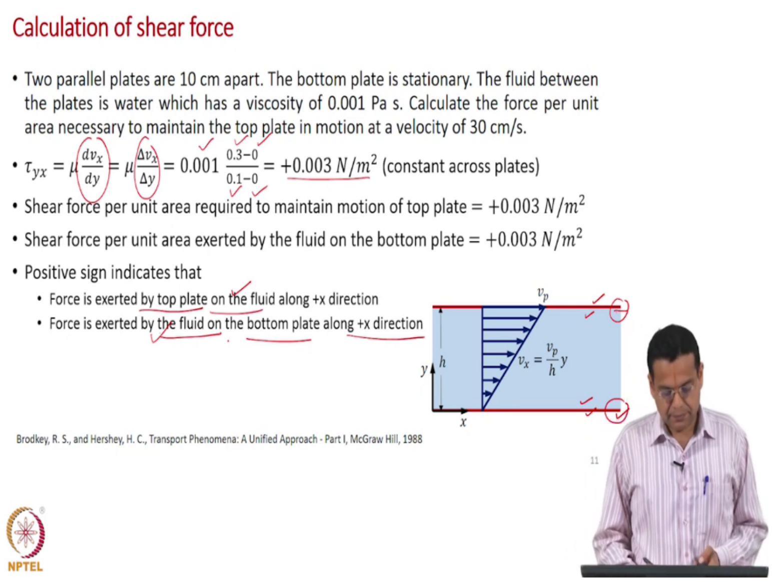
Now, what it says is the shearing force per unit area that is a left hand side is proportional to velocity gradient ok. In the first time we said, force proportional to area, proportional to velocity inversely proportional to the distance, all of them are put together in one simple statement saying that the shearing force per unit area proportional to the velocity gradient and of course, that is what the Newton's law of viscosity says, of course in the 1 dimensional form.

And we used a word saying that experimentally observed. This 1 dimensional Newton's law of viscosity is found to be valid for all gasses and all liquids at molecular weight less than 5000; it is a simple liquids and all the gasses this found to be valid and that itself is a quite wide range of fluids. The units of viscosity is Pascal second.

Let us look at some numbers or viscosity to get a rough idea what is a magnitude at 20 degree centigrade, for air it is roughly about 10^{-5} Pascal second, for water it is 10^{-3} Pascal second much more viscous than air. If you take glycerol it is 1 Pascal second, still more viscous. So, in term of increasing viscosity it is air, water, then glycerol and the graph shown here represents viscosity as a function of temperature and for gasses it increases and for liquids it decreases with temperature .

So, viscosity depends on temperature and pressure; of course, pressure dependency is not so significant, but temperature dependency is significant. So, whenever we look up for value for viscosity, we should be careful about the temperature. Look at the steep decrease in viscosity for liquids, so we should pay attention to the temperature at which we are taking the viscosity data.

Example: (Refer Slide Time: 07:52)



Let us look at some examples of the simple 1 D Newton's law of viscosity. The first example is the flow between two parallel plates. So, two parallel plates are 10 centimeter apart, the bottom plate is stationary, the fluid between the plates is water which has a viscosity of 0.001 Pascal second. Calculate the force per unit area necessary to maintain the top plate in motion at a velocity of 30 centimeter per second.

Solution:

So, very simple application of the 1 dimensional form of Newton's law of viscosity, we have to keep the top plate moving at 30 centimeter per second. So, let us write down the Newton's law of viscosity in the differential form,

$$\tau_{yx} = \mu \frac{dv_x}{dy}$$

Now, replace this differential in terms of difference, substitute the values, (Given data $\mu = 0.001$ and velocity difference is 0.3, because the bottom plate is stationary and distance is 0.1m)

$$\tau_{yx} = \mu \frac{\Delta v_x}{\Delta y} = 0.001 \frac{0.3-0}{0.1-0} =+ 0.003 \frac{N}{m^2}$$

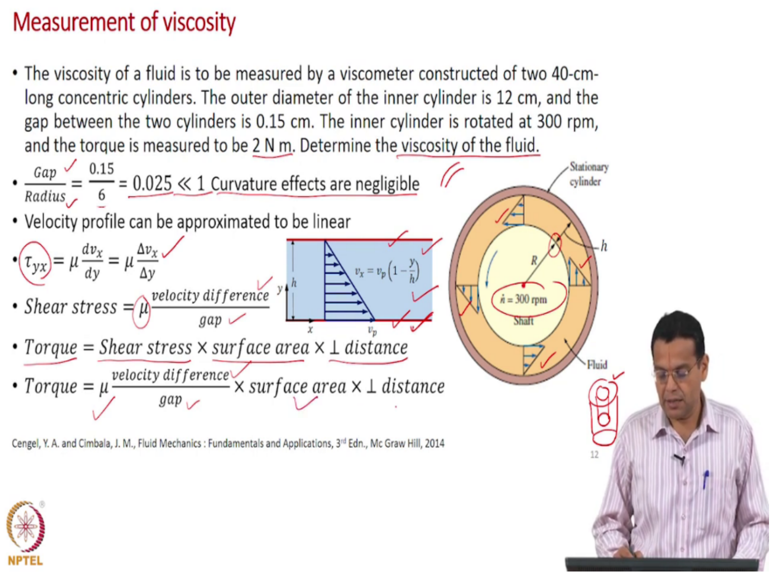
This velocity is velocity of the fluid clinks to the top plate. So, velocity of the fluid is same as velocity of the plate. And so, you get a value of 0.003 Newton per meter square that is a force required to keep the plate moving at 30 centimeter per second. And this value is constant between the two plate, does not depend on the distance between the two plates.

Now, how do you interpret this, shear force per unit area required to maintain motion of top plate. So, we have to apply a force on the top plate in the positive x direction, we have to pull the plate; and so, force acting on the plate in the positive x direction is 0.003. Now when top plate will try to pull the liquid along the positive x direction; at the bottom plate what you have is, this fluid will try to pull the plate in the positive x direction.

So, shear force per unit area exerted by the fluid on the bottom plate is 0.003 and that is what this positive sign indicates. At the top, force is exerted by the top plate on the fluid. So, top plate tries to pull the fluid to the right, force is exerted by the top plate on the fluid in the positive x direction. At the bottom, force is exerted by the fluid on the bottom plate along the positive x direction.

So, sign convention is important; in this case if it is by the fluid, then it will be negative x direction and if it is on the fluid, it will be in the negative x direction. Here we are interested in what is the force on the plate? So, that is on the positive x direction.

Example: (Refer Slide Time: 11:41)



Let us look at another example, a very practical example of measurement of viscosity. Before even going to the example, like to explain the significance of a geometry; what the geometry here? The viscosity of a fluid is to be measured by a viscometer constructed of two 40 centimeter long concentric cylinders. So, we have two cylinders, two concentric cylinders and that is our configuration.

Now, why are we interested in this configuration. Now what I have shown in right hand side, is similar to the example which we have seen earlier; but here the top plate is stationary, the bottom plate is kept in motion. Now this configuration is very simple for two reasons; number one the velocity profile is linear, we always like to have some simple velocity profile, the simplest velocity profile is linear velocity profile.

And then the shear stress, as we have seen in the previous case which is derivate depends on the derivative of velocity is a constant; it does not vary between the two plates. So, one of the simplest configurations is this, the flow between two parallel plates; that is why throughout our discussion we have been taking this example.

We have used this to explain the difference between solids and fluids; that is example we took remember, we took this example to illustrate difference between solids and fluids. We also took this example to illustrate the 1 dimensional form of Newton's law of viscosity. So, in that way very nice example to for such discussion; but now if you want to realize this configuration in lab, it is very difficult. Imagine you have a plate here, another plate here and there is fluid in between and we are setting the plate in motion; how do you realize this in lab that is difficult.

So, that is why we have a configuration where there are two cylinders and there is fluid between the two cylinders and you are rotating the inner cylinder and that is why I took this case, the bottom plate to be set in motion. Now the fluid set in motion between these two cylinders is analogous to the fluid flow between these two parallel plates. But when is it valid?

It is valid when the gap between the two cylinders is very small and it is very small, then you can ignore the curvature and take it that to be equivalent to the flow between the two parallel plates. And you use this arrangement for measuring measurement of viscosity and when you make such measurements, the gap is very small, so that you can assume the velocity profile to be linear. And that is what is shown here, the velocity profiles are shown to be linear here; when is it valid? The gap between the two cylinders is very less.

So, now let us continue the example, the viscosity of a fluid is to be measured by a viscometer; viscometer is instrument used to measure viscosity. See actual viscometer can have different configuration, were the principle is this; constructed of two 40 centimeter long concentric cylinders. The outer diameter of the inner cylinder is 12 centimeter and the gap between the two cylinders is 0.15 centimeter. Look at the value 0.15 centimeter is a gap, but the diameter is 12 centimeter.

The inner cylinder is rotated at 300 rpm and then you are measuring the torque which is 2 Newton meter this is a torque meter to measure the torque. Determine the viscosity of the fluid. A very practical example, which illustrates the use of 1 dimensional form of Newton's law of viscosity; more importantly we have discussed how to realize this flow in a laboratory by having two coaxial cylinders.

First step what we should do is, check really whether the gap is very small. How do you do that? You divide the gap with the radius, to find out the value

$$\frac{Gap}{Radius} = \frac{0.15}{6} = 0.025 \ll 1$$

So, we can conclude that the curvature effects are negligible; then only we can apply the simple 1D Newton's law of viscosity. Velocity profile can be approximated to be linear that is a conclusion, because the curvature effects are negligible.

So, now, let us apply the Newton's law of viscosity first in differential form and then we have written in terms of difference.

$$\tau_{yx} = \mu \frac{dv_x}{dy} = \mu \frac{\Delta v_x}{\Delta y}$$

So, let us put in terms of words becomes easier to proceed further; the shear stress that is a left hand side is equal to μ into the velocity difference by the gap. Now we are measuring torque, we have to find out viscosity. So, what we are going to do now is; find the relationship between torque and then viscosity, we are proceeding towards that.

So, let us write an expression for torque

$$Torque = Shear\ stress\ X\ Surface\ area\ X\ distance$$

Torque is force into the perpendicular distance. The force is the stress into the surface area. We have related shear stress to the velocity difference in the previous equation. So, let us substitute that equation, shear stress is equal to mu into velocity difference by the gap; when I say gap, the distance between the two cylinders multiplied by surface area multiplied by perpendicular distance.

$$Torque = \mu \frac{velocity\ difference}{gap}\ X\ Surface\ area\ X\ distance$$

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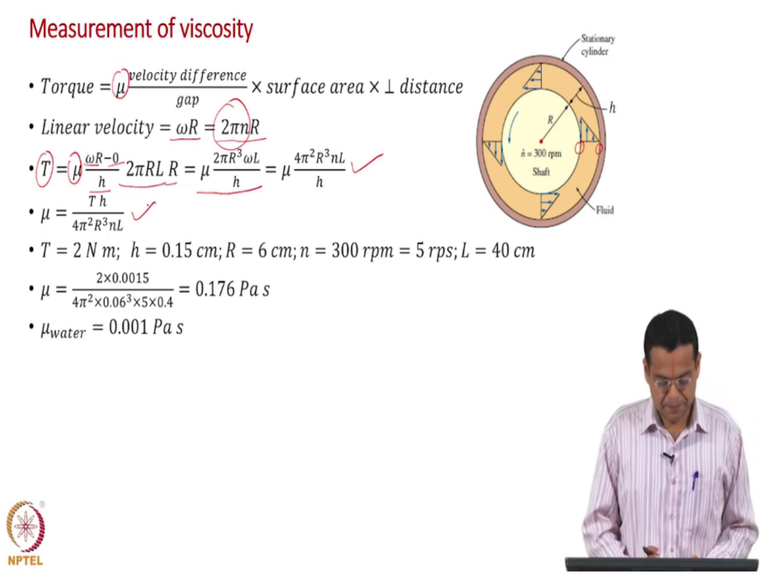

So, let us continue, the same expression is written here.

$$Torque = \mu \frac{velocity\ difference}{gap}\ X\ Surface\ area\ X\ distance$$

Now, when I say velocity difference, whenever you say velocity, we mean the linear velocity. So, how do we convert the angular velocity to linear velocity

$$Linear\ velocity = \omega R = 2\pi nR$$

So, let us substitute this, torque is denoted by capital T here,

$$T = \mu \frac{\omega R - 0}{h} X\ 2\pi RL\ X\ R = \mu \frac{2\pi R^3 \omega L}{h} = \mu \frac{4\pi^2 R^3 nL}{h}$$

So, we have related torque and viscosity. So, let us rewrite this expression for viscosity, because we have to find out the value of viscosity. So,

$$\mu = \frac{Th}{4\pi^2 R^3 nL}$$

So, all the values are known; torque is measured, h is a gap between the two cylinders which is 0.15 centimeter, R the radius is 6 centimeter; n the speed of rotation is 300 rpm. We have to substitute as rps, so 5 rps; L is 40 centimeter, substitute all in terms of SI units and then if you evaluate you will get

$$\mu = \frac{Th}{4\pi^2 R^3 nL} = 0.176\ Pa.s$$

For water it is 0.001; which means, you really have a viscous liquid between the two cylinders.