Continuum Mechanics And Transport Phenomena
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Lecture - 51
Normal Strain and Shear Strain- Part 1

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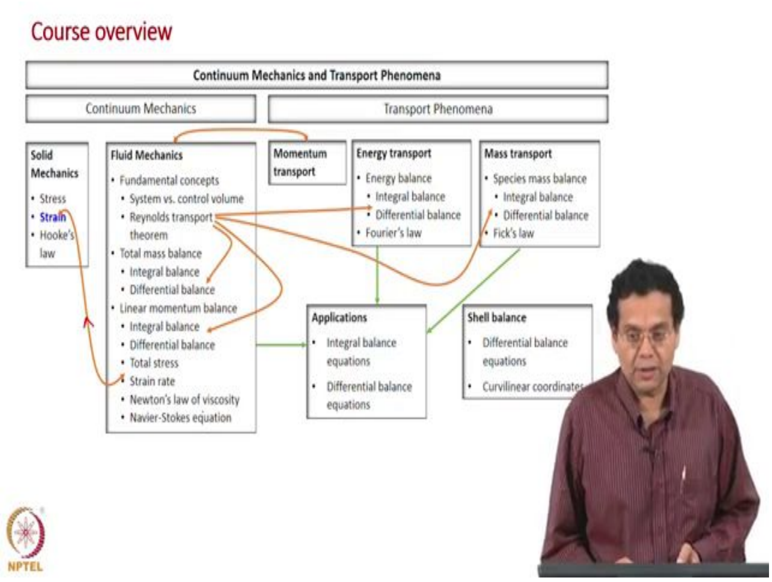

So, we are in the process of deriving the linear momentum balance, we have derived that in the last lecture. But then, we found that the components of the viscous tensor are unknowns. Why are they unknowns? We are proceeding towards to get an equation which is in terms of velocities, we want to get the velocity profile. So, they are unknowns. And we need to express that in terms of the viscous stress tensor components in terms of velocity; velocity gradients to be precise.

And then to understand velocity gradients we need to understand displacement gradients in solid mechanics. So now, we are taking a diversion to solid mechanics. So, in terms of this course overview we are taking a diversion from fluid mechanics solid mechanics and the topic of discussion is termed as strain. What this means will understand as we go along.

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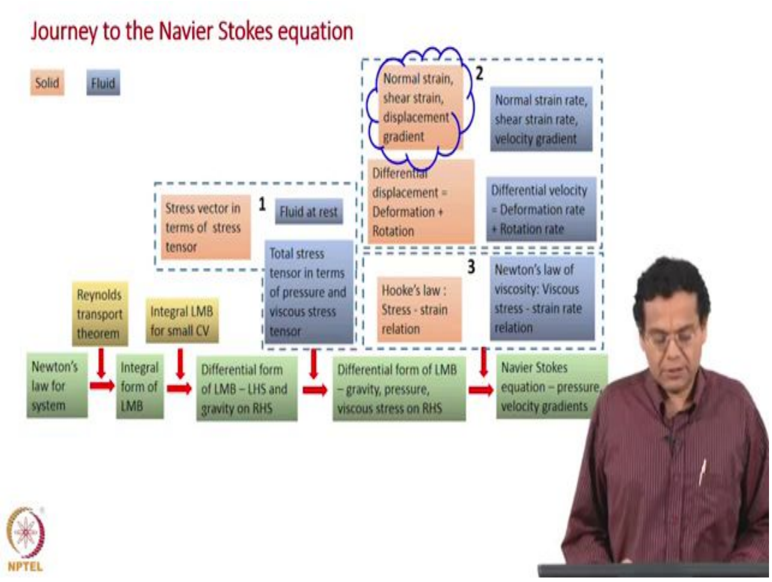

In terms of our journey to the Navier stokes, we started with the Newton's Law and obtain the integral form of linear momentum balance, then obtain the differential form of linear momentum balance the left hand side and the gravity on the right hand side. Understand the surface forces on the right hand side. We took a diversion to solid mechanics understood stress came to fluids understood total stress in terms of pressure and viscous stress in corporate that on the right hand side and derived the differential form of linear momentum balance the complete form. We had gravity, pressure, viscous stress on the right hand side, but we are going to use this differential form of linear momentum balance to get velocity profiles.

So, our unknowns in the equation or the variables in the equation should be of course density, pressure, velocity; we cannot have other unknowns. So, the viscous stress tensor components are unknown. Now, we need to express this viscous stress tensor, the components of that in terms of velocity, really in terms of velocity gradients. Understand this velocity gradients to have better understanding we will understand first displacement gradient, so we take a diversion to solid mechanics and this number 2 indicates that it is our second visit all the solid mechanics.

And then, the two orange blocks or solid mechanics. After discussing that, we will discuss analogously these two blocks in blue fluid mechanics. In that first we are discussing the block 2 on solid mechanics. You look at terms like normal strain, shear strain, displacement gradient, and this is what we are going to discuss the this lecture and next lecture as well. So, let us get started of course after we finish will understand these terms very clearly.

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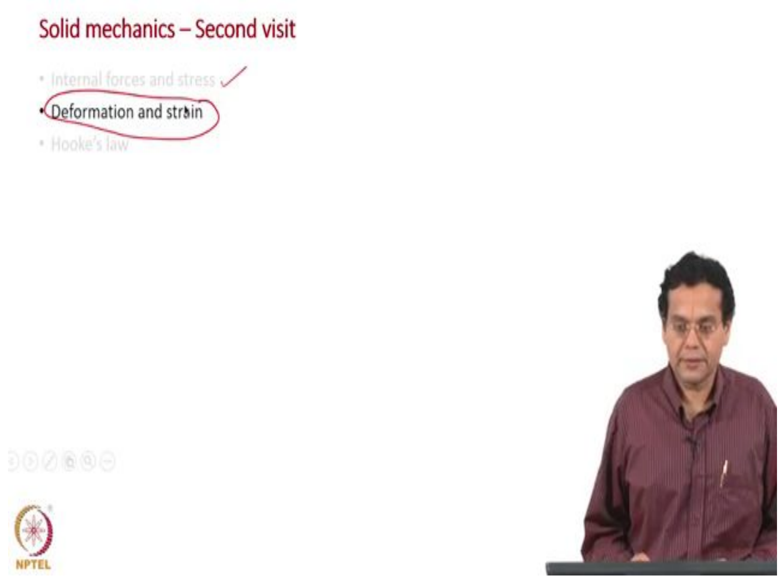

So, just like we had outlined earlier for solid mechanics, the three titles shown here are the titles for the first, second and third visit. We are discussed internal forces and stress, now the second visit the title says deformation and strain, of course third title says Hooke's law. You are going to discuss about the second title which is deformation and strain.

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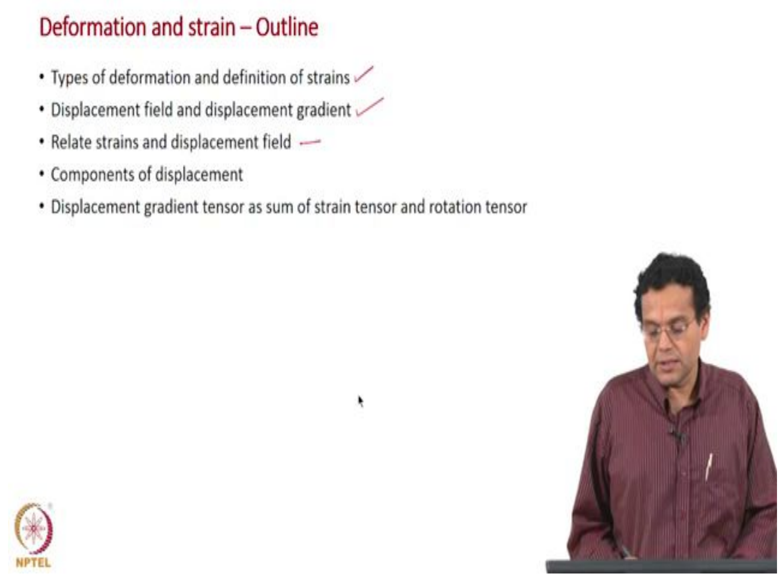

Now, what is a outline for deformation and strain? We are in second diversion to solid mechanics in that this outline is for the second visit we will discuss slowly as we go along, it is a little bit each bullet by bullet. So, what are the points or concepts are going to discuss.

- Going to start with types of deformation and definition of strains. There are lot of terminology is here I am not going to explain them right now, you will understand as we go along. So, let us just read them.
- Second concept is on displacement field and displacement gradient.
- Then we relate strains and displacement field
- Then we look at components of displacement.
- And then finally we split the displacement gradient tensor as sum of strain tensor and rotation tensor. Lot of terminologies will understand very clearly as we go along.

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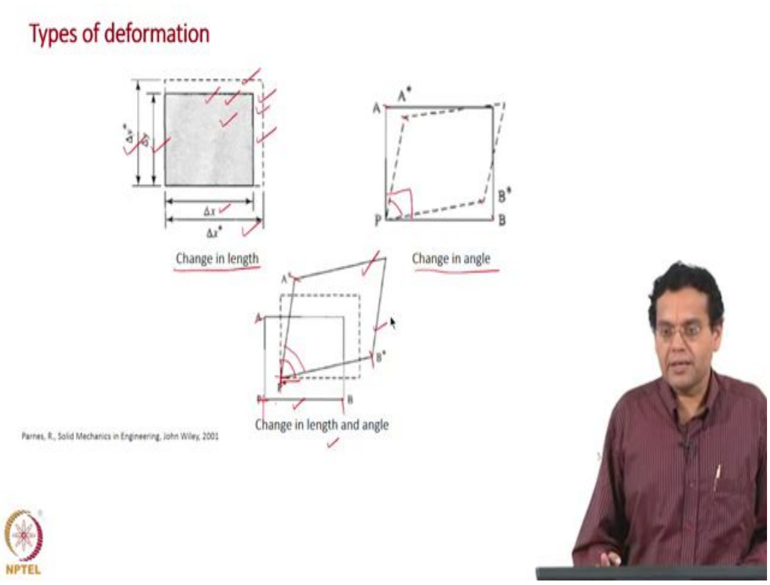

Let us start with types of deformation. How do you understand this visualizes? We take a plate and then let us says this is subjected to some force and because of the action of the force there is change in the dimension along x direction, change in the dimension along y direction. And so, the first type of deformation which can happen is change in length.

So for example, let us say about 3 centimetres becomes 3.5 centimetre, let us say it is 5 centimetre, becomes 5.5 centimetre; so there is increase in length and that can be decrease in length as well. But, the first type of deformation is change in length. Now, that is what is shown in the left side figure. You have a plate of Δx and Δy length and width and that becomes Δx^* and Δy^*.

Most of the times in terms of our figure the continuous line represents the dimension of the solid before you apply the force and the dotted line or dashed line represents after you apply the force. Remember we discussed when we discuss about difference between solid and fluid mechanics, the case of solid it goes from one equilibrium state to another equilibrium state. I have a solid and different names are given for the solid or state before applying the force. We can called as initial state or undeformed configuration. And you apply a force it undergoes deformation. What does it deformation mean; for example, right now change in length.

So, this is the continuous line that represents the plate in the initial state or in the undeformed state and this dashed line represents the same plate after you apply the force it has undergone some deformation, this case change in length and then it has come to another equilibrium state. Remember, for solid mechanics we said solids go from one equilibrium state to another equilibrium state. You continue to apply the force and it changes from one length to some other length in terms of area of course there is change as well. So, and this dotted line represents the deformed configuration.

Now, instead of change in length there could be change in angle as well; you have to take the plate unless you apply a shearing stress shearing force like this. So what happens? The in this particular case this length almost reminds same the length of PA and PA* are almost same similarly PB and PB* almost same, so there is no change in length. But if you look at the angle, the angle between PA and PB was 90 degrees. Now, that angles is reduced and it has become acute angle. So, this also called as deformation, the second type of deformation namely change in angle.

So, whenever we discuss deformation whenever we say deformation; deformation is a very well known word used even day day-to-day activities, but now we have to attach a very specific meaning when we discuss solid and fluid mechanics. So, when we say deformation we say it means that change in length and or change in angle both can happen. That is what is shown in the figure at the bottom, where you have change in length and change in angle. This rectangle represents the state before applying the force initial state or undeformed configuration. Now, this has become almost kind of parallelogram. Now if you see the length PA has increased to P* A*, similarly length PB has increased to P*B*. So, there is increased in length and then look at the angle there is a change in angle as well. This 90 degrees has now reduced. So, this is the example where you have both change in length and change in angle. So, in general you will have both change in angle and change in length as well.

There is one more thing shown here, the plate has moved. If you see here the original plate is here it has moved here and we will discuss about that later, right now we just focus on change in length change in angle and both can happen together. And this we call as deformation ok. Now, we have described it this in words, now we will have to quantify it and that is what we are going to do now. How do you quantify? Change in length change in angle.

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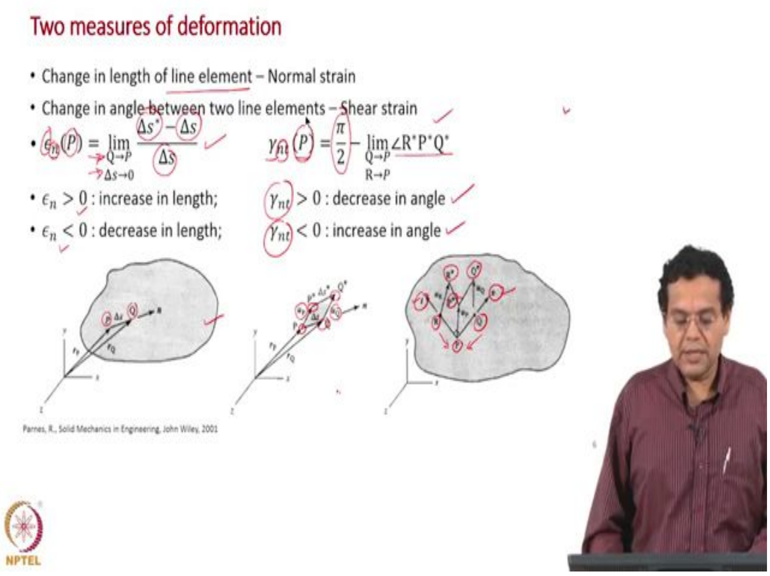

Now, which says two measures of deformation one for change in length and one for change in angle.

- Change in length of line element is quantified in terms of normal strain.
- Change in angle momentum is a angle you need to line elements be quantified in terms of shear strain.

So, let us see how do we do that. What is shown in the bottom figure is a solid object, in that you mark two points P and Q. You mark two points P and Q either you call them as points later on I will call them as particles or solid particles so that you can easily extend to fluid it. Remember you always have to take this two fluids, so we can easily say fluid particles or solid particles. So, we identified two points.

And PQ is a line joining the two points P Q or the two particles PQ and ΔS is the length of the line joining P and Q. Now, I shown one vector n; n represents the direction of the line PQ.

So, what is it we have to done? Taken a plate it is in the initial state, look at the terminology it looks like same as thermodynamics we say know a gas goes from initial state to final state. So, similarly for solid also we said it goes from one equilibrium state to another equilibrium state. That is why our terminology is also initial state or undeformed configuration which means no forces applied.

So, you mark two points P and Q along a direction n, the length of that line is ΔS. Now you apply a force on this. Now, the points you mark or particles identify they get displaced. That is what is shown here. P has moved here to P*, and Q has moved to Q* and this U_P represents displacement of point P, U_Q represents displacement of Q we will discuss this later. Now, focus on the line PQ has become the line P*Q* and the length has become from ΔS to ΔS*.

Now the word strain is not new to you. You would have usually discuss strain for a something like a rod like this and you are defined strain as change in length by original length. Same meaning here as well, but of the small difference that is what we are going to discuss or you would discuss a thread and then change in length by original length ok. Let us do that now.

$$\varepsilon_n(P) = \frac{\Delta S^* - \Delta S}{\Delta S}$$

$$\varepsilon_n > 0 \ : \textit{increase in length};$$

$$\varepsilon_n < 0 \ : \textit{decrease in length};$$

Let us define normal strain in this way I will explain what is the meaning of this definition of the shear strain, will come to that shortly. Now, these are the conditions, we will discuss that as well. Now, we are defined the normal strain which is represented by ε and then we have subscript n, why is a subscript n, this line element means that the line joining the points PQ. Now, we said the line PQ is along direction n that is why we used subscripts n here, I will come to the P within bracket shortly.

Now, we defined normal strain as change in length by original length same definition. So, ΔS^* is the let us say that final length, ΔS is the initial length divided by ΔS, initial length. Now, what is the difference between what you already know and this definition? What you have usually know; you took this original length and then the final length took the difference divided by the original length this gives you normal strain for averaged over the entire length of the rod or thread.

But remember, we are going to go back to our linear momentum balance all of them are valued at every point. So, we need a point form of strain is that; what you know is the definition of average strain, if there is variation of strain along the length of the rod, then we need a definition of normal strain which is valid every point; what do we do here we took P and Q slightly separated make $Q \to P$ and that is what this means. Limit of $Q \to P$ what will happen $\Delta S \to 0$. Same definition in the limit of; what does it mean you have just P Q very near to each other then this becomes the definition of normal strain. And that is why here within bracket have written at P.

So, what does the definition tell you? Normal strain at P and of course, you require little more imagination if you imagine at a point but a small line along n it is not one point then you are at a point and imagine a small line along direction n. And what is the change in length for that line, of course divided by the original length gives you the definition of the local normal strain.

Significance is known to you change in length by original length, only difference are of course that is it required for this our scope of this course is that we have defined normal strain locally, we will see an example in fact. Using this we can find the normal strain at every point along the rod or along the length of the thread. And that is why we are taken $Q \to P$ and $\Delta S \to 0$.

So, several aspects to be looked to be focused on in this definition. First is ε_n is normal strain; n does not represent normal strain, n is used represent the direction of the element for which you are defining the normal strain. Please keep that in mind. And on the right hand side change in length by original length in the limit of $Q \to P$ and $\Delta S \to 0$.

$$\gamma_{nt}(P) = \tfrac{\pi}{2} - \angle R^{*}P^{*}Q^{*}$$

$$\gamma_{nt} > 0 \ : \textit{decrease in angle};$$

$$\gamma_{nt} < 0 \ : \textit{increase in angle};$$

Now, let us move on to shear strain. Shear strain is used to quantify change in angle. Moment I say change in angle I need two line segments which means I need three points, that is what is shown in the right bottom figure. In the undeformed configuration initial state I have points P, R and then Q. So, I take a plate and then mark points P R Q, and then I marked such that the angle between them is 90 degrees.

Now, P R can have one direction, P Q and can have one direction; that is why we have two directions n and t. For the case of normal strain it has only one line element, so only one direction is enough that is why we denoted as by ε_n. But when we come to shear strain you are talking about angle between two line elements which means that two directions are required. That is why we have two directions n and t.

So, what is that we have done? We have taken line elements P R and then P Q there along directions n and t and the angle between them is 90 degrees. Now, we apply a force, the body gets deformed, goes to another equilibrium state and then stops deformation. In the deformed state, in the final state R has got displaced to R* and then P has got displaced to P *, and Q has got displaced to Q*.

Now there is a change in the angle between the two line elements the angle between RPQ is 90 degrees, the angle between R*P*Q* is in this case lower than 90 degrees; of course can be more than that. This difference change in angle is quantified as shear strain.

Now, let us look at the definition. The shear strain is represented by the letter γ and two subscripts are require subscripts are n and t, and I will come to that point P as we discussed earlier. It tells about change in angle, the initial angle is $\frac{\pi}{2}$, the final angel is R*P*Q*. As we have discussed the case of normal strain I need definition of shear strain at a point. So, what should happen? This Q should become as close as possible to P, R should become as close as possible to P and this change in limit of $Q \to P$, $R \to P$ is your definition of shear strain.

Now, how do you imagine once again? You imagine at a point, two small line segments and though which are at 90 degrees to each other. Why small? We are saying $Q \to P$, $R \to P$, but still at a point you are imagining too small line segments perpendicular to each other. Of course, a little difficult to imagine we are seeing point, but is telling two lines etcetera as you will go along will get practice to that. The case of normal strain at a point imagine a small line, the case of shear strain at a point imagine two small line segments perpendicular to each other.

And what happens to their angle, why this small point because we need definition of local normal strain, definition of local shear strain. We want to describe what happens to that length change in length change in angle at every point in the body that need not be same. In the case of a simple rod it may be same throughout the rod or throughout the thread, but in the general body when subjected to force this normal and shear strain can vary or will vary most of the time throughout the body. And that is it that is why we want a definition of local normal strain and shear strain.

Coming back to this point P; I said imagine a point at that point we have two line segments. So, this rep finally it is shear strain at point P only, though we have P Q R etcetera finally the definition is for shear strain shear strain it point P only. Now, we have define this normal strain along line element which is along n and shear strain for two elements along n and t. What we are going to use in our discussion is only line elements along our coordinate axis. And that is what you are going to see now.

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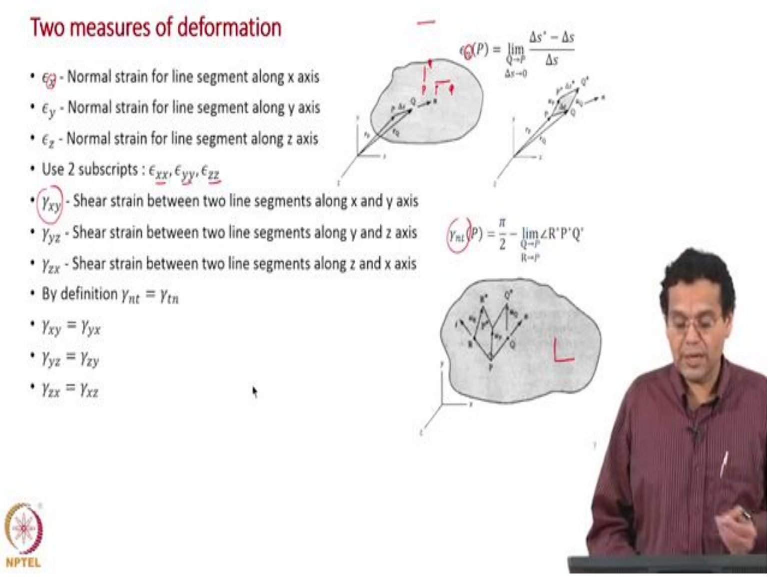

What is the analogy in term in terms of stress discussion? Remember we said any plane and then we are a stress vector resolved along the particular axis. Then we said we are going to resolve along x, y, z axis, then we took planes which are parallel to our coordinate planes. Similarly here, we have a very general definition so we got we took n, t etcetera. But now, what is it going to use subsequently are line elements which are along x axis, along y axis. So, just let us write down the normal strain for line segment along x axis.

Let us write down the definition of normal strain for line segment along x axis.

$$\varepsilon_x = \textit{Normal strain for line segment along x axis}$$

What is what do you mean by that? Take a line segment in a object you mark two points along x axis P Q, earlier it was any direction n now you will be marking along x axis only and then you will be finding out what is the change in length by original length.

$$\varepsilon_y = \textit{Normal strain for line segment along y axis}$$

$$\varepsilon_z = \textit{Normal strain for line segment along z axis}$$

Similarly, along y axis and then similarly along z axis. These are specific forms of the earlier expression and we will be using only these normal strains for line segments along x axis, y axis, and z axis

Now, for the case of shear strain we had two subscripts but for the case of normal strain we have only one subscript one subscript is sufficient. But to be uniform we will represent normal strain using two subscripts namely ε_{xx}, ε_{yy}, ε_{zz} Because there is only one direction here, but to have uniformity will use two subscripts. So, our nomenclature for normal strain for line segment along x axis will be ε_{xx} accordingly other directions.

Now, we have looked at a definition for shear strain for two line elements along n and t direction. Instead of that we will take two line elements along x axis and y axis. So, our nomenclature becomes

$$\gamma_{xy} = \textit{Shear strain between two line segments along x and y axis}$$

Similarly,

$$\gamma_{yz} = \textit{Shear strain between two line segments along y and z axis}$$

$$\gamma_{zx} = \textit{Shear strain between two line segments along z and x axis}$$

Now, shear strain represents the difference in angle between initial state and final state. So,

$$\gamma_{xy} = \gamma_{yx}; \quad \gamma_{yz} = \gamma_{zy}; \quad \gamma_{zx} = \gamma_{xz}$$

So, we have defined normal strain, shear strain locally that is important. For elements along any direction n and two elements along n and t we have adapted that or applied that for line elements along x, y, and z axis; for normal strain adapted that for line segments two line segments along x and y axis, y and z axis, z and x axis.