Continuum Mechanics And Transport Phenomena
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Lecture - 39
Stress tensor - Part 2

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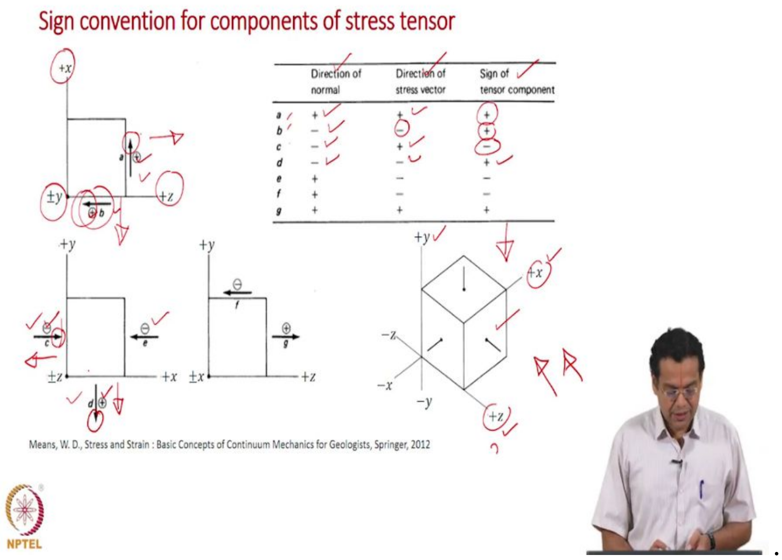


Just to explain or little more understanding in the sign convention, once again it is a very nice example from this book Stress and Strain by W D means; nothing new here little more example to understand the sign convention. You have a stress element and x axis, y axis and z axis and we are considering several cases like a, b, c, d etcetera and for each case we are going to discuss what are the direction of normal, may be few cases we can discuss direction of normal.

	Direction of normal	Direction of stress vector	Sign of tensor component
a	+	+	+
b	-	-	+

c	-	+	-
d	-	-	+
e	+	-	-
f	+	-	-
g	+	+	+

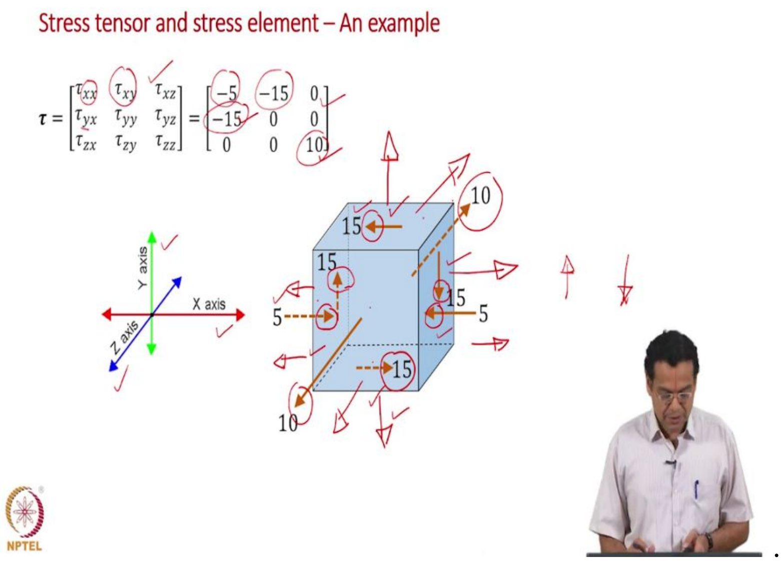
Let us take the case of a, look at the axis you are looking from the top that is why it says $\pm y$; and then you have x axis and then z axis and y axis becomes $\pm y$, which means we have looking from the top. Now, if you are considering this phase, the normal to this phase is along the positive z axis; and then suppose if you represent your component along the positive x axis, the normal to the plane is also positive, the direction of the force also positive. Hence corresponding sign of tensor component is also positive.

Now let us take the case of b, in this side the normal to this phase is along the negative x axis, the direction of the force is along the negative z axis. So, look at this, direction on normal is negative, direction of the force is also negative once again this stress component is positive.

Now let us take the case of c, for this phase the normal is along the negative x axis. Now coming to this left side phase, the normal to the phase along the negative x axis and the direction of the force is along the positive x axis; that is why the tensor component is negative. That is what we shown here; direction of normal is negative, the force is positive, resulting in a negative sign for tensor component.

So, if you take let us say d, here the direction of normal is along negative y axis, the force is also shown along negative y axis. So, you have a positive component of stress tensor and that is why both are negative here resulting in a positive sign. So, likewise you can discuss all other cases and of course, these are shear stress components and these are normal stress components; whichever is perpendicular normal stress components and others are shear stress components, then you can check the other case of e, f, g easily.

Example: (Refer Slide Time: 03:37)



Numerical example just to understand the sign convention, the stress tensor is

$$\tau = [\tau_{xx}\ \tau_{xy}\ \tau_{xz}\ \tau_{yx}\ \tau_{yy}\ \tau_{yz}\ \tau_{zx}\ \tau_{zy}\ \tau_{zz}] = [-5\ \ -15\ 0\ \ -15\ 0\ 0\ 0\ 0\ 10]$$

Let us see how do you represent, as usual we have x axis, y axis and z axis. What is shown here, is the stress element not a control volume. Why do we choose a stress element? See if it is a vector we represent by an arrow mark; you want represent this 9 components in a pictorial representation that is where we choose the stress element. We cannot represent this stress tensor with 9 component by arrow mark; if it is three component if it is vector is an arrow mark represent, because there are 9 components and you want to represent pictorially, we use a stress element.

Now, let us look at this -5. Now it says τ_{xx} which means, the x component of stress vector acting on the x phase; and then two ways of representing it. If you take the right hand side phase, the normal to the phase along the positive x axis; because the stress tensor component is negative, the direction of the force should be along the negative x axis.

Suppose if you choose to same -5 only, suppose if you want to choose the left hand side phase, the normal to this phase is towards the negative x axis. So, direction of force should be along the positive x axis. If you look at the stress element, we mention only the numerical value, magnitude only is a represented; the direction tells what is the sign of that, component in the stress tensor.

Now let us take the $\tau_{zz} = 10$ component; which means we are talking about the z component of stress vector acting on the z plane. So, we have to consider the front phase and the rare phase or one of them; if you consider the front phase then the normal is along the positive z axis, so direction of force also along positive z axis. Consider the rarer phase the normal is along the negative z, so the direction of force also along negative z axis, because the stress tensor component is positive.

Now let us take the shear stress component. Now, if you take $\tau_{xy} =- 15$, we have to consider the x phase, component of stress vector acting along y direction. How do you represent? Once again you can take the either left phase or the right phase, let us consider the right phase. And for right phase the normal to the plane is along the positive x axis; so because τ_{xy} is negative, the direction of force should be along the negative y axis. If you take the left phase, the direction of normal is along the negative x axis; because sign of the stress tensor component is negative, the direction of force should be along positive y axis. So, right hand side this arrow, left hand side this arrow or both equivalent; both represent only -15.

Now let us take $\tau_{yx} =- 15$; what does it tell you? The first subscript is y, which means we are considering components of stress vector acting on the y plane; which means we consider the top plane or the bottom plane. For the top plane, the normal is along the positive y axis, because it is negative the direction of force along the negative x axis. And if we consider the bottom plane, the normal is along the negative y axis and hence the direction of force along the positive x axis, because it is -15. I think this is a very good simple example to clearly understand the sign convention, probably to it would have been little more confusing in the previous slide; now become very clear.

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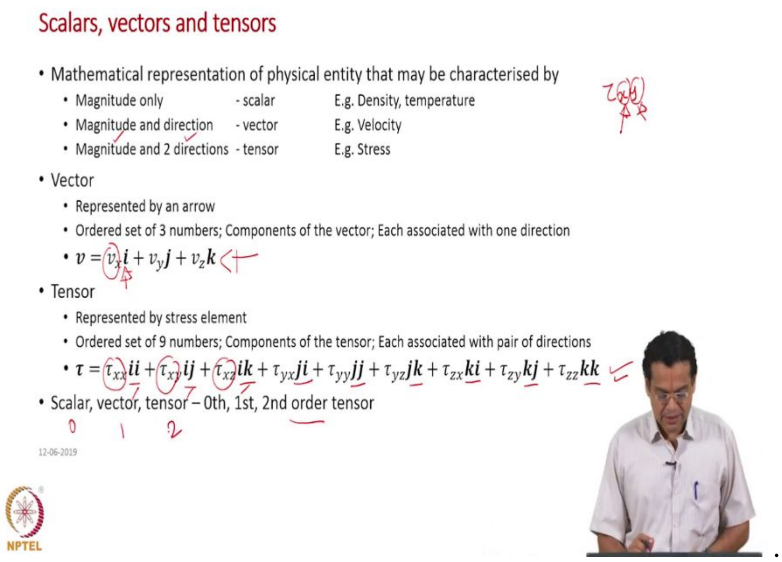

We have been talking about tensor, probably new for most of us. So, quick very brief preliminary introduction not even I would say introduction, so very brief preliminary introduction, so that have some idea what tensors are.

Now what is the motivation behind introducing tensors? The motivation comes from the physical quantity, physical entity. We have different types of physical quantities for example; we have density, temperature. It is enough you would say densities let us say 1000 kg/m^3, temperature is 100 degree centigrade that is all is required and completely describes density are temperature.

So, there are physical variables which can be described only by magnitude; of course, they are scalars. There are physical quantities like velocity; it is not enough if you say so much meter per second; immediately you should say what is the direction, so there are physical quantities like velocity which require a magnitude and direction also.

Now we are come across another quantity, namely stress which requires 2 directions; that is why we had the stress tensor components had two subscripts let us say xy, yx etcetera. So there are why do we require 2 directions? One to represent the direction of the plane, one to direction of the force.

So, to represent such physical quantities which require 2 directions, you cannot just leave saying stress that is incomplete; moment you say stress you should say, what is the direction of the plane and what is the direction of the force, etcetera. So, such quantities require another mathematical representation in terms of tensor that is a origin or requirement for tensor.

Now, vector can be represented by an arrow, graphically; immediately you draw a vector by an arrow. Or you can represent as a ordered set of 3 numbers, you want more represent as a set of numbers you can represent that, which are nothing but the components of vector; each associated one direction. For example, if you take velocity

$$v = v_x i + v_y j + v_z k$$

Now why do we discuss this, so that we can discuss analogously tensor. You can not represent tensor by an arrow mark; that is why we discuss about stress element. So, we represent tensor by a stress element, we have seen that it is ordered set of 9 numbers and they are the components of the tensor; now each associate to the pair of directions. If you say τ_{xy}, it is a pair of directions, namely i and j are involved in it. So, if you want represent tensor. So, first of all we will have 9 entries

$$\tau = \tau_{xx} ii + \tau_{xy} ij + \tau_{xz} ik + \tau_{yx} ji + \tau_{yy} jj + \tau_{yz} jk + \tau_{zx} ki + \tau_{zy} kj + \tau_{zz} kk$$

This is the actual representation of the tensor. You can represent it as a matrix, because several operations as similar to that of matrix operation. So, our purpose we can even say just matrix; but otherwise formally you represent tensor in this particular way.

What is a big difference between vector and tensor, you have only one unit vector in vector either i, j or k; but in tensor, you have a pair of unit vectors ii, ij, ik etcetera.

Now if you want to unify all of them, we can say that scalar is a 0^{th} order tensor, vector is a first order tensor and then of course, tensor can be of any orders what we are discussing second order tensor. What does order represent? Order represents the number of directions, for scalar it is 0 and vector it is 1, tensor it is 2; it can be higher also anyway that is not within our scope.

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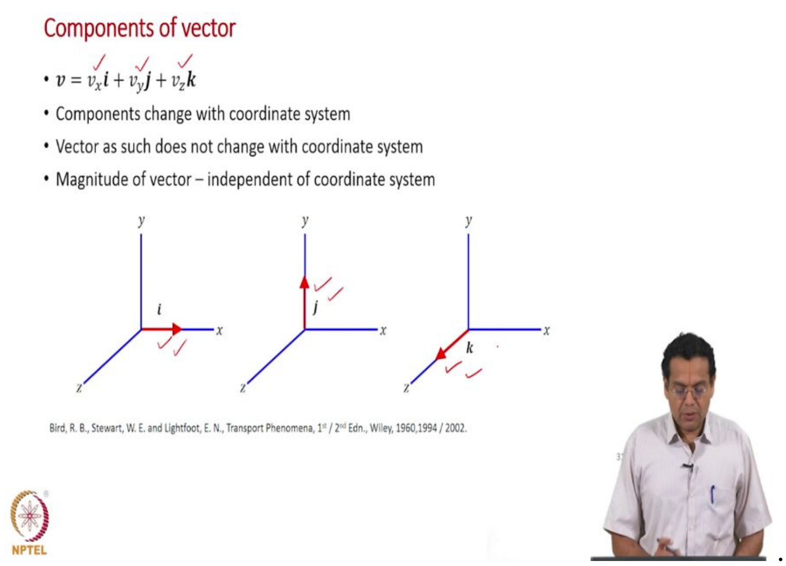

The same discussion just little more graphically here; components of vector of course, we know that we have components along x axis, y axis and z axis that is how we represent the vector.

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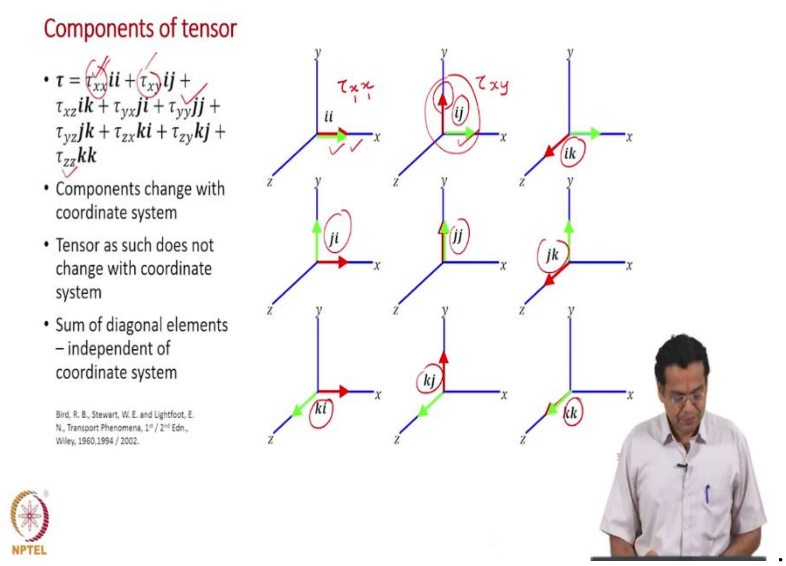

Now, let us have small discussion on what happens when you change the coordinate axis. We have this x, y, z axis and if you rotate the orientation, what happens, remember vector still remain same that is not going to change; but all these components will be different let us say

v_x', v_y', and v_z'. Depending on the orientation with coordinate axis vector remain same, but the components change.

Now we also know that, as you discussed vector as such does not changed with coordinate system and then; obviously, the magnitude of vector is independent of coordinate system. Whatever you do, some whatever magnet let us say a magnitude of a velocity that does not change; but v_x, v_y, v_z can change. Of course, this known to us why your discussing as a prelude to discussion on tensor; pictorially we represent among x axis, y axis, z axis. So, now, just analogously we will extend to tensors.

$$\tau = \tau_{xx}ii + \tau_{xy}ij + \tau_{xz}ik + \tau_{yx}ji + \tau_{yy}jj + \tau_{yz}jk + \tau_{zx}ki + \tau_{zy}kj + \tau_{zz}kk$$

Now, we have seen this representation of a tensor. Let us understand that; we said one component is associate with the pair of directions and those pair of directions, 9 combinations are shown here, first is both are i vector i vector. So, this τ_{xx} is associated with this pair of directions, both unit vectors along the x axis.

The second representation is; one the unit vector along x axis, one unit vector along y axis and the component associate with this is τ_{xy}.

Two directions are required, that is why every component of stress, every component of the tensor in general are describe stress tensor associate with the pair of directions. What are the pair of directions? For our case the first subscript represent the pair of direction of the normal and second for the it force.

Now, as we have discussed the last slide for vectors, these components will change with coordinate system. If you rotate then these components becomes τ_{xx}', τ_{xy}' all the components will change. Now tensor as such whatever we can imagine only differences is vector we can easily imagine as an arrow mark, that advantage is not here for the case of tensor; tensor whatever you can understand, imagine that is not change.

And then we said, for the vector the magnitude does not change; similarly for tensor there are some properties which will not change when you change the coordinate system. We are mentioning only one of them, three of them are there which is little more relevant to us, we just mention here. We represent it has a matrix, the sum of the diagonal elements is not depend on coordinate system; what is that, this $\tau_{xx} + \tau_{yy} + \tau_{zz}$ that sum will not change, even if you change the coordinate axis.

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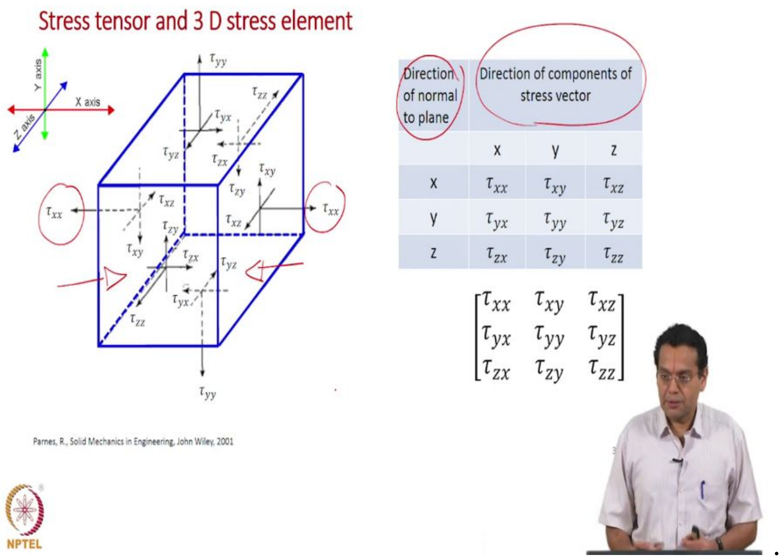

So, let us just apply this for the case. We said tensor has two directions, the tensor which you have seen as an example now is a stress tensor; or put the other way we introduce this stress and that physical quantity required two directions, hence tensor is introduced that is correct way to put it. And as you have seen in the last slides, two directions are required to represent each component of the stress tensor; one for the direction of normal to the plane and other the direction of the components of the stress vector direction of the force, only then it is completely specified.

And just like we mention, that based on the sign convention what we have chosen look at the two directions of τ_{xx}, the tensile forces become positive, compressive forces become negative. If you draw τ_{xx} in the negative direction $[\rightarrow\tau_{xx}\leftarrow]$, both are equal representation, it is a compressive force that becomes negative; tensile force we are pulling that becomes positive. That is why pressure in our sign convention is negative, I said pressure is compressive; according our sign convention it is negative minus p.

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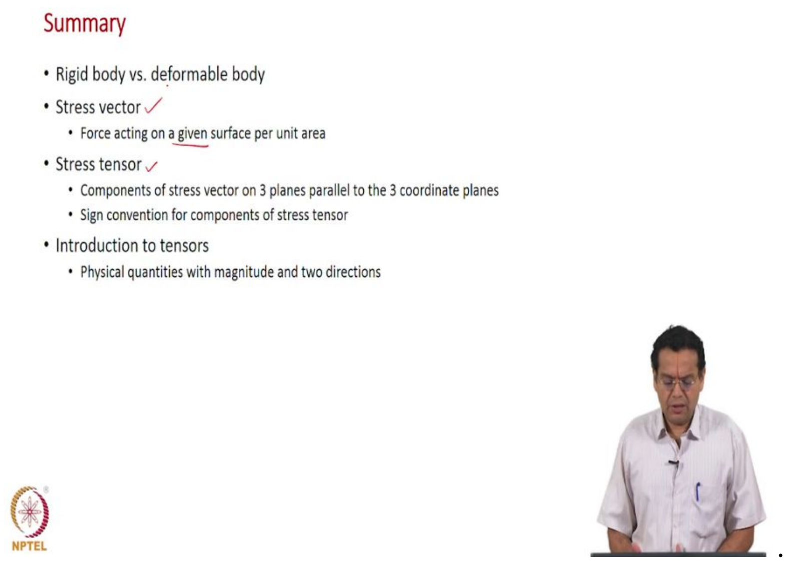

So, let us summarize what we have discussed. We started differentiating between rigid body and deformable body. We are slowly advancing in that sense, moving from rigid body to deformable body; because we need take principles of deformable solid mechanics to fluid mechanics. We discussed what a stress vector is, we defined it; force acting on a given surface per unit area. As I like to mention, why it becomes stress vector there and then a stress tensor here? The key word here is the given surface, moment I give a surface it becomes a vector; because no more further direction for the surface.

You have a surface I say, what is the force acting on this surface becomes a vector; but stress tensor is more generic it accounts for the vectors acting on all planes at that particular points. This sentence will understand later, that is why in this case second case, stress tensor; first case when you introduced we said take a plane, force acting on that plane that becomes a vector, because you specify the direction. When you allow the direction of the force and the plane to vary it becomes a stress tensor; this particular point you understand a later much clear ok.

So, of course, we introduced the stress tensor; what is the stress tensor, what are the components. The 9 components there are; components of 3 vectors 3 stress vectors, those stress vectors are all acting on planes parallel to our coordinate planes. We also discuss sign convention for the components of stress tensor. Looked examples also, you also had a very quick introduction to tensors; these are physical quantities with magnitude and two directions.