Continuum Mechanics And Transport Phenomena
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Lecture – 07
Lagrangian and Eulerian Descriptions Part 2

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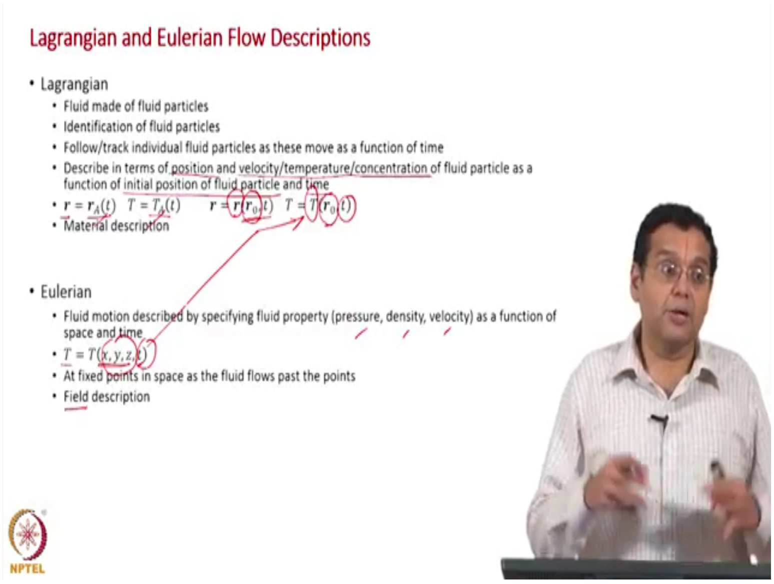

Having seen so many examples of two different ways of flow descriptions, Eulerian and Lagrangian, we will put them in down terms in terms of words. Now, you notice a difference here, the order of the title says Lagrangian and Eulerian. All of an example the way in which I discussed, first I discussed Eulerian and then I discussed Lagrangian.

You will see this reasons for that shortly. First is Lagrangian. Now, a Lagrangian approach is based on the our imagination that fluid is made of fluid particles even to start describing Lagrangian I said let us start taking a fluid particle. So, the concept of fluid particle is integrated with the discussion of Lagrangian. Then what did we do? We said we will identify the fluid particles. So, Lagrangian approach involves identification of fluid particles, we said we will identify by coloring them.

And then we said we are following the fluid particles as they move as a function of time that is what we did. In all the cases, we track the fluid particles as they moved in the region of interest. And then describe in terms of position and velocity, temperature, concentration of fluid particle as a function of initial position of fluid particle and time. The one dependent variable is either velocity, temperature, concentration which you are interested in because we are tracking them. Another dependent variable is a position which automatically also gets measured because we are tracking the fluid particle. What are the independent variables as I have told you the initial position of fluid particle which we denote as r_0 and time.

In terms of expression, r the position of fluid particle is for a particular particle temperature for a particular particle as I have discussed earlier instead of saying A, I will say r_0. So, the position of the particle and then the temperature of the particle those are the dependent variables. Independent variables are initial position and time.

Another name for Lagrangian description is material description. Why is that, because you are following a material particle following a fluid particle and that is why we called Lagrangian description as material description, because we are following a fluid particle.

What is the Eulerian description? Now, for Eulerian description, I need not talk about fluid particles. Though we said it is equivalent to sensing different fluid particles falling through, the point to begin with I will not talk about fluid particles. How do you describe the fluid motion here? By specifying the fluid property. What are the fluid properties? Pressure, density, velocity, temperature, etcetera as a function of space and time that is what we did. For example, in this case, the temperature as a function of space and time; so the dependent variable is temperature, it could be pressure, density, velocity, concentration and independent variables are spatial location and time.

So, I think we should compare these two expressions. The independent variables are the initial position and time in Lagrangian description and in Eulerian description, the spatial location and time are the independent variables. The independent variables are different, because the way in which you do measurements are different. And these are done at fixed point in space as the fluid flows pass this point that is what we said several fluid particles pass through that point at a fixed point, we have a fixed point. The first one is following a fluid particle. Second case, we have to first fixed point, then different fluid particles pass through that point.

Other name for Eulerian description is field description. Whenever we say velocity field, temperature field means that we are describing the temperature variation in a space, velocity variation in a space, and that is why it is called field description. We come across very frequently the velocity field is given by this expression; temperature field is given by this expression.

What do you mean by field? For example, let us take this particular room. As I told you earlier temperature near that condition may be lower, near the entrance may be a slightly warmer which means that there is spatial variation of temperature in this room. Similarly, velocity varies let us say in this room. I want to describe this T temperature as a function of x, y, z that is called temperature field. Similarly, the velocity as a function of x, y, z that could also vary with time that is why we say it is a field description.

So, when you come across first time field, it may not make much sense, but as you become used to that, the word field just tells you that spatial variation in terms of x, y, z and could be time as well. We do not say field for Lagrangian description, we say only field for Eulerian description. The moment I say field let us say velocity field automatically implies that I am talking about the Eulerian description.

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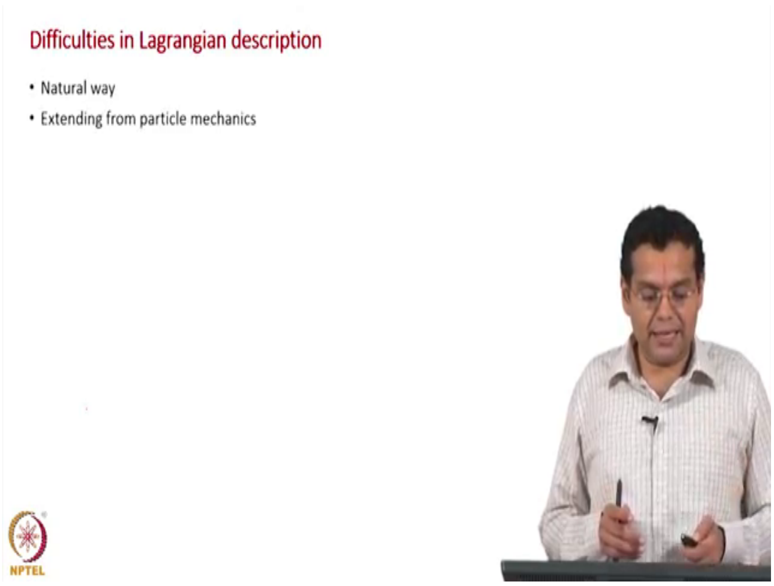

What are the difficulties in Lagrangian description? Natural way, natural way is not a difficulty. This natural way is one the reason why the previous slide I listed Lagrangian first and Eulerian second. why is it naturally we will see that shortly. If you say naturally, if you want to list the sequence, I should have discussed first Lagrangian, then only I should have discussed Eulerian. The reason I discussed Eulerian first is that, that is more easily understandable. The case of a chimney first I said put a temperature sensor, make a measurement, so it is easy to understand. If I say follow a fluid particle and follow that, and measure the temperature, little more difficult to imagine that is why in all the examples we discussed Eulerian first that is how we do an experiment in a real in a lab, but Lagrangian is more natural. Why is Lagrangian more natural?

Now, all of us would have studied particle motions, solid motion in physics. So, for example, we would have discussed about studied about Newton's second law of motion for a solid particle. Moment I say solid particle easy to imagine. Now, our Lagrangian description remember every description was same following a fluid particle we had some position of particle for differentiate to get velocity, differentiate to get acceleration, everything almost same only difference is instead of a solid particle like a ball or an object, it has a fluid particle here that is why the Lagrangian description which is based on description of a fluid particle is a natural way.

Why, because the laws of physics etcetera are all for a solid particle, also should be applicable for a fluid particle. So, the natural way, natural sequences first Lagrangian, and then Eulerian. Just because we are so used to Eulerian way of measurement measuring variables, I discussed Eulerian first and then Lagrangian second.

That is why in the previous slide when we discuss summarized everything in terms of words, I first discussed Lagrangian and then I discussed Eulerian. So, natural order is Lagrangian and Eulerian in terms of physics, in terms of theory. In terms of measurement, Eulerian becomes first, because we are very used to that easy to measure Eulerian also. I said that image a fluid particle, attach a temperature sensor to that, so it also difficult to imagine. But if I say take a thermocouple, put it a point a measure become easy to measure also and see also not even imagine, that is why in terms of reality in terms of measurement Eulerian is first. In terms of theory, in terms of principles, Lagrangian is first. Please keep that in mind.

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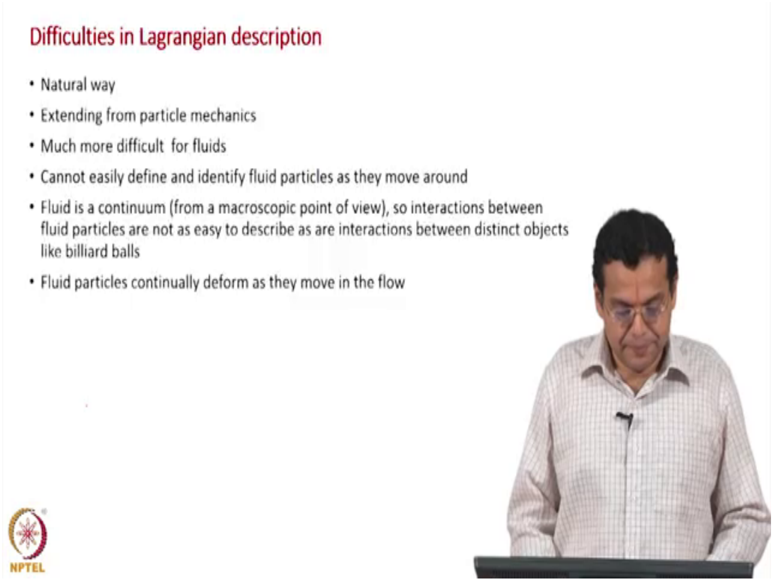

Difficulties in Lagrangian description natural way. Now, we have seen why it is natural way just the laws of physics are all for Lagrangian description only. It extends straight from particle mechanics. Whatever you did for part solid particles, you have to just extend for fluid particles yeah and that is where the difficulty starts. When I say take a solid object, take a ball, easily imagine. You want to play with it, you can play or you want to analyze the force acting on it easily you can do. But now fluid particle, you first spent enough time defining a fluid particle. We had one slide, we had animation all that were required to tell you what a fluid particle is, but for a solid particle, moment I say solid in one second or few seconds is enough to tell you what a solid particle is, but just to make you clear what a fluid particle is, we took enough time to explain what a fluid particle is, so much more different for fluid particles, though the laws should be valid both for solid particle and fluid particle. So, keep in mind our objective of discussing solids and fluids analogously under the team of continuum material or continuum hypothesis. And the continuum hypothesis whatever we discussed usually, discussed taking fluid as example, because these are discussed in a fluid mechanics books, but all those are applicable both for solids, gases etcetera, but easy to understand for a liquids or fluids.

So, We cannot easily define and identify fluid particles as they move around that we have seen very clearly. And fluid is a continuum that is what we have seen from macroscopic point of view. So, interactions between fluid particles are not as easy describe as the interaction within solid objects. If two particles collide you can write a force balance, momentum balance, all those are applicable no doubt, but easy to write for them.

You can find out what is the velocity momentum before collision, after collision, etcetera, you wrote an examples in that physics course, but, same are applicable fluid particles much more difficult to do for fluid particles. So, interaction between fluid particles, why interactions, we have considered fluid particles throughout the region, so they can interact as well. So, describe those interaction becomes difficult not as easily as you do for solid objects.

And we have seen this as well. Fluid particles continuity form as they move in the flow this hardly does not happen for solid objects when two balls collide they hardly deform they are very almost rigid. So, you not worry about deformation at all. But fluid particles no way possible almost invariably they deform and so you will have to account that as well. So, all these make the like make the Lagrangian approach difficult. Keep that in mind what we should keep in mind is that the loss of physics are from a Lagrangian point of view or for a fluid particle only, but description in terms of Lagrangian becomes difficult for us.

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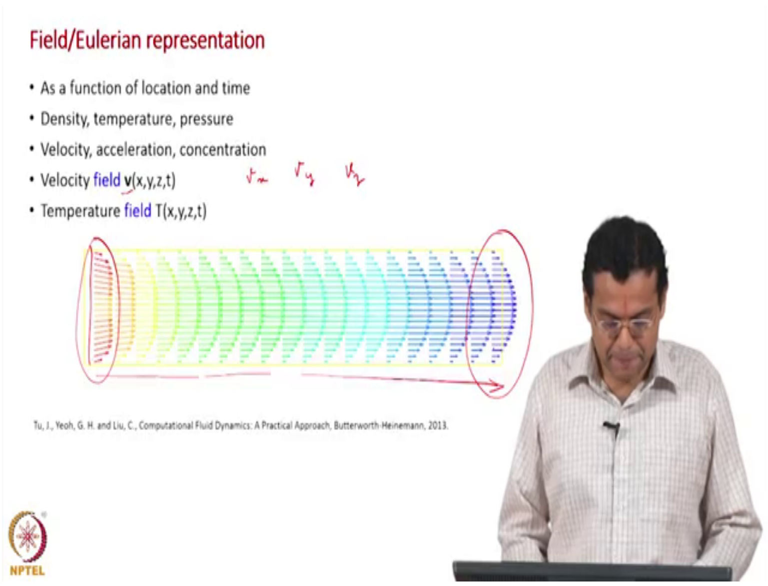

What I shown here is field and Eulerian representation discussed this as a function of location and time, spatial location and time ok. And it could be the variable could be density, temperature, pressure, or could be velocity acceleration concentration any of these variables. As I told you sometime back velocity field, when I say velocity field, v is a vector here which means that it could have different components v_x, and then v_y and then v_z, and that is a function of x, y, z, and time or we can also call as a temperature field.

So, the word field is associated with Eulerian representation, it just means that variation across spatial along spatial coordinates. What is shown here is a velocity field for flow between the two parallel plates is called the vector plot. It shows the magnitude of velocity is shown by the length of the vector, length of the arrow here. And then it shows the how the velocity varies between the distance between the plates along the lateral direction, and then how velocity varies along the direction of the flow.

You look at how velocity varies here, and then as the flow passes through this plate. Look a look at the velocity how it varies between the two plates. And this is called a velocity. When a when I want to say this vector plot, I will say that this figure represents the velocity field at different distances along the plate. So, velocity field, temperature field, just tells about variation spatial variation and also time as well in this case just a steady state.

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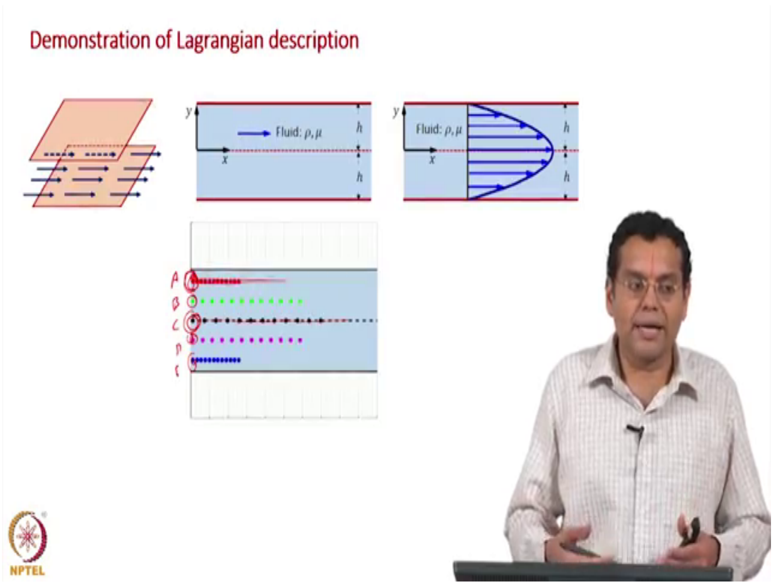

So, first to demonstrations to represent the Lagrangian description, Eulerian description. Think the geometry and the plates are same as I discussed earlier, flow between two parallel plates. Now, let us straight away run this simulation ok. What we say Lagrangian description is identifying a fluid particle and tracking it, and that is what has been done here. We have identified a fluid particle, and then we are tracking that as it flows through the domain.

For example, let us say this black particle identify with black dye and tracking it as it flows through the region. Same example or same demonstration was done for defining a fluid particle also. Fluid particle Lagrangian or analogous to each other that is why same simulation being shown, but there I used it to demonstrate what a fluid particle is.

Here I want to show say illustrate or demonstrate that a Lagrangian description we identify fluid particle and track that particular fluid particle. And as we have seen we can track different fluid particles as of a nomenclature this is A, this is B, C, D etcetera. So, different fluid particles we said instead of A we will use initial position. So, for example, I can let us say this is initial position 0, let us say some 0.2, 0.4, similarly let us say minus 0.2, minus 0.4. So, I can give some initial positions to identify the fluid particles ok, instead of saying A, B, C, D etcetera.

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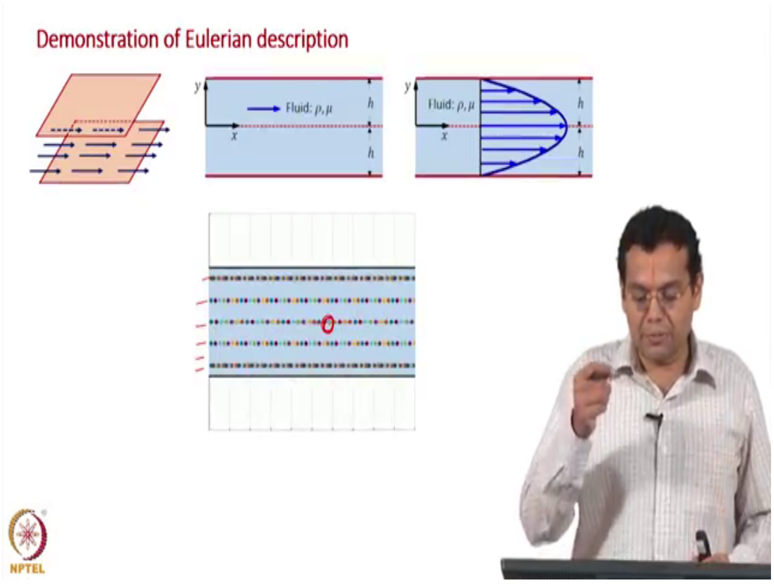

Example to illustrate Eulerian description, we will have a small demonstration here once again the same physical system. So, now, let us the flow between two parallel plates. Let us run this video and see what is happening. What I shown here are fluid part fluid flow entering and then leaving as shown different mean fluid flow is entering here, represented here as fluid particles here.

What do we say in Eulerian description I put my sensor at one loc one location and measure the velocity? So, in Eulerian description, I put my sensor at one particular location and we said different fluid particles pass through that point. This for example, as we said may be a pitot tube, I put a pitot in this region it gives a velocity, but the velocity is representative of different fluid particles passing through the same point. So, this is what we do in Eulerian description. We are we are fixing our sensor at the particular location and making measurement of a particular point by sensing different fluid particles passing through that point

In the earlier case, we dyed one fluid particle and followed it this is a Lagrangian description. In this particular case, different fluid particles pass through your measuring point, your measuring point has fixed. And you make the temperature and velocity measurement as the case may be.