Continuum Mechanics And Transport Phenomena
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Lecture - 68
Strain Rate: Example 1

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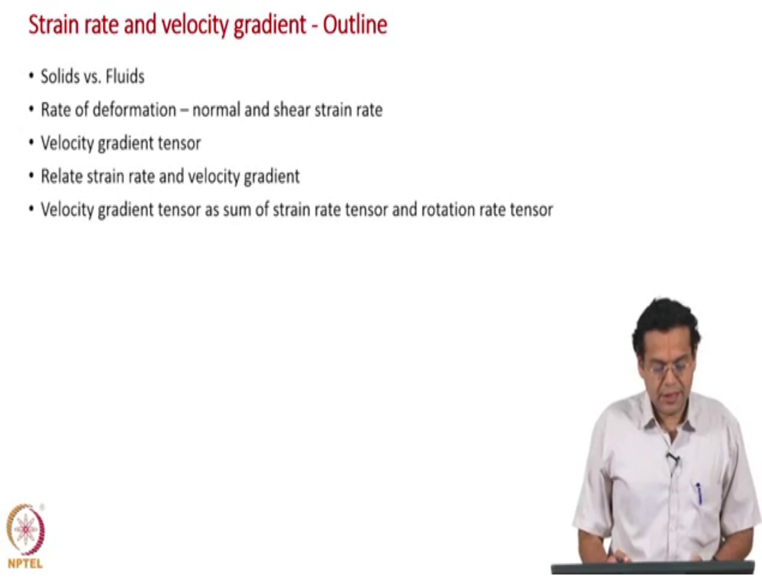
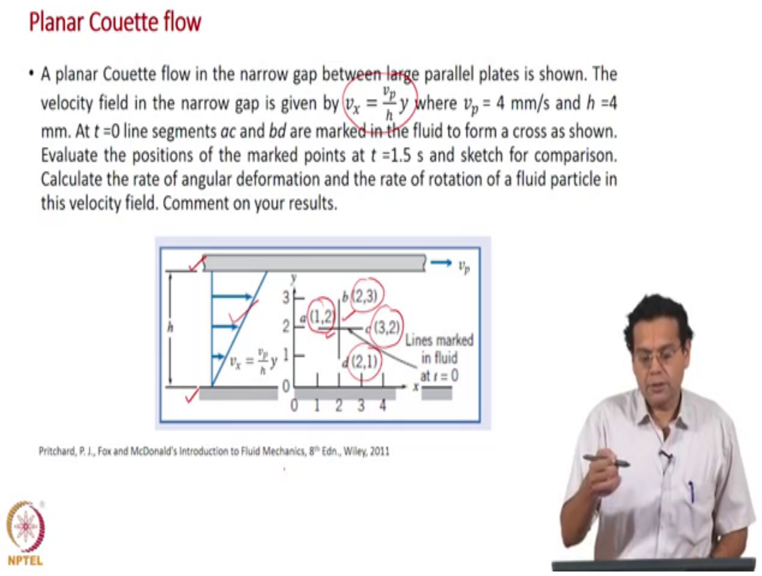


We are discussing Strain Rate in Fluid Mechanics and we have discussed these topics, we started distinguishing fluids from solids and found that deformations for solids and rate of deformation for fluids and rate of deformation could be in terms of normal and shear strain rate and we also discussed about the rate of rotation, and introduced the velocity gradient tensor, analogous to the displacement gradient tensor for solids and then we related strain rate to the velocity gradient and then finally, expressed the velocity gradient tensor as sum of strain rate tensor and rotation rate tensor, which we called as a decomposition of fluid motion.

Example: (Refer Slide Time: 01:23)



Now, we look at applications or examples where we evaluate them, interpret them physically, that is the objective. So, let us take the first example.

So, let us take this example on Planar Couette flow which is well known to us. We have two plates, and the top plate is moving at a velocity of v_p. So, planar Couette flow in the narrow gap between large parallel plates is shown. The velocity field in narrow gap is given by

$$v_x = \frac{v_p}{h} y$$

Here, v_p is the velocity with the plate and h is the vertical distance between the two plates, where given the velocity with the plate as $v_p = 4$ millimeter per second and the height to be $h = 4$ millimeter.

So, now at time t = 0 line segments ac and bd are marked in the fluid to form a cross as shown. So, we are going to track these lines. That is what is shown here, ac is the horizontal line and bd is the vertical line and evaluate the positions of the marked points at t equal to 1.5 seconds and sketch for comparison.

So, how will the position of a, b, c, d look like after 1.5 seconds; And we will also has to calculate the rate of angular deformation and the rate of rotation of a fluid particle in this velocity field And then comment on your results.

So, now the coordinates of a, b, c, d are given, a is (1, 2), and c is (3, 2). Same y-coordinate and differing in x-coordinate. And compare for the vertical line bd, the coordinates are (2, 3) and (2, 1) of course, x-coordinate is same and differing in the y-coordinate. So, what you should imagine is as if you have a color dye and then marking this is the lines line at time t = 0.

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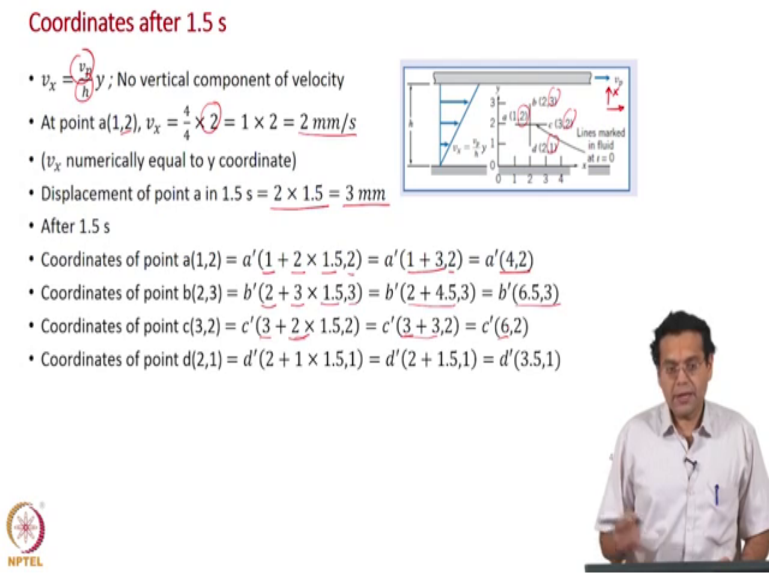

Now, we ask to find out coordinates after 1.5 seconds. So, let us do that. So, how do you find coordinate? The original coordinate plus the displacement which is happened during the time 1.5 second, that is all we will have to do.

$$v_x = \frac{v_p}{h} y; \qquad v_y = 0$$

So, in this case, there is only the x component of velocity, there is no y component of velocity.

Now, let us take at a point a (1, 2), and the velocity will be

$$v_x = \frac{v_p}{h} y = \frac{4}{4} x2 = 2 \frac{mm}{s}$$

I said taking numerically v_p and h are equal gives as the velocity at a point as numerically equal to the y-coordinate.

So, if you look at this 4 points, the velocity depends only on the y-coordinate. Not only that because of this particular example numerical values chosen the velocity is same as the y-coordinate. So, you can quickly get to know the velocity from the coordinates. So,

$$At\ point\ a(1, 2),\ v_x = \frac{v_p}{h} y = \frac{4}{4} x2 = 2 \frac{mm}{s}$$

$$At\ point\ c(3, 2),\ v_x = \frac{v_p}{h} y = \frac{4}{4} x2 = 2 \frac{mm}{s}$$

$$At\ point\ b(2, 3),\ v_x = \frac{v_p}{h} y = \frac{4}{4} x3 = 3 \frac{mm}{s}$$

$$At\ point\ d(2, 1),\ v_x = \frac{v_p}{h} y = \frac{4}{4} x1 = 1 \frac{mm}{s}$$

The point a moves horizontally at a velocity of 2 millimeter per second. Point c also moves at the same velocity 2 millimeter per second because they are at the same y-coordinate.

But if you compare b and d, b moves at 3 millimeter per second which is at a higher velocity of course, because it is closer to the plate which is moving and if you look at d is moving at 1 millimeter per second which is closer to the stationary plate. So, now v_x is numerically equal to y-coordinate. That will be easier to understand as we go along.

So, now, as I said we will have to find out the displacement of point a in 1.5 second is the velocity into the time, so

$$Displacement\ of\ point\ a\ in\ 1.5\ s = v_x x\ time = 2x1.5 = 3\ mm$$

Similarly,

$$Displacement\ of\ point\ b\ in\ 1.5\ s = v_x x\ time = 3x1.5 = 4.5\ mm$$

$$Displacement\ of\ point\ c\ in\ 1.5\ s = v_x x\ time = 2x1.5 = 3\ mm$$

$$Displacement\ of\ point\ d\ in\ 1.5\ s = v_x x\ time = 1x1.5 = 1.5\ mm$$

So, after 1.5 second what are the coordinates of point a,

$$Coordinates\ of\ point\ a(1, 2) = a'(1 + Displacement, 2 + Displacement) = a'(1 + 3, 2 + 0) = a'(4$$

Remember there is no change in y-coordinate. Only if there is a vertical component was velocity component along the y direction that will change, but we do not have that..

Now, let us do repeat this for the other points b is (2, 3)

$$Coordinates\ of\ point\ b(2,3) = b'(2\ +\ 4.5\ , 3) = b'(6.5, 3)$$

Now, similarly we can find out the other coordinates for example, c is

$$Coordinates\ of\ point\ c(3,2) = c'(3\ +\ 3\ , 2) = c'(6, 2)$$

Similarly, for the point d,

$$Coordinates\ of\ point\ d(2,1) = d'(2\ +\ 1.5\ , 1) = d'(3.5, 1)$$

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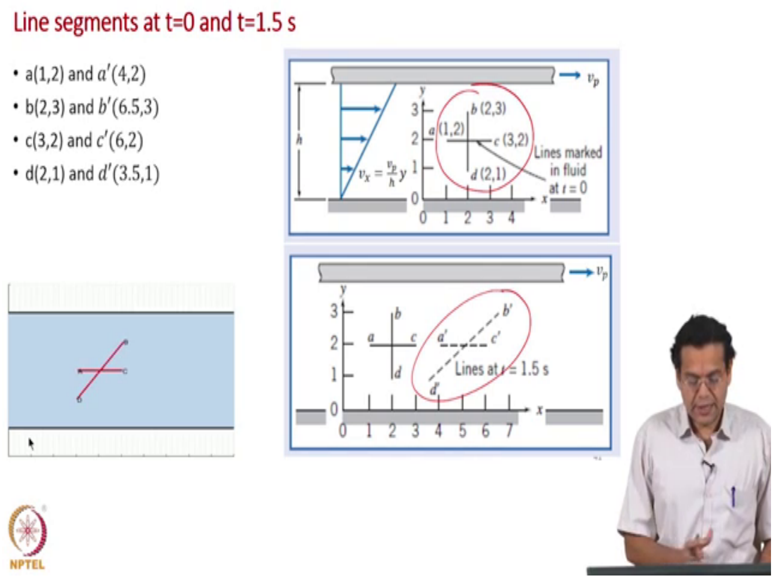

So, this is how the lines look at t equal to 1.5 seconds. So, this top figure shows the two lines at time t equal to 0 and the lines have become as soon in the diagram below. The line ac is become a'c' in the line b d as become b'd'. So, these are the new coordinates;

$$a(1,2)\ and\ a'(4,2)$$

$$b(2,3)\ and\ b'(6.5,3)$$

$$c(3,2)\ and\ c'(6,2)$$

$$d(2,1)\ and\ d'(3.5,1)$$

Now, let us run this animation which will show you how these lines move as a function of time. This is after a particular time interval of 1.5 seconds. What we will see is how this initial the cross, two lines are crossing each other, how they move as the time progresses

So, this is how they move and what of course, this is qualitatively shown here and what you see here is the position of the lines at time t = 1.5 seconds but as if you going to track from 0 to 1.5 seconds this is how the line moves, the both the lines move.

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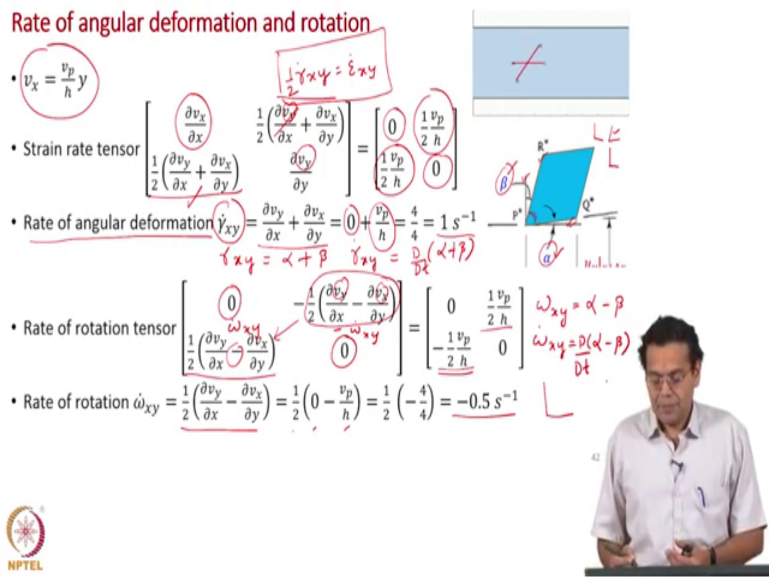

We introduced few tensors and with physical significance. So, we will calculate those tensors, also find out the rate of angular deformation, rate of rotation which is asked in the question.

$$v_x = \frac{v_p}{h} y$$

Now, let us calculate this strain rate tensor for this particular velocity field. So far we have been working in terms of the expression for the strain rate tensor, but now because we are given the velocity field we can evaluate the strain rate tensor.

$$Strain\ rate\ tensor = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{1}{2}\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right) & \frac{1}{2}\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right) & \frac{\partial v_y}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\frac{v_p}{h} & \frac{1}{2}\frac{v_p}{h} & 0 \end{bmatrix}$$

So, if you calculate the rate of angular deformation remember rate of angular deformation is

$$\dot{\gamma}_{xy} = \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} = 0 + \frac{v_p}{h} = \frac{v_p}{h} = \frac{4}{4} = 1\, s^{-1}$$

How do you interpret this; what is the meaning of this. Now, remember, we had $\gamma_{xy} = \alpha + \beta$ based on this diagram which we have seen earlier. Now, $\dot{\gamma}_{xy}$ remember it is just taking time derivative. So, if you want to write formally it is $\dot{\gamma}_{xy} = \frac{D}{Dt}(\alpha + \beta)$. So, this rate of angular deformation tells you that rate at which these two angles $\alpha + \beta$ together are changing, that is a significance of 1 second inverse.

γ_{xy} is just shear strain, $\alpha + \beta$. $\dot{\gamma}_{xy}$ is the material derivative of that which is following the fluid motion. So, in this case the angles are $\alpha + \beta$, and we are looking at the sum of this and how that sum of that angle changes the respective time.

So, in this case the sum is positive which means that α *and* β are increasing, which means that the angle between let us say a time t = 0 you marked these lines, at some time little later the angle would have decreased. So, rate of angular deformation tells you about a rate at which $\alpha + \beta$ are changing, and if it is positive tells you that the two line segments let us say initially they were P Q and P R, now it is P*Q* and P*R*, they are approaching towards each other.

We already seen that earlier remember shear strain is positive means acute angle. In this case we cannot talk about angle remember, we can always talk only about the rate at which the angle changes. So, 1 per second tells you rate at which the two line segments approach each other towards each other.

Now, let us calculate the rate of rotation tensor is,

$$Rate\ of\ rotation\ tensor = \begin{bmatrix} 0 & -\frac{1}{2}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\frac{v_p}{h} \\ -\frac{1}{2}\frac{v_p}{h} & 0 \end{bmatrix}$$

Now, let us find out the rate of rotation. Rate of rotation is

$$\dot{\omega}_{xy} = \frac{1}{2}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) = \frac{1}{2}\left(0 - \frac{v_p}{h}\right) = -\frac{1}{2}\frac{v_p}{h} = -\frac{1}{2}\frac{4}{4} = -0.5\ s^{-1}$$

So, what does this tell you? Remember, when we introduced $\dot{\omega}_{xy}$ we introduced that as $\alpha - \beta$. We said α is counter clockwise, β is clockwise rotation. So, we attached sign to that because rotation takes about the sense of rotation. Now, if you want to calculate $\dot{\omega}_{xy}$ then it is $\frac{D}{Dt}(\alpha - \beta)$ the material derivative.

So, now, this tells you the what is the net rotation, so this rate of rotation tells you rate at which $\alpha - \beta$ changes. So, tells about the net rate of rotation of the element and in this case it is negative which means it is rotating the clockwise direction. Remember, in both the cases at every instant we are taking two perpendicular line at let us say $t = 0$, and then to $t = 0.1$, you take two perpendicular lines and they approach each other as given by the rate of angular deformation.

Similarly, at next instant you cannot start with those lines which have come together. Once again start with lines which are perpendicular to each other because always you are talking about $\dot{\gamma}_{xy}$ which represents rate of angular deformation for lines with our long x and y axis. So, every instant you always considering two perpendicular, two perpendicular lines and what is happening to the angle between them, that is what we have seen, ok, rate at which they have come together.

So far we have been looking at analytical expressions, this example gives us a chance to understand the values numerically, ok; and also like to mention that what you see here thus the fluid flows, you are marked; you are marked two lines and then remember the fluid is continuously flowing and this two marked lines are also let us say they are flowing along the fluid but what you see is the representation of all the effects put together.

What is that? Normal strain rate of course, it is 0 and this particular case and so, it is a combined representation of normal strain rate, shear strain rate and rotation rate. What you see it is a combined effect. You do not see individual effects there because we calculated them using the velocity field, something equivalent to calculate using the velocity gradient tensor not using strain rate tensor or the rotation rate tensor.

What you have calculated is use; is equivalent to calculating using the velocity gradient tensor. Hence, what you see is a representation of the rate of deformation and the rate of rotation. That should be kept in mind.