Continuum Mechanics And Transport Phenomena
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Lecture – 76
Stress Strain Rate Relation: Introduction

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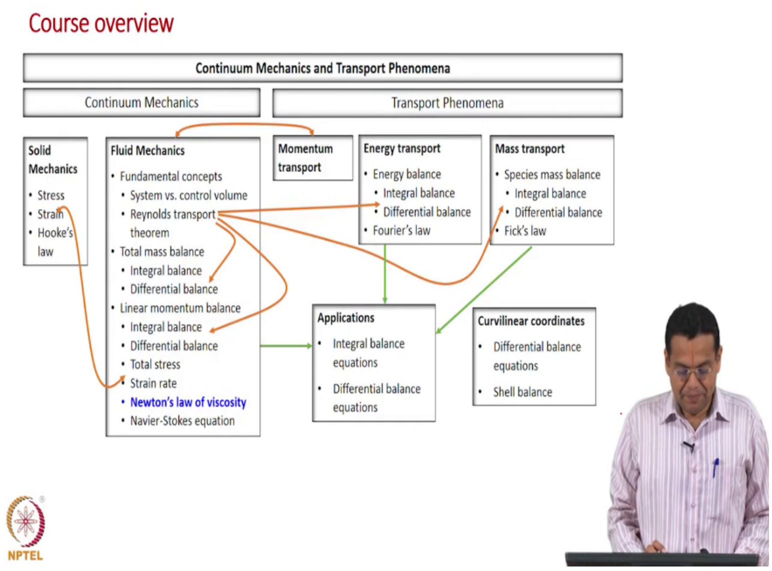

We are going from the linear momentum balance to the Navier-Stokes equation and we have shuttle back and forth fluids and solids few times. To understand total stress we went to solid mechanics, understood stress, came back to fluid mechanics. To understand strain rate once again we went to solid mechanics, understood strain and then came back.

Now, to understand Newton's law of viscosity, we once again went to solid mechanics, discussed Hooke's law there as a previous lecture now we have come back to fluid mechanics discuss Newton's law of viscosity that is where we are in the overall outline.

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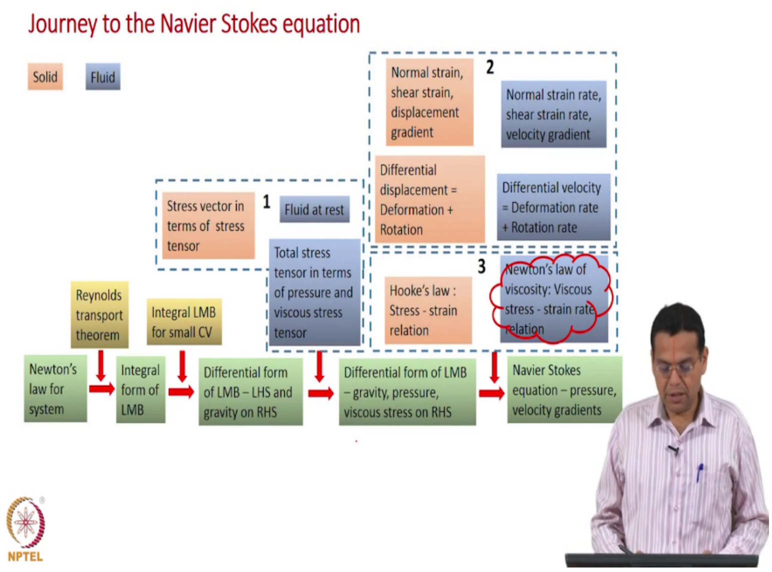

So, in terms of our journey looks like we are almost close to the end and first time we went to solids to understand forces, stress etcetera, second time to understand strain and we are now visiting third time. What was the whole objective we wanted to relate the viscous stress to the velocity gradients or the strain rate, which is the Newton's law of viscosity, we just went to solid mechanics to understand Hook's law and now we have come back to fluid mechanics to almost write down the Newton's law of viscosity.

I would not say we can derive you will see that, we will we understood Hooke's law very clearly. We already discussed the analogy between solid and solids and fluids so, we will use this use that analogy to almost write down Newton's law of viscosity which is the relationship between the viscous stresses and strain rate ok. This is what we want to do for quite some time and that we will achieve in today's lecture.

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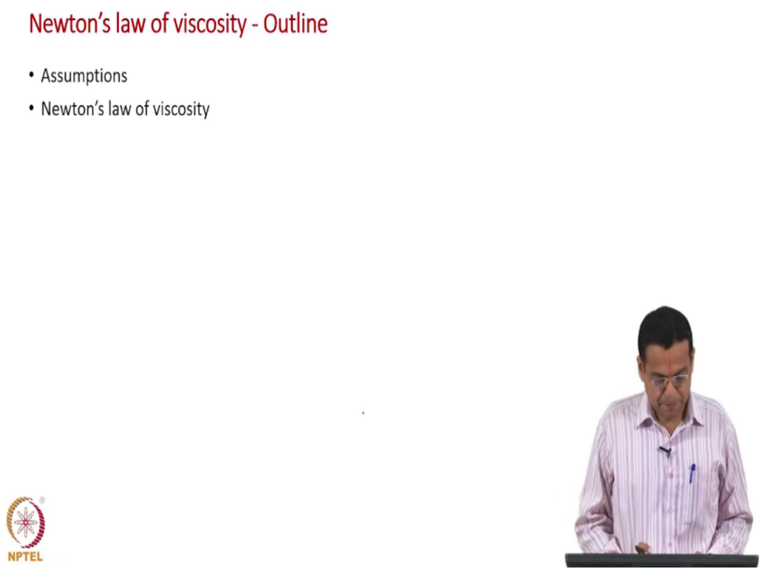

In terms of outline, this is the outline. If you look at this outline, this analogous to what we discussed for Hooke's law. It is said Hooke's law outline, here we have Newton's law of viscosity outline; there we said assumptions, here also assumptions; there we said Hooke's law, here we have Newton's law of viscosity. So, this slide is analogous to what we have done what we have discussed for or what we have seen for solids.

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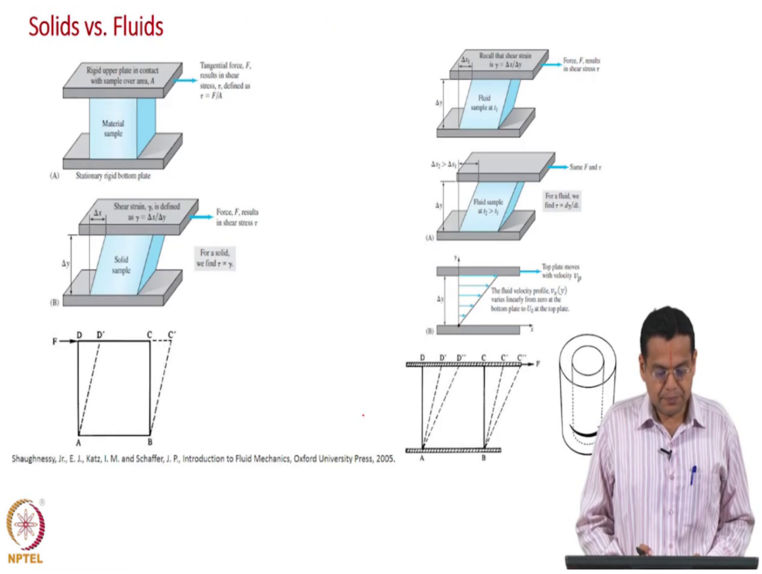

Now, just to quickly recall the next few slides are recall slides. It is better to recall the difference between solids and fluids. So, I would suggest you to go to the earlier discussion where we have discussed the difference in detail. We will just recall them major difference now.

Solids deform and stop deforming. Fluids continue to deform and the difference can be nicely explained using the cylinder example. If you take a solid material inside, the force required to turn the cylinder depends on how far the solid is deformed from some initial state.

But, if we have a fluid in between the two cylinders the force required to rotate the cylinder depends on how fast you are rotating, how depends on the rate of deformation. The first case was force depended on deformation how far you are from initial state. Second case for the case of fluids depends on the rate of deformation how fast you are rotating.

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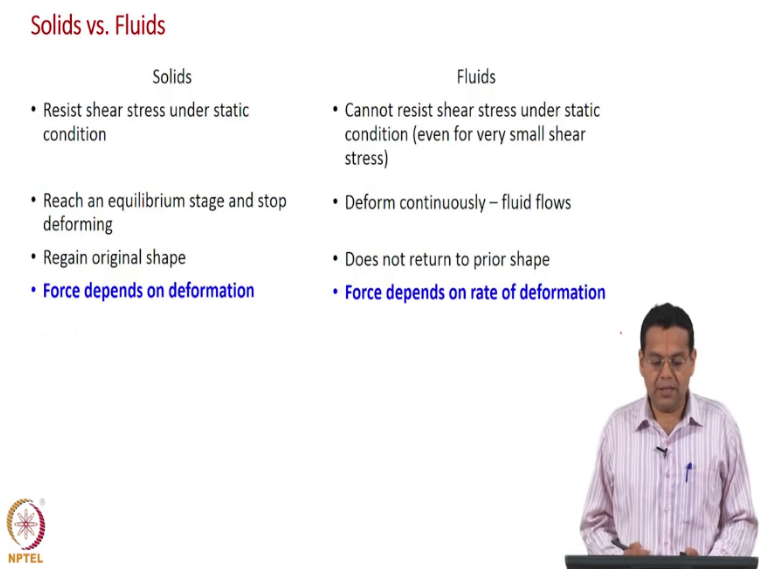

And, we looked at the difference between solids and fluids and we highlighted this difference when we discussed about strain rate for fluid mechanics. Force depends on deformation for solids; force depends on rate of deformation.

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Solids vs. Fluids

Solids	Fluids
Deformation	Rate of deformation
Translation Rotation Normal strain Shear strain	Translation rate Rotation rate Normal strain rate Shear strain rate
Displacement field Displacement gradient	Velocity field Velocity gradient
$\epsilon_{xx} = \frac{\partial u_x}{\partial x}$	$\dot{\epsilon}_{xx} = \frac{\partial v_x}{\partial x}$

For the case of fluids we also discussed this table which brings the analogy between solids and fluids. We had deformation for solids, rate of deformation for fluids; and we had translation, rotation, normal strain, shear strain for solids and just we added rate for fluids. Of course, we know the actual significance, but quickly understand we said translation rate, rotation rate, normal strain rate, shear strain rate. We had displacement field displacement gradient for solids and velocity field, velocity gradient for fluids.

And, then more importantly the normal strain was related to gradient of displacement for solids, and normal strain rate related to the gradient of velocity for fluids. So, if you recall this it will be easier to understand what we are going to discuss now.

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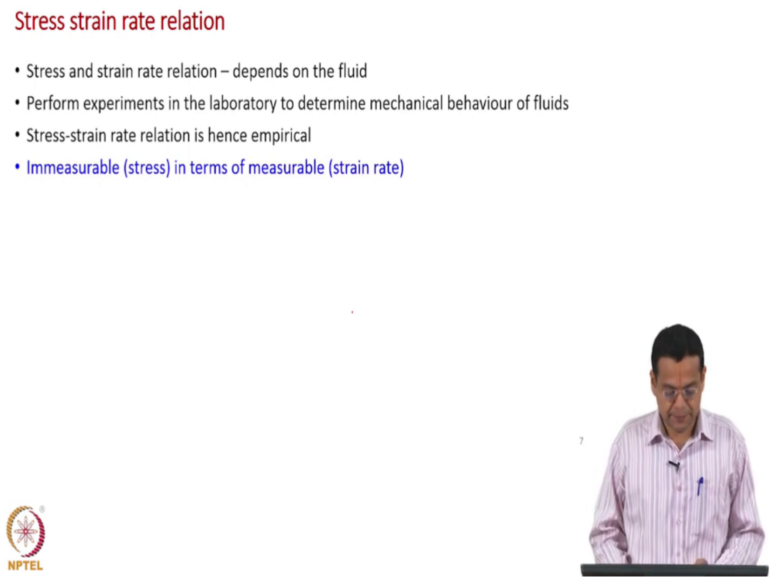

So this slide is also analogous to the slide for solids. What is the first slide? why we should discuss about assumptions. Same reason stress strain rate relation depends on the fluid. When we discuss about total stress for fluids, we discussed about stress under hydrostatic condition, additional stress because the fluid is moving and during the entire discussion on a stress vector, stress tensor whereas, no necessity to discuss about the material at all. It is applicable for any fluid, the whole discussion is applicable for any fluid.

When we came to strain rate, we discussed that strain rate as a material derivative of strain and we discussed about normal strain rate, shear strain rate once again the whole discussion is applicable for any fluid be at air, be it water, be it any oil etcetera. But, now we are going to relate stress and strain rate and that depends on the material, in this case the material is a fluid, exactly this is what we discussed for stress strain analogously.

This relationship is determined by performing experiments in laboratory to determine the mechanical behavior; there it was mechanical behavior of solids now it is mechanical behavior of fluids. Hence, stress-strain rate relationship is empirical it is experimental you cannot derive theoretically, but can only empirically relate these two.

And, as we have done for solids we are going to relate an immeasurable quantity namely stress in terms of the measurable quantity, there it was strain now it is strain rate that is all. Strain rate is measurable why? Strain rate is in terms of velocity gradients. Velocities are measurable, velocity gradients are measurable that is why stress which is immeasurable is expressed in terms of strain rate which is measurable.

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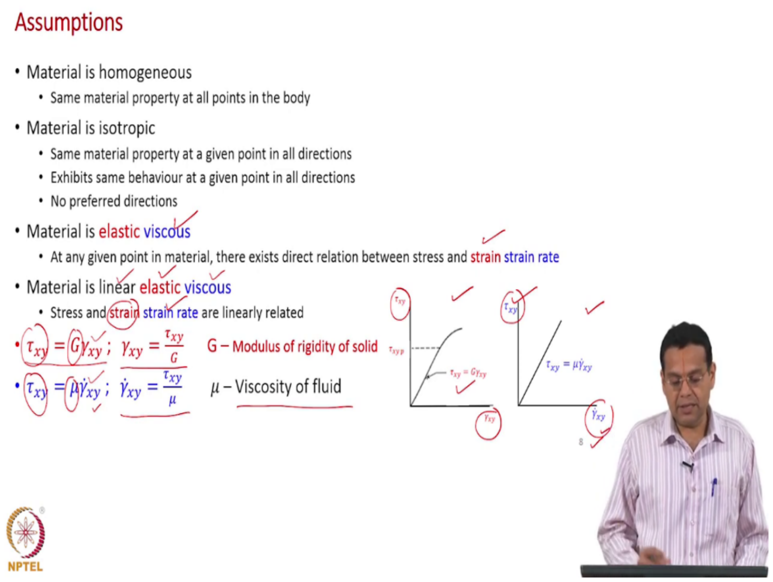

Look at the flow just analogous to what we discussed for solids, we are now going to discuss about assumptions. Of course, quickly we can go through because we have discussed in detail assumption for solids.

- The first assumption is that the material is homogeneous. When I say material here it is a fluid material. What does it mean? Same material property at all points in the body. Viscosity is the material property which is known to us. So, let us say viscosity is same at every point in the fluid; every point in the fluid.
- Second, material is isotropic same significance. Same material property at a given point in all directions. We are not going to say viscosity depends on direction it is all independent of direction. So, exhibits same behavior so, the fluid is going to exhibit same behavior at a given point in all directions.
- No preferred directions, that is why we say it is isotropic.

Now, these two terminologies are same both for solids and fluids; homogeneous solid homogeneous fluid; isotropic solid, isotropic fluid. Now, there is a difference in nomenclature.

- We said material is elastic when we discuss about solid. Now, we are going to say material is viscous. So, we should say material is viscous. What does it mean? At any given point in material, there exist direct relationship between stress and strain rate. Earlier it was stress and strain for the case of solids, now it is stress and strain rate for the case of fluids.
- Now, next we said material is linear elastic for the case of solids, now it is linear viscous for the case of fluids. Of course, this terminology in fact, also same both for solids and fluids. Only the terminologies which are same are homogenous isotropic and linear the terminology which is different is whether it is elastic solid or viscous fluid.

Now, we went on to explain what do we mean by linear elastic condition or linear elastic relationship. Now, for linear viscous fluid the stress and strain rate are linearly related; earlier, we said stress and strain are linearly related now we are going to say stress and strain rate are linearly related. That is why it is very advantageous to discuss about solids, so that we can easily extend to fluids that we have been maintaining throughout our discussion.

Now, let us put this more mathematically or in form of a expression. For the case of solids we discussed the relationship between shear stress and shear strain and we took a linear relationship between shear stress and shear strain and that is written here.

$$\tau_{xy} = G\gamma_{xy}; \quad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

This for the case of solids a linear relationship between shear stress and shear strain and the proportionality constant or the property was the modulus of rigidity of the solid or shear modulus of solid. We also expressed that as an expression for shear strain as shear stress by G.

Now, what is the analogous relationship for the case of fluids and that is the graph that is shown here, where we have plotted τ_{xy} the shear stress as a function of shear rate. That is why we discussed the comparison between solids and fluids in the previous three slides as a recall, we said whatever is strain for solids, it is strain rate for fluids. So, now, we can understand the x-axis should become strain rate or more precisely shear strain rate.

And, now we take a linear relationship between shear stress and shear strain rate

$$\tau_{xy} = \mu\dot{\gamma}_{xy}; \quad \dot{\gamma}_{xy} = \frac{\tau_{xy}}{\mu}$$

Now, the proportionality constant is μ which is the viscosity of fluid. So, the role played by viscosity for fluid is the same as the role played by G for the case of solids. G and μ are analogous properties; G is for solids, μ is for fluids.

But, remember left hand side is force or force per area, right hand side for solids it is shear strain, but for fluids it is shear strain rate. We said force depends on deformation for solids and for the case of fluids we said force depends on rate of deformation and that is reflected in the stress-strain relationship also for the case of solids and stress-strain rate relationship for the case of fluids. Of course, we can write that as an expression for strain rate also shear-strain rate as the shear stress divide by μ. So, this nicely compares the analogous relationship between elastic solid and viscous fluid.