Continuum Mechanics And Transport Phenomena
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Lecture - 52
Normal Strain and Shear Strain - Part 2

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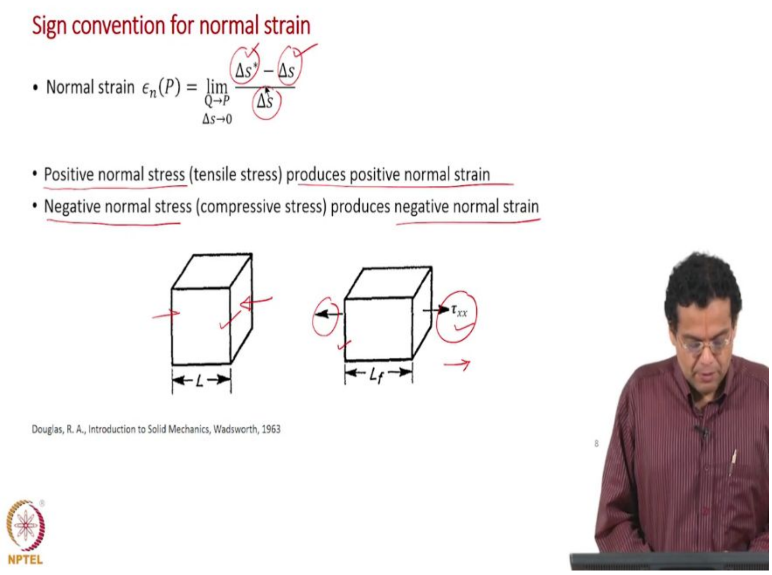

When we discussed stress we discussed about sign convention. Similarly, we are going to discuss about sign convention for normal strain. It looks very obvious, but there is a sertile point here. The sign convention for strain goes in line with the sign convention for stress. Let us see how does it reflect here.

Now, if you take a solid object and apply a tensile stress; tensile stress pulling away; and according to our sign convention tensile stresses are positive, on a positive face along positive axis so, they represents positive stresses. When you apply a tensile strength then the length will increase and the normal strain will be positive value and that is what we are seeing here.

Normal strain was defined as

$$\varepsilon_n(P) = \frac{\Delta S^* - \Delta S}{\Delta S}$$

Now, as we have seen, if you subject a object to a tensile stress tensile stresses are positive because on a positive x face the force is along x axis, negative x face force is along negative x-direction and of course tensile forces are positive and that results in a expansion. So, ΔS^* will be larger than ΔS; you will result in a positive normal strain.

So, we can conclude that positive normal stress tensile stress produces positive normal strain. Negative normal stress, what does it mean? Suppose, you apply a compressive stress then the length will decrease. When length decreases ΔS^* will be less than ΔS this will be a negative value. So, negative normal stress, also called as compressive stress, produces negative normal strain just like positive normal stress produces positive normal strain.

So, we could have define for example, $\Delta S - \Delta S^*$, but we are defined as of course, final length minus initial length which looks more logical anyway, but the sign convention associated with the strain goes in line with that for the stress as well. Like to mention here we could have left that change in length alone just like we left that change in angle, but change in length is not normalized. That is why to normalize, expressed as a fraction we divided by original length otherwise it could be 2 centimetre, 5 centimetre, it is not normalized.

To normalize we divided by the original length and get a normalized expression of course, that is not required for the shear strain, it is just a change in angle.

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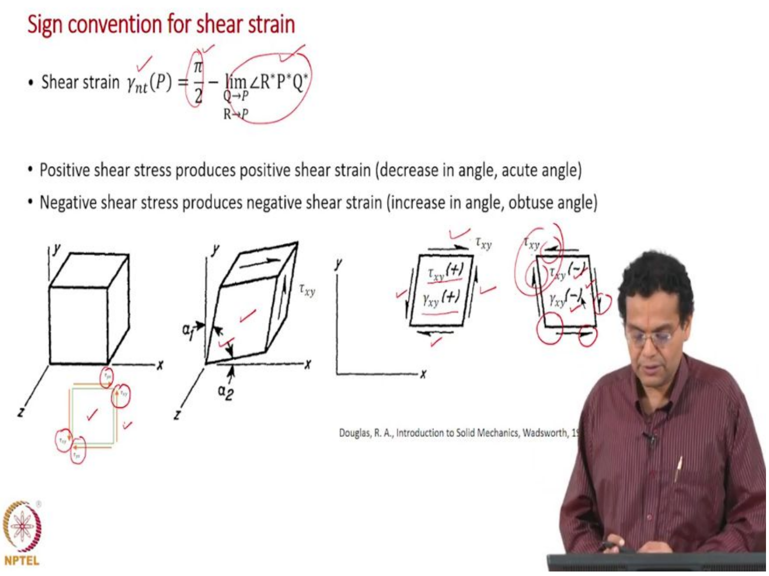

Now, let us see the sign convention for shear strain. What is shown here is a stress element with only shear stresses all of them in the positive sense, what does it mean? On a positive x face force along positive y direction, positive y face force along positive x axis. On a negative x face the direction of forces along negative y axis; on a negative y face force along negative x axis. Now, this combination of forces or combination of direction of forces represents all of them in a positive shear stress. What does it mean?

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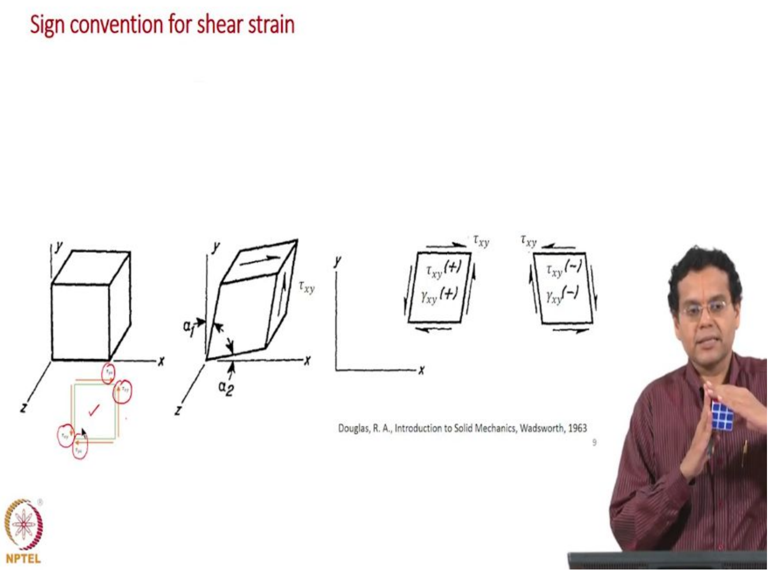


Take an object your shear stresses are the forces which will act in the opposite directions. It will make the object to shear in a particular way that is what is shown (in the above figure) which means the angle which has original 90 degrees will now decrease and we know that from the definition of a shear strain that the decrease in angle results in a positive shear strain. So, positive shear stress results in a positive shear strain. I will just repeat it.

We are shown as a 2D stress element with all the shear stresses in the positive sense. When you represent in positive sense the forces are such a way if you have a cube they act in the opposite way. So, they make the cube such a way that this angle which was originally 90 degrees will now reduce and that is what is shown here according to our definition of shear strain,

$$\gamma_{nt}(P) = \frac{\pi}{2} - < R^* P^* Q^*$$

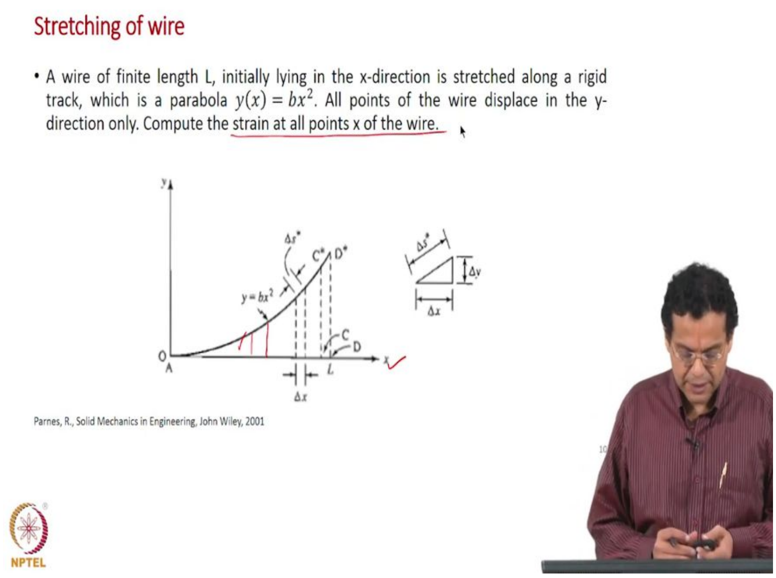
The shear strain is 90 degrees minus the final angle and the final angle is reduced so, the shear strain is positive. So, positive shear stresses result in positive shear strain and that is what is shown here as well. The stresses are shown in the positive sense and that is what this indicates, positive shear stress. So, that is the first statement we make here; positive shear stress produces positive shear strain. Of course, decrease in angle or acute angle.

Now, let us discuss the are the reverse condition and that is what is shown in the right hand side figure. Now, all the shear stress are negative here because on a positive face the forces along negative direction, on a negative face the forces along positive direction. So, these the action of forces according to this direction are all negative shear stresses.

So, you have a cube and they act in this direction. So, now, the angle will increase it will become an obtuse angle. What will happen if you have an obtuse angle? You have $\frac{\pi}{2}$ and an obtuse angle. So, you have a negative shear strain. So, in negative shear stress results in negative shear strain. So, it is that is the statement next statement, negative shear stress produces negative shear strain increase in angle or obtuse angle.

The important point in these two slides is that the sign convention for stress and strain are related. The sign convention is such a that such a way that you adopted is that positive stresses results in positive strain be it normal or shear that is a conclusion from these two slides.

Example: (Refer Slide Time: 07:25)

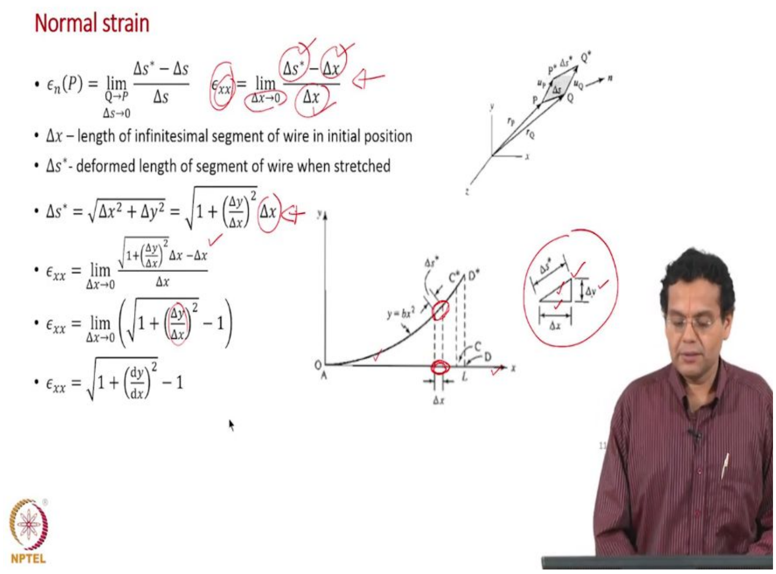


Let us take an example of a stretching of a wire. This example is to illustrate the calculation of local strain. You would have done the same example. You are given a wire, you are given the initial length, you will be given the final length, you will find difference in length divided by original length; you would have calculated strain that is that gives the average strain over the entire length but, now we are going to apply our expression, now our definition of local normal strain and going to see how can how local strain varies locally along the length of the wire that is example let us read this.

A wire of finite length L, initially lying in the x-direction. So, initially a wire is along the x axis is stretched along a rigid track which is a parabola. So, you are stretching it along this track, it is a parabola. This is given as $y = bx^2$; all points of the wire displace the y-direction only.

All points of the wire displace in the y-direction only. Compute the strain at all points of the wire that is the difference between what you already known what it discussing now.

Solution: (Refer Slide Time: 08:57)



So, let us do that. Let us start with the definition of normal strain

$$\varepsilon_n(P) = \frac{\Delta S^* - \Delta S}{\Delta S}$$

Now, we are considering the wire which is lying originally along the x axis. So,

$$\varepsilon_{xx} = \frac{\Delta S^* - \Delta x}{\Delta x}$$

Now, ΔS^{*} is a final length and initial length is Δx because along the x axis, similarly initial length is Δx and instead of $\Delta s \to 0$; I write as $\Delta x \to 0$ and instead of ε_n I write as ε_{xx}. We said for the normal strain of line elements along x axis are denoted as ε_x, the two subscript notation they are ε_{xx} and that is what is shown here.

Taken a small line element along the wire the original configuration that has become delta s star in the deformed configuration this is the un-deformed configuration or initial state of wire. This curve; the parabola is the final configuration of the wire or a deformed configuration. We are told that every point that displaced only vertically. So, that is why this vertical line shows.

Now, because we are considering a small segment of the wire you can approximate this Δs^{*} be a straight line and the x axis distance is Δx, let us call the y axis distance is Δy.

So, Δx is the length of infinitesimal segment, we are taking a small segment of wire in the initial position and Δs^{*} as we have discussed is the deformed length of segment of the wire when stretched which you called as a final configuration.

So, from this figure (right hand side in above slide image) we can write

$$\Delta s^{*} = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x$$

Now, we substitute this expression in this definition of normal strain

$$\varepsilon_{xx} = \frac{\Delta S^{*} - \Delta x}{\Delta x}$$

$$\varepsilon_{xx} = \frac{\sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x - \Delta x}{\Delta x}$$

Now, you can cancel out in Δx and that leaves us

$$\varepsilon_{xx} = \left(\sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2} - 1\right)$$

Now, we take limit $\Delta x \to 0$ which means

$$\varepsilon_{xx} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2} - 1\right)$$

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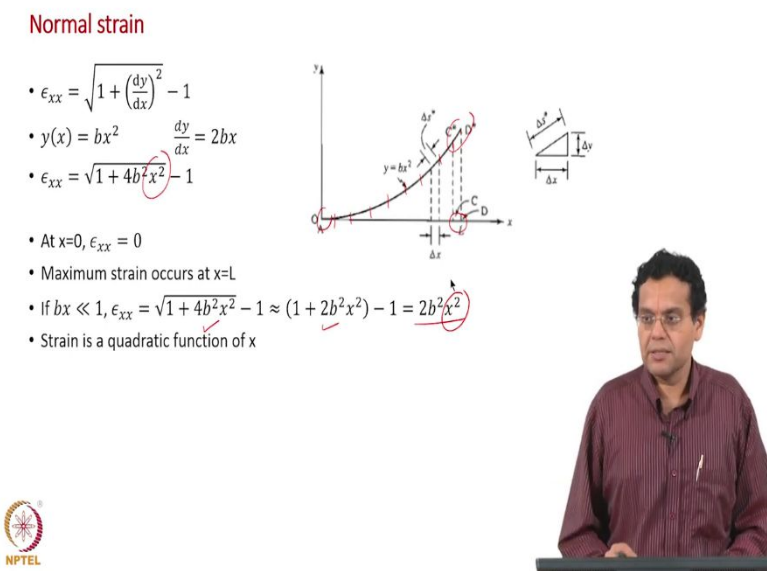


Now, let us proceed further we are given the expression for y which is

$$y\,(x) = bx^2$$

We will have to just differentiate that to get $\frac{dy}{dx}$,

$$\frac{dy}{dx} = 2bx$$

Then, substitute in the expression

$$\varepsilon_{xx} = \left(\sqrt{1 + 4b^2x^2} - 1\right)$$

This expression gives you the local strain that gives you the strain at every point along the wire that is the difference between what you already learned and extra information from this.

Of course, the strain remember the strain varies along the length of the wire, it is not constant it is the function of x; and,

$$at\,x\ =\ 0\,;\ \varepsilon_{xx} =\ 0$$

So, that if you take a small point at A, there is no change in the length at all it is just there and maximum strain where does it happen? If you see here this segment CD has become C star D star.

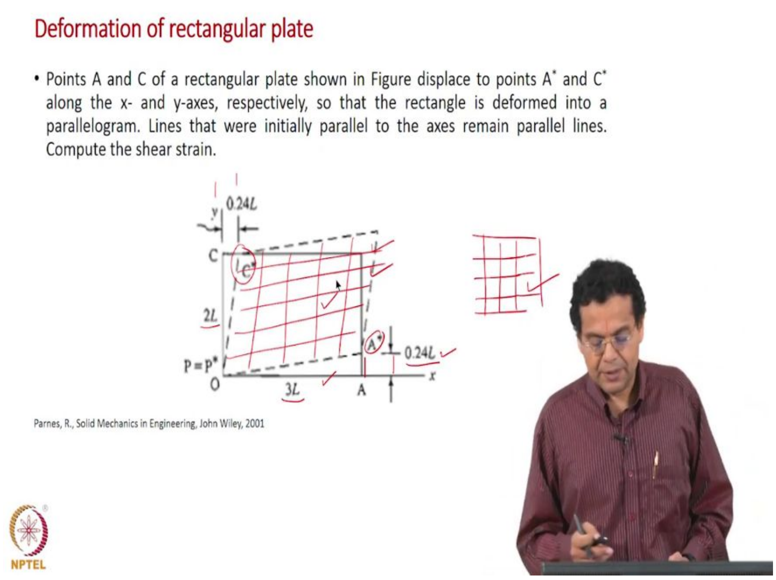
So, the maximum strain occurs at x = L and found simplify for example,

If $bx \ll 1$ then you can approximate this using the binomial theorem

$$\varepsilon_{xx} = \left(\sqrt{1+4b^2x^2} - 1\right) \approx \left(1+2b^2x^2\right) - 1 \ = 2b^2x^2$$

So, to a quick approximation the normal strain varies quadratically with the position or with x, strain is the quadratic function of x. So, only one main concept is example we evaluated normal strain locally in a wire that is being stretched.

Example: (Refer Slide Time: 14:31)



We will take an example for evaluation of the shear strain. Now, the what is shown here is a plate and you are given the un-deformed configuration you are given the deformed configuration, initial state and final state. Always remember, in all these cases a force is applied and it continued to apply and then it is it undergoes deformation.

The original configurations are P, C and then A the original points are P, C and A; point P has not moved, it is remained at P*. So, P = P*. A has moved A* and C has moved to C*. Now, the vertical displacement of a this length is given as 0.24L, the dimensions are given in terms of L; L could be taken as unity as well. So, the length is 3L, the width is 2L and the vertical displacement of A is 0.24L, the horizontal displacement of C is 0.24L.

Now, let us read the problem. Points A and C of a rectangular plate shown in figure displace to points A* and C* along the x and y axis respectively, so that the rectangle is deformed to parallelogram. If you look at P *C*A* the dashed line is a parallelogram. Lines that are initially parallel to the axes remain parallel lines. Why this statement is required? Just to simplify the problem saying that the shear strain is same throughout the plate.

You need not find shear strain at every point in the plane, if you are finding one value of shear strain the change in angle that is applicable throughout the plate. That is why, line parallel to the axis they remain parallel in the final configuration as well. So, let us evaluate the shear strain.

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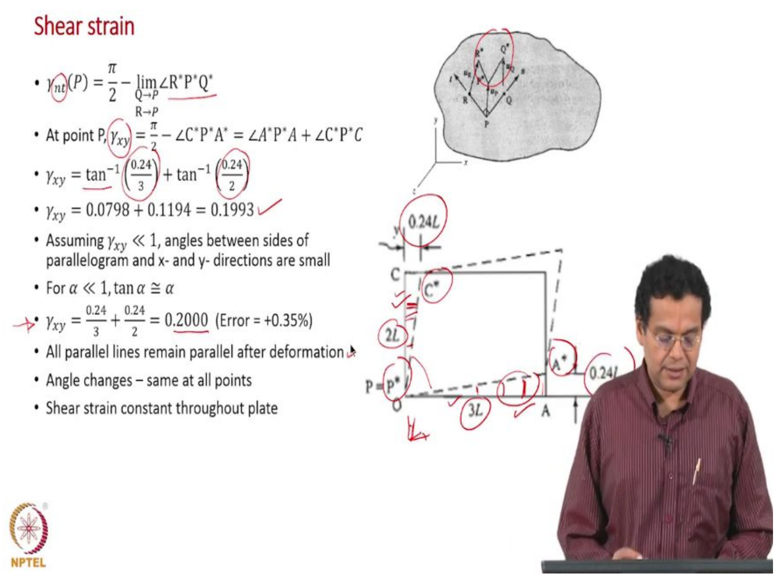

Let us start with the definition of shear strain.

$$\gamma_{nt}(P) = \tfrac{\pi}{2} - < R^*P^*Q^*$$

We are considering point P, the two line segments are along x axis and along y axis. So,

$$\gamma_{xy} = \tfrac{\pi}{2} - < C^*P^*A^* = < A^*P^*A + < C^*P^*C$$

The $R^*P^*Q^*$ if you look at the original diagram it represents the angle between the two line elements in the final configuration. What is the corresponding angle in this diagram? It is $C^*P^*A^*$ is it that is a final angle. So, $\frac{\pi}{2}$ is the original angle CPA that is angle 90 degrees minus the final angle is $C^*P^*A^*$.

Now, based on the dimensions given displacement is given we can evaluate γ_{xy}. What is the angle A^*P^*A; it is just $\left(\frac{0.24}{3}\right)$, L of course cancels out. Now, for the other angle it is $\left(\frac{0.24}{2}\right)$, L cancels out. Now, if you evaluate

$$\gamma_{xy} = \left(\frac{0.24}{3}\right) + \left(\frac{0.24}{2}\right) = 0.0798 + 0.1194 = 0.1993$$

This is exact value of the shear strain. Now, if you assume $\gamma_{xy} \ll 1$, a small value if you assume.

Remember we discussed rigid body and then the deformable body. You can quickly approximate bodies to be rigid but we are discussing deformable bodies that is why we are discussing about strain etcetera and I told you if you apply mega Pascal of force per area the change in length will be very small value of millimetre or 10^{-6} meters will be in terms of microns.

Similarly, if you apply mega Pascal of let us a shearing a force the angle change will also be very small that is why we are discussing the assumption of $\gamma_{xy} \ll 1$, which means that these angles will be very small. So, assuming $\gamma_{xy} \ll 1$, angles between sides of parallelogram and x and y directions are small.

And, now we know that for $\alpha \ll 1$, any angle less than less than 1, $\tan tan\ (\alpha) = \alpha$ which means that $\alpha = (\alpha)$. let us do that and see what is the value.

$$\gamma_{xy} = \frac{0.24}{3} + \frac{0.24}{2} = 0.2000$$

Now, this is an approximate value, because we have not taken but look at the error a very small rough 0.35 % is introduced. We will be using as we go along only this second value. It is almost accurate only a small error is introduce and we will be following this second approach as we go along. Just to illustrate numerically why do we do that you will understand later on.

So, we have found exact value of shear strain an approximate value but that is very good approximation for the exact value.

Now, the question mentioned that the all parallel lines remain parallel of deformation. We have illustrated in the previous slide. So, we have calculated angle changes for the boundary between the two boundaries of the plate x boundary and y boundary.

So, applicable at any point inside the plate so, the angle changes are same at all points in the plate; that is why if this is not given then we cannot come to that conclusion. That is why that line was mentioned because it is very simple example; angle changes are same at all points in the plate and which mean that the shear strain is constant throughout the plate; angle changes are nothing but shear strain.