Continuum Mechanics And Transport Phenomena
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Lecture - 44
Total stress tensor for fluids

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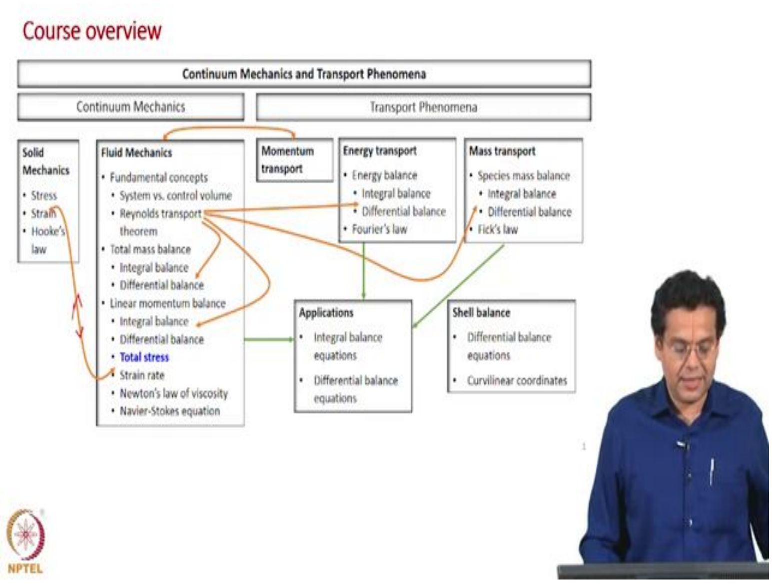

We are in the process of deriving the differential form of the linear momentum balance. Then, to understand the surface forces in the right hand side we took a diversion to solid mechanics and now that we understood the internal stresses through solid mechanics, we are traversing back through this arrow mark and coming back to fluids. And, then we are going to discuss total stress. We will understand that is we go along or in general internal stresses in fluids.

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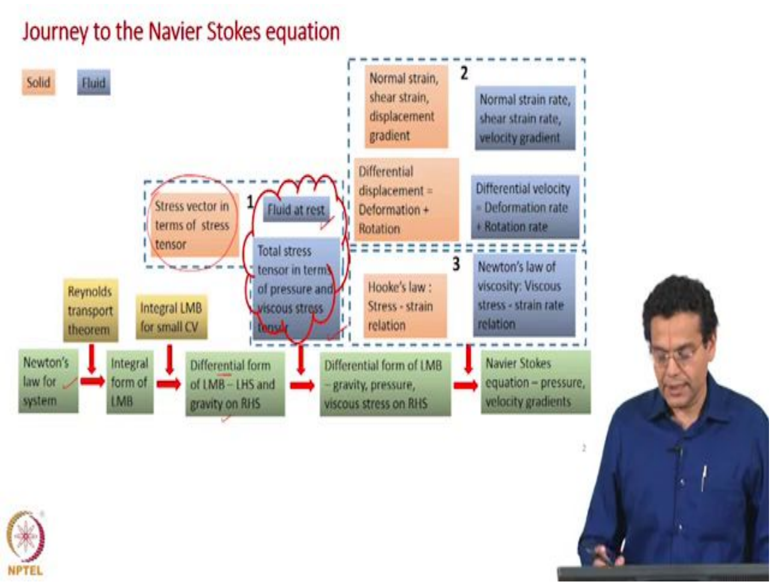

In terms of our journey, we start with Newton's second law of motion, apply the Reynolds Transport theorem, derive the integral form of the linear momentum balance and then apply it there for a small control volume, derived the differential form, the left hand side of that and then we considered the body force the right hand side and then there are surface forces. To understand that only we went to we took a diversion to solid mechanics.

We discussed stress vector, we discussed stress tensor, also relationship between these two, we also looked at properties of stress tensor. Now that is the block in orange. Let us move to the two blocks shown in terms of blue color it is applicable for fluids. What do they mean we will understand later, but should know that in terms of our journey to the Navier-Strokes we are in this highlighted portion. Once we do that we will be able to complete the differential form of linear momentum balance.

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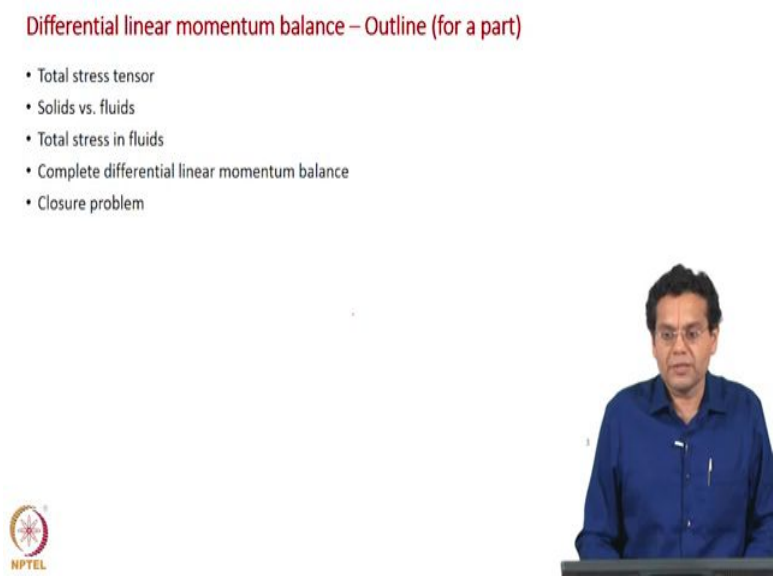

So, that is the outline of course for a part of the differential linear momentum balance. We will introduce total stress tensor for fluids and then discuss solids versus fluids. What are the differences between solids and fluids? Some properties are known to you, very well known to everybody. We will discuss specifically in terms of properties were relevant to our scope from a mechanics point of you.

And, then we quantify total stress and fluids write in terms of expression and then we will be in a position as I told you to complete the differential linear momentum balance, namely the surface forces in the right hand side. And, then you would feel that we are almost there, but then we will come across a new problem which call as a closer problem. We will discuss that as well.

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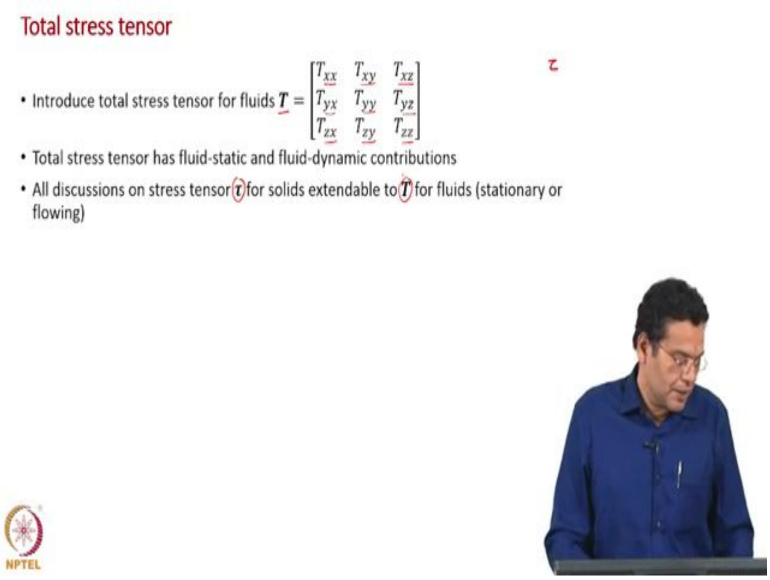

I used the word introduce because we have discussed stress tensor for solids. Now, for the case of fluids introduce another nomenclature T. In the case of solids it was τ. Now, introduce T as a total stress tensor. If you look at the components, they are analogous to our components of τ

$$T = \left[T_{xx}\ T_{xy}\ T_{xz}\ T_{yx}\ T_{yy}\ T_{yz}\ T_{zx}\ T_{zy}\ T_{zz} \right]$$

Now, why these total stress tensor? Of course right now just a statement you understand as we go along; when a fluid is stationary, some stresses are acting, when a fluid flows additional stresses are acting. So, you should combine both this and that is why we call as total stress.

We will discuss and you will understand clearly what this statement means as we go along, right now just introduction. All discussions on stress tensor τ for solids are extendable to T for fluids whether it is stationary or flowing. This is what you are going to discuss now, going to be a just it is going to be a just revision of what you done for solid mechanics with a change in nomenclature from τ to T. Putting it the other way, suppose if you are not gone to solid mechanics to understand surface forces. We have remained in fluid mechanics and discuss surface forces in fluids itself that is this is how I would have started.

But just have better clear understanding and also to understand analogy between solids and fluids, we went there understood clearly I would say and then we come back. If we have starting straight away with fluid mechanics let us say stress and fluids, then this is how we would start.

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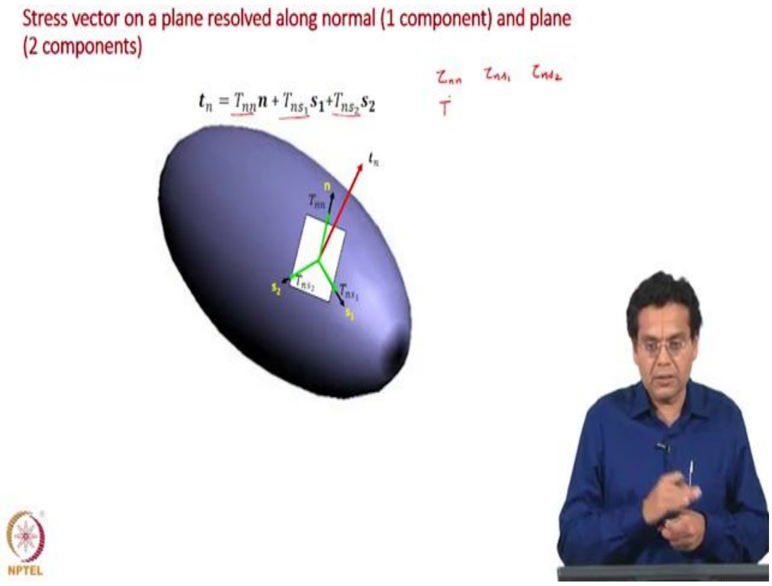

The slides we are going to see now are same slides as we have come across for solids. Only difference is the nomenclature not alone that when you now imagine visualize you should imagine this is a fluid body. Because, it is little difficult to imagine as a fluid body it was very easy for us to let us say take a solid look at a section etcetera. We went to solid mechanics and we are come back.

So, now of course a color also matches more the fluid rather than a solid I would say. So, now what did we say? We began our solid mechanics on stress saying that I have a solid make a section. All these are applicable for a fluid and then we said at a point there is any surface. Just imagine as of this is a fluid body and then we are at a particular point and there is a surface at a particular angle.

So, now what did we do? We defined a stress vector. We can define for fluids as well and then we said we will introduce three directions one normal to the plane and two tangential to the plane n, S_1, S_2. Only difference now is when I resolve the stress vector I resolve in terms of T_{nn}, T_{nS_1} and T_{nS_2}.

$$t_n = T_{nn} n + T_{nS_1} S_1 + T_{nS_2} S_2$$

Earlier there are in terms of τ_{nn} , τ_{nS_1} , τ_{nS_2} ; that is why I told you whatever you discuss for tau, it is all applicable for the T tensor.

So, now the components are denoted as T_{nn} , T_{nS_1} and T_{nS_2} the significance remind same normal stress and then shear stresses, normal stress perpendicular to the plane shear stresses on the surface of the plane. So, this whatever I discussed is completely applicable for fluids.

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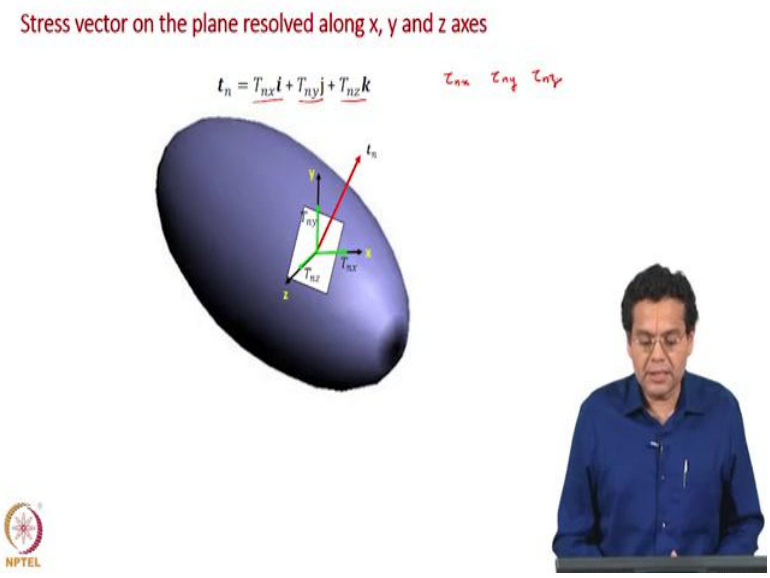

Now, what do we do next? We said we will consider any plane, but resolve the stress vector along x, y, z axis. Exactly it is applicable now also, but now as usual of course as I told you imagine as the fluid body and then now the components are T_{nx} , T_{ny} , T_{nz} . Earlier we had τ_{nx} , τ_{ny} , τ_{nz} . So, instead of that now we have T_{nx} , T_{ny} , T_{nz}

$$t_n = T_{nx}i + T_{ny}j + T_{nz}k$$

The discussion still remains valid instead of resolving the stress vector along coordinates associate with the plane. We are resolving the stress vector along our usual Cartesian co-ordinate axis namely x, y, z. So, same first subscript represents the direction of the plane, second subscript represents the direction of the component of.

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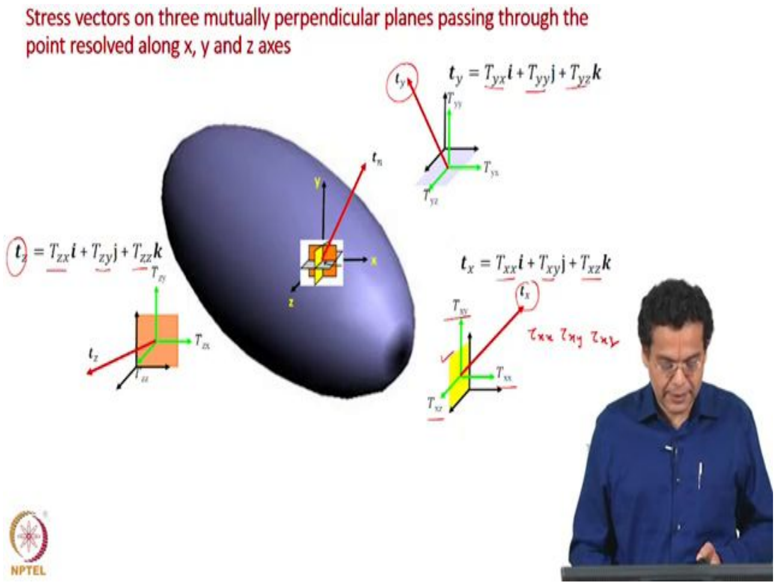

Now, we said instead of considering the plane I will consider three planes which are parallel to the coordinate plane same discussion applies now also, but now these point is inside a fluid domain and this planes are inside the fluid domain. Now, what did we say? We considered stress vectors acting on these three planes one by one, we consider for example, the x plane. Now, the stress vector acting on the x plane is resolved into the components T_{xx} , T_{xy} , T_{xz} instead of the our earlier τ_{xx} , τ_{xy} , τ_{xz} .

$$t_x = T_{xx}i + T_{xy}j + T_{xz}k$$

Similarly, stress vector acting on the y plane and stress vector acting on the z plane are

$$t_y = T_{yx}i + T_{yy}j + T_{yz}k$$

$$t_z = T_{zx}i + T_{zy}j + T_{zz}k$$

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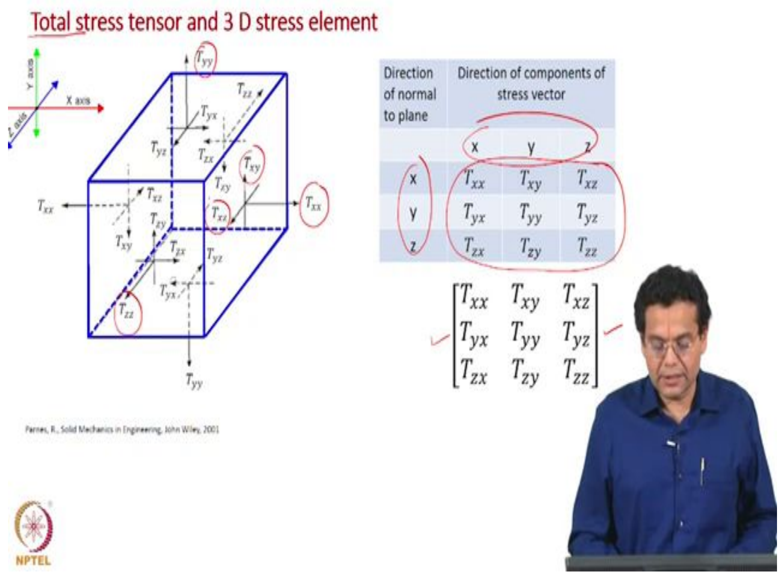

So, what we will do just like we did for solid; we will put together all the three components of each of the stress vectors acting on these coordinate planes into a stress tensor.

$$T = \left[T_{xx}\ T_{xy}\ T_{xz}\ T_{yx}\ T_{yy}\ T_{yz}\ T_{zx}\ T_{zy}\ T_{zz} \right]$$

This is the total stress tensor. The same slide as we have discussed for solids I shown here. Instead of stress tensor I use the word total stress tensor and then all the discussions are applicable here as well, we can represent that using a stress element where now we have 9 components. On the other surface we have other 9 components. You can choose any one of them.

And, then in terms of significance the first subscript represents the direction of the normal to their plane and second subscript represents the direction of the force. So, as we have seen for τ , we have this 9 stress tensor components, total stress tensor components which we said total stress tensor or simply let us say matrix, and in terms of sign convention it is applicable and in terms of significance the diagonal elements are the normal stresses and off-diagonal are the shear stresses. So, every discussion is applicable for fluids what I discussed for solids.

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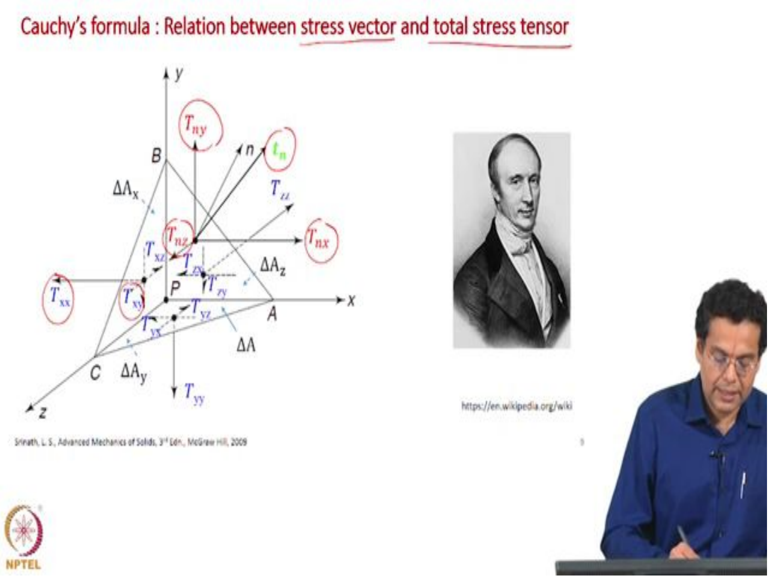

Then, we wanted to relate the components of stress vector to the components of the stress tensor and we discussed the Cauchy's law. Now, the title says relationship between stress vector and total stress tensor. It was simple force balance. So, it applicable now as well but only thing only difference as I told you is that this tetrahedron to begin with is inside of fluid body, inside of fluid domain where fluid is flowing not necessarily fluid stationery.

We have a domain through which fluid is flowing and this tetrahedron is inside the fluid domain and you are interested in any surface in the fluid domain. And then we took a force balance and then we shrink the tetrahedron to the point. All those are exactly valid here as well. Now, what is the difference in terms of nomenclature in this diagram I have shown? The stress factor of course is t_n, but now the components are T_{nx}, T_{ny}, T_{nz} and then the stress tensor components as we have seen are T_{xx}, T_{xy}, T_{xz} and analogously for other phrases y phase and z phase.

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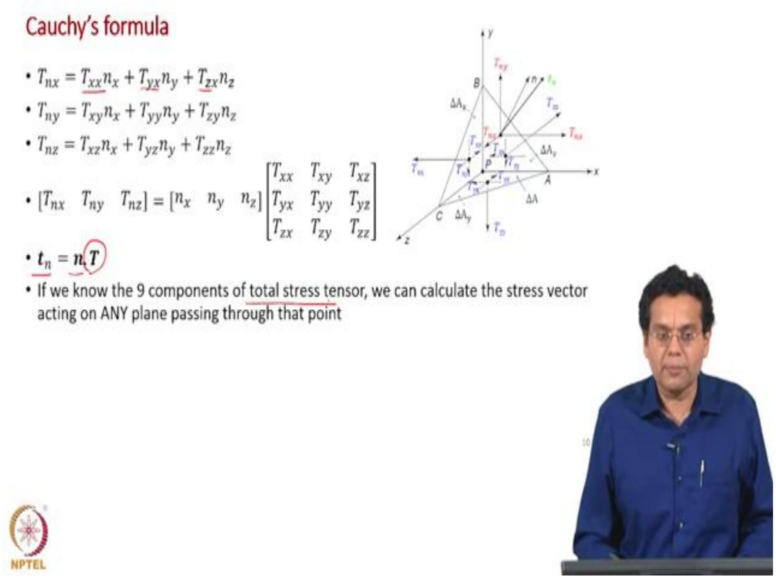

Now, we derive the Cauchy's formula. So, simple force balance certainly applicable now as well. Now,

$$T_{nx} = T_{xx}n_x + T_{yx}n_y + T_{zx}n_z$$

$$T_{ny} = T_{xy}n_x + T_{yy}n_y + T_{zy}n_z$$

$$T_{nz} = T_{xz}n_x + T_{yz}n_y + T_{zz}n_z$$

T_{nx}, T_{ny}, and T_{nz} are related to the components of the total stress tensor. So, let us put and then we express that in form of a simple matrix multiplication, same is applicable now as well.

$$\left[T_{nx}\ T_{ny}\ T_{nz}\right] = \left[n_x\ n_y\ n_z\right]\ \left[T_{xx}\ T_{xy}\ T_{xz}\ T_{yx}\ T_{yy}\ T_{yz}\ T_{zx}\ T_{zy}\ T_{zz}\right]$$

$$t_n = n.T$$

Now, the difference is that their stress vector is equal to the normal dotted with or matrix multiplication between n vector and total stress tensor, analogous description only this nomenclature change. Physically what it means we will discuss now.

So this statement is applicable now; if you know the 9 components of total stress tensor; we can calculate stress vector acting on any plane passing through that point in a fluid domain. You can just add saying that in a fluid domain.

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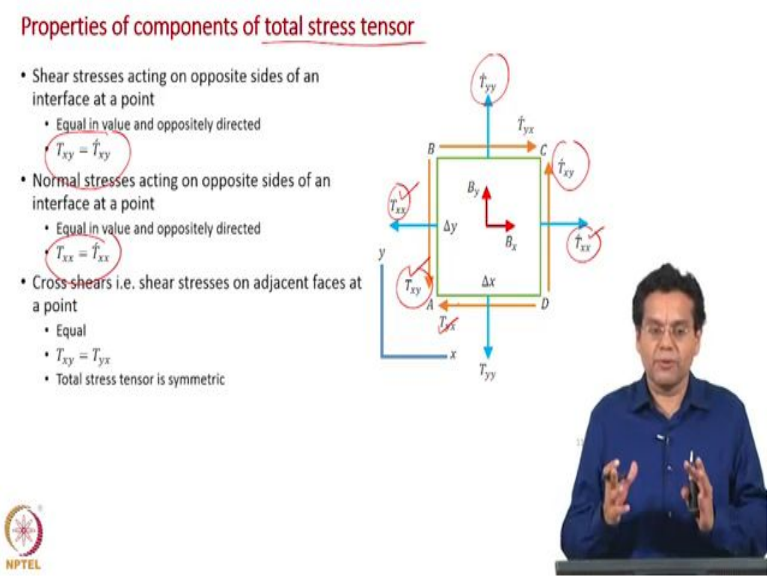

Now, we derived the properties of the components of stress tensor. Now it is total stress tensor. All the properties are applicable. Now, look at the advantage of going to solid mechanics, solid mechanics I could say that this is a plate. Consider a plate or imagine as a 2D region in a inside a solid.

Now, you can easy now we extend that it is a two dimensional region or a plane in a fluid domain. That is advantage of one big advantage of going to solid mechanics. I can show you something. I cannot show you fluid domain like this in a region of fluid domain, but I can easily show you like this in a solid and so, now it is easy for us to extend all the discussion to case of fluids and look at the nomenclature all in terms of T, components of the total stress tensor. So, let us quickly list whatever you have discussed earlier

- Shear stresses acting on opposite sides of interface at a point. So, you have a interface. Now this interface is inside of fluid domain. On opposite sides of interface we have seen that the shear stresses are equal in value oppositely directed. So, in our nomenclature now it is

$$T_{xy} = \dot{T}_{xy}$$

- Now, normal stresses acting on opposite sides of an interface at a point Once again a plane inside a fluid domain, normal stresses are equal in value and oppositely directed. So, in our nomenclature in terms of T

$$T_{xx} = \dot{T}_{xx}$$

- Finally we are said the cross shears, the shear stresses acting on adjacent face at a point; if we have a two adjacent faces, the shear stresses acting on adjacent face at a point once again inside a fluid domain are equal. So, in terms of our nomenclature

$$T_{xy} = T_{yx}$$

Similarly other components are equal. So, the total stress tensor is symmetric just like we said τ is symmetric, the total stress tensor. As of now what we have done is, we have something called τ for solids, we have something called T for fluids. What actually means we are going to discuss now, but right now as I told you; if we would started discussion for fluid straightaway, we would have set worked in terms of T. That is what I have done quickly, having discussed all the concepts using solid mechanics.