Continuum Mechanics and Transport Phenomena

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Lecture – 71
Material Properties

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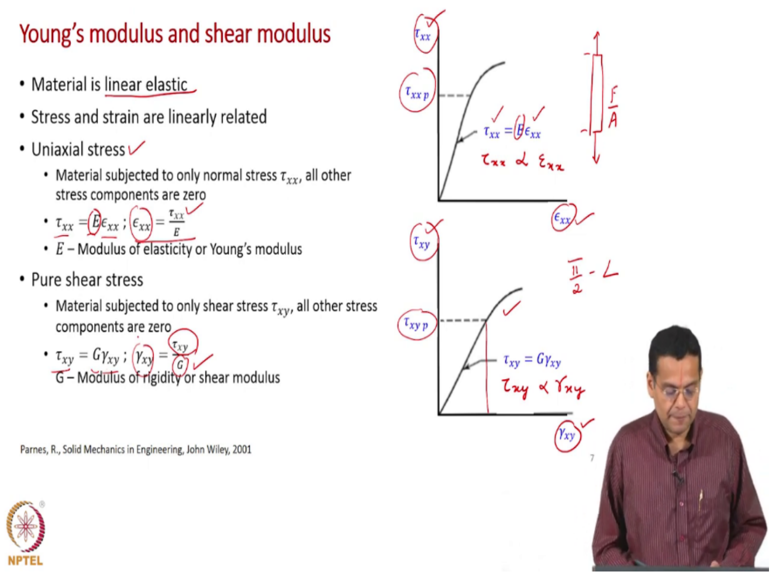

In the previous case we said relationship exists between stress and strain. Now we said could be of any form now we are going to restrict further saying that it should be linear. So, homogeneous, then we said isotropic, then we said elastic, but not general elastic it should be linear elastic. So, material is linear elastic not any general elastic what does it mean?. So, we will discuss this linear elastic with respect to normal stress and shear stress those the two graphs shown here. The first graph plots normal stress versus normal strain a general relationship is shown here as a curve here.

But the relationship is linear up to a point $\tau_{xx,P}$ that is a limit within which τ_{xx} is the normal stress is linearly related to the normal strain beyond that it becomes non-linear. So, our limit of applicability of Hooke's law is within that region where the stress strain relationship, normal stress normal strain relationship is linear. Now how do we experimentally get this data, remember we said, we are going to find the constants the material properties by means of experiment. As I told you,

$$\tau_{xx} \propto \epsilon_{xx}$$

$$\tau_{xx} = E\epsilon_{xx}$$

And the proportionality factor is the Young's modulus which is E, but you determine this by experimentation, we are not going to predict young's modulus, we are going to experimental determine Young's modulus. So, let us see how do we do this we consider let us say rod and then subjected to Uniaxial stress. What do I mean by Uniaxial stress? You apply a normal stress along only one direction, that is why it says Uniaxial stress.

Material subjected to only normal stress tau xx and then all other stress components are zero. We have τ_{yy}, τ_{zz}, τ_{xy} there are no other normal stresses there are no other shear stresses also; that is what we mean by Uniaxial stress. There is only one normal stress along one particular direction in this case we are considering τ_{xx}. Now how do we carry out the experiment you apply a force and then and divide by the area of cross section you get this stress. So, which is your y axis and for every force applied you determined the change in length by original length which will give you the strain and you plot this graph.

So, you take the let us say a rod apply a tensile force and express the force as stress dividing with the cross sectional area and measure the change in length and change in length by original length will give you the strain. Now when you plot as we have discussed we will restrict only to the linear portion of the curve and where $\tau_{xx} \propto \epsilon_{xx}$ and the proportionality factor or in this the slope of this curve is E; which is a Young's modulus. So, as we proceed we will express stress in terms of strain and strain in terms of stress. So, we also express the this relationship as

$$\epsilon_{xx} = \frac{\tau_{xx}}{E}$$

E is the modulus of elasticity or Young's modulus which is very well known. In fact, the Hooke's law which is very well known to everybody is this is this form of the Hooke's law is very well known to us, we are going to extend this to three dimensional case. This is for a one dimensional case subject to only Uniaxial stress there is only subjected only normal stress.

There are no other normal stresses, there are no normal stresses in other direction there is no shear stress, we are going to extend this a three dimensional case where there are normal stresses acting in all the three directions and shear stresses as well. So, that will give you an idea of what is it you are proceeding towards. Now we will discuss analogously for pure shear stress. We discussed the case where there is only normal stress along only one direction we will discuss now for the case of pure shear stress. Now what do we mean by that the material let us say you have a plate.

What do we mean by pure shear stress, we have a material and then we subjected to pure shear stress of course, something like squeezing this. So, material subjected to only shear stress and all other stress components are zero. So, we have only τ_{xy}, other stresses like τ_{yz}, τ_{zx} they are not there and the normal stress as τ_{xx}, τ_{yy}, τ_{zz} they are also not there. Taking a plate and subjecting it to only shear stress, let us say the xy plane.

Now let us say you apply the force express per unit area and that is the shear stress on the y axis and you measure the change in angle, remember we defined the γ_{xy} as the difference between $\frac{\pi}{2}$ and the angle in the final state. So, you measure this angle which is the shear strain and that is the x axis. So, you plot a graph between these two that is the shear stress which you have applied and the shear strain which you measured and you get a relationship.

Now just like the graph between normal stress and normal strain you get a general relationship which could be non-linear, but like in the previous case we restrict to a point where the shear stress shear strain relationship is linear and that relationship is expressed as

$$\tau_{xy} \propto \gamma_{xy}$$

$$\tau_{xy} = G\gamma_{xy}$$

The proportionality constant is denoted by G, which is the modulus as of rigidity also called us shear modulus.

So, in terms of understanding Young's modulus very well known to us which relates normal stress and normal strain, G is a modulus of rigidity or shear modulus which relates shear stress and shear strain. Just like we express the previous relationship for normal strain we are also express this relationship for shear strain.

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

First expression was for the shear stress in terms of shear strain, second expression is for shear strain in terms of shear stress. As I told you as we go along will use both the relationship few times. And in terms of significance let us say a material as very high Young’s modulus what does it mean for the same.

Suppose let us say you subject two material to the same τ_{xx} the material will higher Young modulus will have only a very small value of normal strain. Similarly if you have two material and then subject to the same shear stress the material with higher modulus of rigidity will show a smaller change in angle. So, that is the significance of E and G. Higher the value lower the strain.

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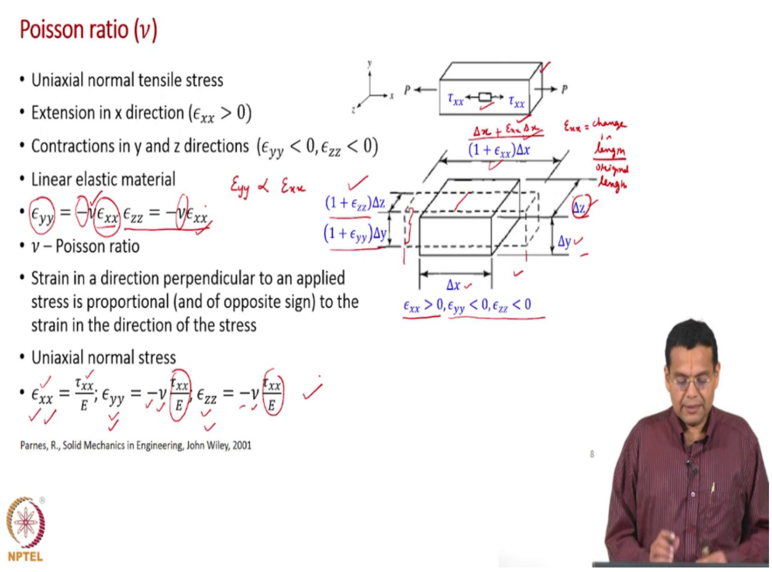

Now, we are discussed two material properties one is Young’s modulus, other is shear modulus. We will discuss one more material property which is Poisson ratio. So, now, I think it should be clear to you what we really meant by material property, when we discussed homogeneous material and then isotropic material. We discuss about material property these are the material properties Young’s modulus, shear modulus, Poisson ratio etcetera.

Now to explain about Poisson ratio we will once again consider let us say a cuboidal block, a block is cuboid shape subjected to Uniaxial normal tensile stress. Uniaxial meaning there is only one normal stress and in this case it is tensile stress ok; that is what is shown here. And you are shown an element which is subjected to Uniaxial normal tensile stress.

And that is shown magnified here the length of the element is Δx, the height is Δy and the width is Δz. Now that is the those are the dimensions in the initial state. Now this object to normal tensile stress and then it undergoes defamation in the final state the dimensions have changed. Now let us look at the length there is an increase in length Δx has become $(1 + \epsilon_{xx})\Delta x$. How did I write that Δx is the original length and ϵ_{xx} is change in length by original length.

So, $\epsilon_{xx}\Delta x$ will give you the change in length when you sum up will give you the final length, just let me repeat. Express this as Δx the first term which is the original length ϵ_{xx} is the change in length by original length. So, the change in length is ϵ_{xx} into the original length which is Δx. So, when you sum up these two you get the changed length or the new length.

Now what happens if you subject let us say a cuboidal to tensile stress along x direction there is decrease in the height and the width along the perpendicular direction. So, will use two terminologies; one is direction of stress other is proponent direction. So, in this particular case there is elongation there is increase in length along the direction of stress, there are two perpendicular directions one is y, other is z along those directions there is decrease in the dimensions of the height and the width.

Let us see that if you look at the height the original height was Δy and that has become this height which is $(1 + \epsilon_{yy})\Delta y$. Coming to the width the original width was Δz and the new width is $(1 + \epsilon_{zz})\Delta z$. These have been written similar to what I have explained here. So, to begin with we had a cuboid dimension Δx, Δy, Δz. You applied a Uniaxial normal stress along the x direction, the direction of stress and there was elongation along the x direction increase in length.

Because of increase in length along x direction there was a decrease in the height and the width along y and z direction. Now, what we had discussed is for the case where $\epsilon_{xx} > 0$, because increase in length; and $\epsilon_{yy} < 0$ and $\epsilon_{zz} < 0$ other case are is also possible. For example, when you are applying let us say a com compressive stress $\epsilon_{xx} < 0$ and other two ϵ_{yy} & ϵ_{zz} can be greater than 0.

The strains in the other two directions can be greater than 0, but throughout our discussion we will consider the case where $\epsilon_{xx} > 0$. That is what we are discussed extension in x direction, contractions in y and z direction.

Now, for a linear elastic material it has been experimentally observed that the normal strain in the perpendicular direction is proportional to the normal strain in the direction of stress.

$$\epsilon_{yy} = - \nu\epsilon_{xx}; \qquad \epsilon_{zz} = - \nu\epsilon_{xx}$$

What is the direction of stress; x direction epsilon xx is the normal strain in the direction of stress. What is the perpendicular direction; y direction one of the perpendicular direction is y direction.

So, left hand side is normal strain in the perpendicular direction. Experimentally it has been observed that $\epsilon_{yy} \propto \epsilon_{xx}$ what is the proportionality constant is ν which is the Poisson ratio. Now we have seen that when ϵ_{xx} is positive and ϵ_{yy} is negative. So, to account for that, we have a negative sign here. So, that the material property which is Poisson ratio is positive.

We do not want a material property which is negative. We have a positive Young's modulus, you have positive shear modulus, so we do not want to have a negative Poisson ratio. So, we take that into account in writing the relationship itself. So, just to repeat this we said stress strain relationships are empirical. Why? Because it is based an experimental observation, we cannot derive this linear relationship. Only based an experimental observation right this linear relationship and the proportionality factor ν has to be found by doing experimentation.

So, both the form of the expression and the constant is based on experimental observation. And so to put this in statement, ν is a Poisson ratio. So, strain in a direction perpendicular to an applied stress is proportional and of course, opposite sign to the strain in the direction of this stress.

So, in this particular case strain in the direction of the stress is ϵ_{xx}, strain in the direction perpendicular to applied stress is ϵ_{yy} they are proportional and the proportionality constant is ν. Negative sign is introduced to take care of one undergoes extension other undergoes contraction. Or when you have extension in one direction there is contraction other direction ok. Similarly you can write for the normal strain in the z direction is proportional to the normal strain in the x direction along which you applied the stress proportionality constant is ν.

And remember we said isotropic material, see we have use the same ν for both the relationship we have not a given two different ν which means that we are assuming ν the Poisson ratio to be independent of direction. If we would had depending then we should use different ν here because we are assuming material properties to be independent of direction. Because we are assuming material is isotropic we are using the same ν.

Now suppose if we subject body to a Uniaxial normal stress what are all the strains. Along the direction of stress we can write

$$\epsilon_{xx} = \frac{\tau_{xx}}{E};\ \epsilon_{yy} = -\ \nu\frac{\tau_{xx}}{E};\quad \epsilon_{zz} = -\ \nu\frac{\tau_{zz}}{E}$$

I told you we will use both form of the relationship, we will write stress in terms of strain also strain in terms of stress. So, this is the relationship which you have seen in the previous slide writing normal strain in terms of normal stress.

So, this is the normal strain along the direction in which the stress has been applied. Now there are two perpendicular directions. So, the normal strain along y direction is $-\ \nu\frac{\tau_{xx}}{E}$. Similarly, you can write for the z direction which is the which is the another perpendicular direction. So, strain in the z direction is equal to $-\ \nu\frac{\tau_{zz}}{E}$.

So, what is the meaning of the statement; if you have a let us say cuboidal element and subjected to Uniaxial normal stress along x direction, then the normal strain along x direction is given by this expression and normal strain along y and z directions are given by these two expressions. So, that is the meaning of the last statement.

So, to summarize the last two slides we discussed what we mean by linear elastic material. We said linear elastic material is a material which has a linear relationship between stress and strain. We discussed the linear relationship for normal stress and normal strain also for shear stress and shear strain. These two introduced two material properties namely Young's modulus and shear modulus.

Now we discussed another material property namely Poisson ratio which relates strain in the direction of stress to the strain in the perpendicular direction ok. And what we have seen this line the equations here are the new subject body to a Uniaxial normal stress. What are all the strains in the three directions; one strain along the direction of stress and strain along the perpendicular directions.