Continuum Mechanics And Transport Phenomena
Prof. T. Renganathan
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture - 38
Stress tensor - Part 1

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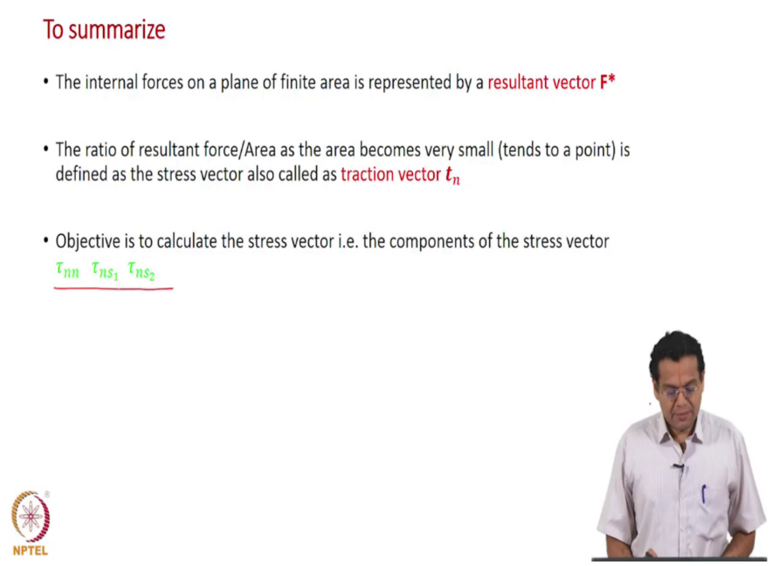

This is the summary which we have seen, I will summarize more pictorially whatever we have written in terms of sentences here.

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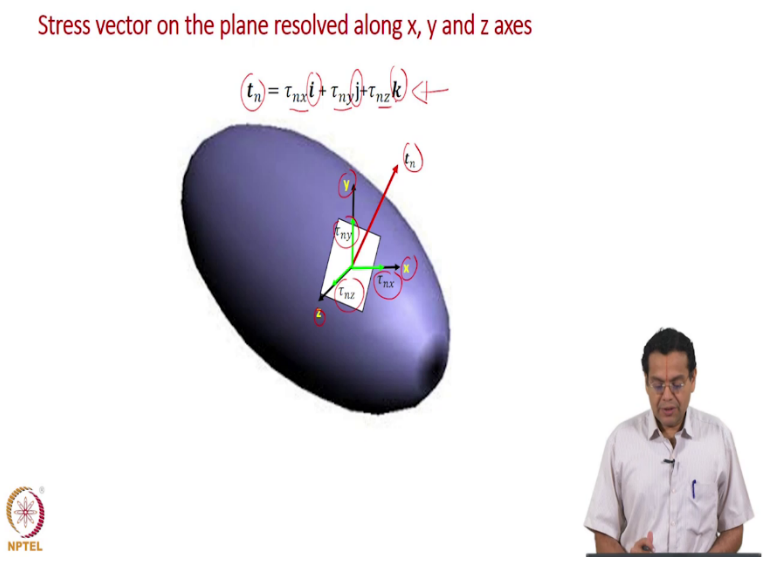

So, we began with the plane and then whose normal was n and we had a stress vector acting on that plane and we introduce a coordinate system associate to that plane or normal to the plane and then the two directions on the surface of the plane. Then we resolved stress vector acting on this plane along these three directions and those are τ_{nn}, τ_{nS_1}, τ_{nS_2} and represented vectorially.

That is where we started any plane, stress vector acting on this plane and resolved along the coordinate ax axes attach to the plane namely n, S_1, S_2 and one is the normal stress and other two together are the shear stresses. Now, next what we did remember same plane, same stress vector. But now our coordinate axes are x, y and then z very well-known coordinate axes. Resolve the same stress vector, same plane, same point remember the plane shown there is exaggerated should imagine a very small plane because remember stress vector is at a point.

And now that stress vector has now been resolved along x axis, y axis and z axis represented vectorially, the same stress vector. Now, the unit vectors are i, j and then k components are τ_{nx}, τ_{ny}, and τ_{nz}.

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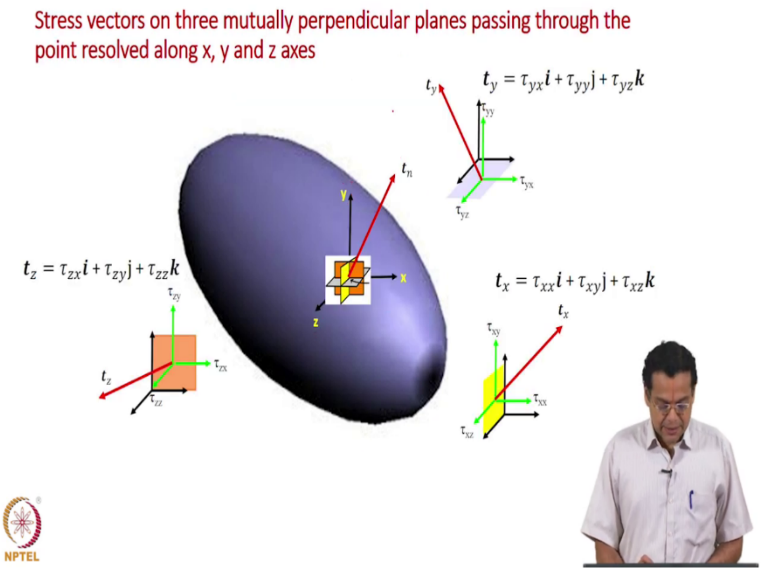

Look at the figure the slides have been made so that the same location we replace with three planes let me go back and come back. So, that you have clear understanding at the same point, we have some arbitrary plane of any direction n vector instead of my objective is finite to find out the stress vector and then the components.

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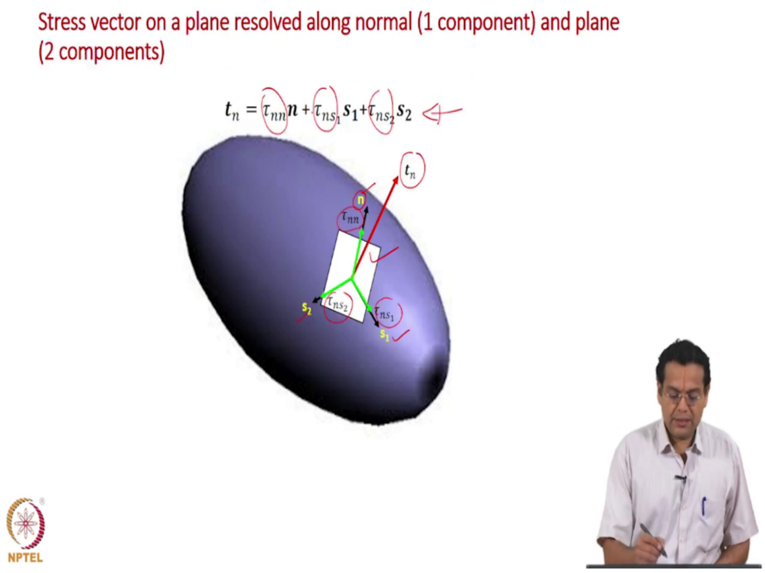

Because on a plane I am interested in the normal force and the tangential forces that is my objective, because there is straight away difficult be resolved along x, y, z axis. Because this is also straight away difficult we said I will consider three planes at the same position. Instead of this inclined plane any plane at the same point I consider three well defined planes. When I say well defined those planes are parallel to our usual x, y, z coordinate planes which we denote as x plane, y plane, and z plane.

So now, let us take the x plane; the x plane is shown by the yellow colour and that is what is shown magnified. So, now t_x is a stress vector acting on this x plane remember that is t_n is stress vector acting on the n plane and t_x is the stress vector acting on the x plane shown here with it is components τ_{xx}, τ_{xy}, and τ_{xz}.

So, this is the normal stress acting on x plane and the shear stresses acting on the x plane, the very clearly shown in terms of normal and then tangential directions, vectorially you represent as,

$$t_x = \tau_{xx} i + \tau_{xy} j + \tau_{xz} k$$

Now, similarly you take the y plane, the y plane is shown in light shade of blue colour and then t_y is the stress vector on the y plane and then the components are τ_{yx}, τ_{yy}, τ_{yz} and shown vectorially here.

$$t_y = \tau_{yx} i + \tau_{yy} j + \tau_{yz} k$$

Similarly, the z plane, the z plane shown by an orange colour and then the stress vector acting on that z plane and the components are τ_{zx}, τ_{zy}, τ_{zz} and then shown vectorially here.

$$t_z = \tau_{zx} i + \tau_{zy} j + \tau_{zz} k$$

We started with the t_n, we have to find out that, but now we have come to t_x, t_y, t_z how are we going to that we will see shortly. Now, once again I will summarize this in another way I would say completely in the reverse direction, we went in one direction forward. Now, next slide will summarize the entire discussion in a reverse direction let us see that.

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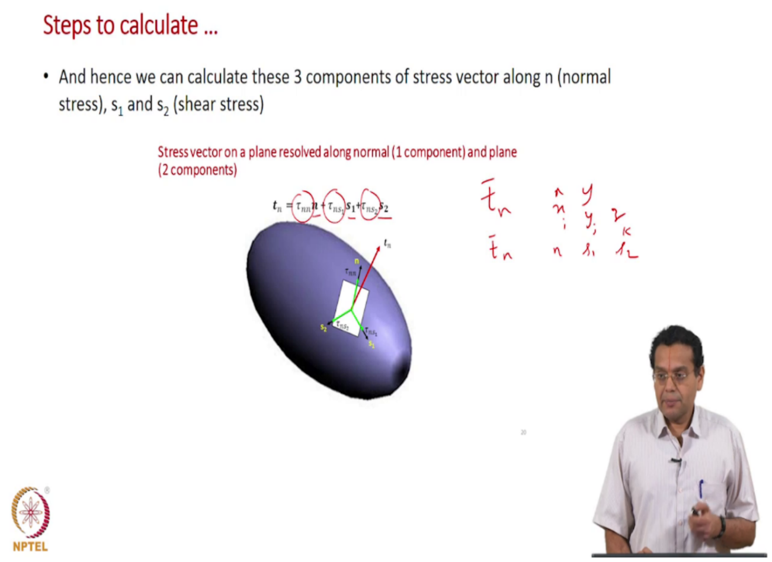

Now, what are the steps to calculate? If we know these 9 components, remember this is where we ended we are starting from there. If we know this 9 components, what are the 9 components, τ_{xx}, τ_{xy}, τ_{xz}, τ_{yx}, τ_{yy}, τ_{yz}, τ_{zx}, τ_{zy}, and τ_{zz}.

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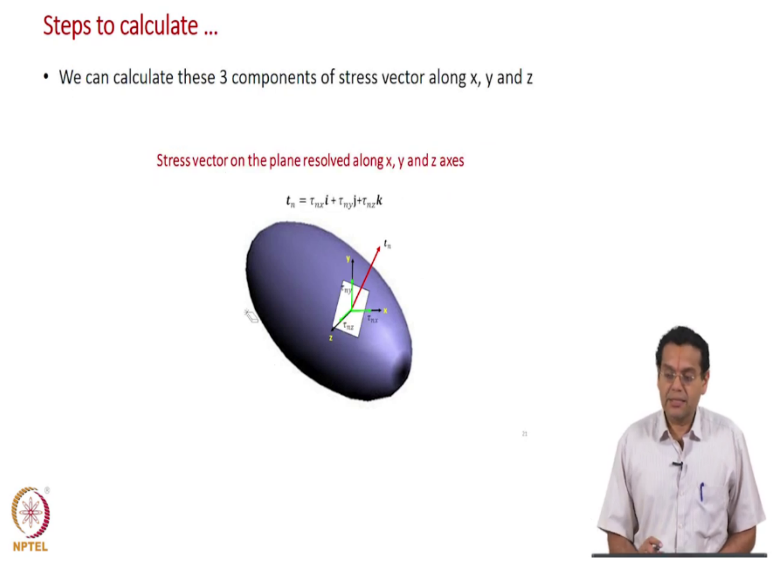

So, if we know these 9 components we can calculate the three components of stress vector. But these components of stress vector are along x, y and z axis. We get the component of stress vector, but the components along x, y, z axis.

How do we do that we will discuss later. Suppose, we are able to get these components I can calculate the three components along the normal to the plane and tangential to the plane, this is what we are interested in. Finally, we need to calculate these three components which are along n, S_1 and S_2 and τ_{nn}, τ_{nS_1}, τ_{nS_2}.

Now, second step is simple that is pure mathematics, if you are having a vector resolved along x, y, z axis then you can resolve easily along any other coordinate axes that is the second step. So, if you know t_n resolved along x, y and then z axis you can resolve t_n along n, S_1 and S_2 directions. We have resolved t_n along i, j, k and we are resolved along n, S_1, S_2. Now, second step is easier first step we will discuss later.

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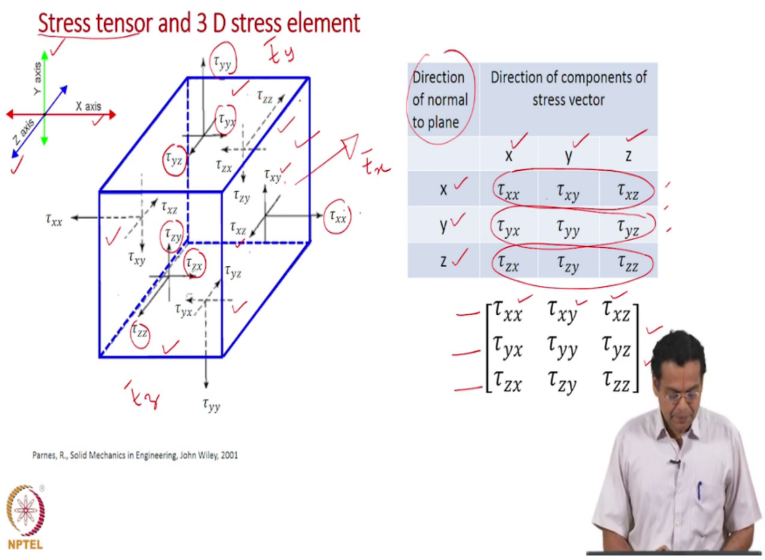

Now, what we will do is collect all the components and put in a more structured way and that is shown in the form of pictorial representation here. That is what is shown in the left hand side let us first understand the axes; x axis, y axis and z axis. What is shown is a cuboidal shape I want emphasize that it is called stress element it is not a physical volume. We have seen control volumes earlier this is not a control volume this is only a representation of all the nine components which are discussed earlier. So, once again it emphasize this is just a pictorial representation not a control volume it is called as stress element, what is it being shown there.

Let us take the right hand side face and it is a x face because x axis perpendicular to that and you have a stress vector acting on this face which is not shown. But what they shown are the components of t_x, remember we discussed about stress vector acting on x plane. What is shown here are the components of the stress vector along x direction, y direction and z direction. So, the three components of stress vector acting on x plane are shown here.

Similarly take the top face once again the stress vector t_y is not shown, but the components of that stress vector τ_{yx}, τ_{yy}, and τ_{yz} are shown there.

Similarly take the front face a stress vector t_z that stress vector is not shown. But the components are shown here along x axis τ_{zx}, τ_{zy}, and τ_{zz}.

So, totally 9 components are shown, 3 components for t_x, 3 for t_y, 3 for t_z and that is what is also summarized in this table. Direction of normal to the plane x, y, z we have considered planes whose normal are x, y, z and then the direction of components are once again x, y, z that is why we result in combination of three by three nine components are there.

Direction of normal plane	**Direction of components of stress vector**		
	x	y	z
x	τ_{xx}	τ_{xy}	τ_{xz}
y	τ_{yx}	τ_{yy}	τ_{yz}
z	τ_{zx}	τ_{zy}	τ_{zz}

So, if you look at the first row they are all components of t_x, whatever you see on the right side face and then if you take the second row they are all components of the vector acting on the y plane, that top plane and then if you look the third row they are all components of stress vector acting on the front face. So, these three rows with three elements is can be represented as a matrix. Whatever represented table all the 9 components have been arranged in a form of a matrix.

Now, I called as a matrix, but a more formal representation is called is called a tensor the 9 components have been put and what is called as a stress tensor as the title indicates.

$$Stress\ tensor = \left[\tau_{xx}\ \tau_{xy}\ \tau_{xz}\ \tau_{yx}\ \tau_{yy}\ \tau_{yz}\ \tau_{zx}\ \tau_{zy}\ \tau_{zz}\right]$$

Now, about the stress tensor you will gradually understand as you go along the course. As far as this course goes when are not going to discuss in detail about tensors but we will have a small introduction to that you will slowly understand what tensors are as we go along. So, right now you can imagine as a matrix which has 9 elements in it and remember each row represents the 3 components of the stress vectors three stress vectors and each com each row we are used to represent the 3 components of the stress vectors. Now that is we have 9 components here.

Now, if you look at this diagram, there are nine other components shown with dashed lines if you look at the left face some three components are shown, if you look at the bottom face some three components are shown, if you look at the rear face some three other components are shown. They are shown in opposite direction they are shown in dash line.

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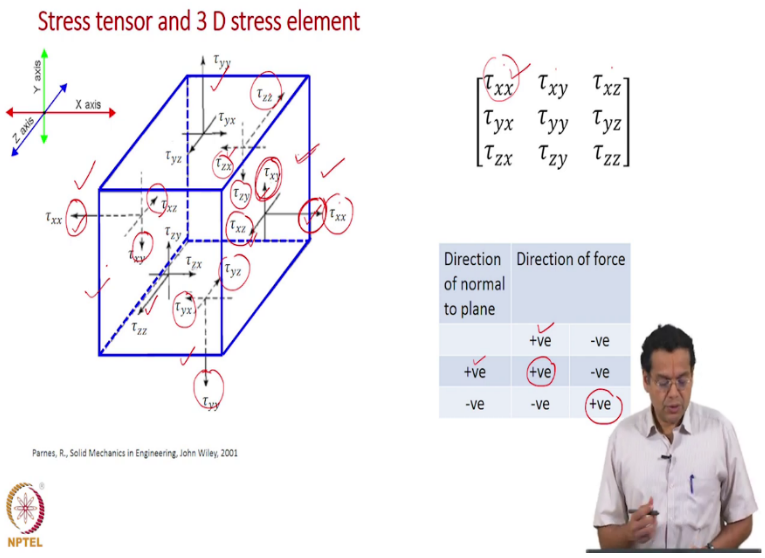

Now, to understand that what are they let us look at the next slide. Now for representing stress element we follow a sign convention. What is the sign convention for a stress component? You consider a stress component, component of stress tensor to be positive under two conditions, what are the conditions. If we have a positive face what is a positive face you have a plane whose axis is towards the positive x axis, so this is a positive face. In that positive face if the direction of force is also along the positive axis then this component is the positive value. So, if the direction of the normal to the plane is positive and then the direction of the force is also positive. Then the direction of stress component is positive.

Direction of normal to plane	**Direction of force**	
	$+ve$	$-ve$
$+ve$	$+ve$	$-ve$
$-ve$	$-ve$	$+ve$

What is the other possibility if the direction of the face is negative that is what is shown here, when I say face is negative, then the direction of the normal to the face along the negative x axis and the direction of the force is also negative, then once again you represent a positive component of the stress tensor.

So, the right hand side representation and the left hand side representation are both equivalent. If you take a face whose normalize along positive x axis all your arrow marks are along the positive coordinate direction this component is along positive x axis, this component is along positive y axis and this component is along positive z axis.

Now if you take the left hand side face the normal to the plane is along negative x axis. Now, look at the all the arrow marks they are towards the negative axis negative x axis, negative y axis, negative z axis, but both have both represent the same tau xx the same τ_{xx}, and then same τ_{xy}, $\tau_{xy_{xz}}$ that is same only. It is like, you have a plane you are looking at opposite sides of the same plane. Remember it is not as I told it is not a control volume this same plane you are looking from this side same plane you are looking from other side same equivalent representation.

So, what we have discussed at the earlier is for the top face, for the bottom face the normal to the face along negative y axis that is why all the directions of the force are also along negative y axis. See τ_{yx} is along negative x axis τ_{yy} along negative y axis τ_{yz} along negative z axis. Similarly we have discussed earlier for the front face, now for the rear face the normal to the face along negative z axis.

So, all the direction of the force are also along negative axis τ_{zx} along negative x axis, τ_{zy} along y axis, τ_{zz} along negative z axis. So, though 18 components are shown, but they are equivalent to just 9 components, only two ways of representing it. Just one discuss the other way also, if one of them is opposite direction, that is if you have a positive face and the force is along negative direction then the stress component is negative or if the direction of the normal to the face along negative axis and the direction of the force is along positive axis, then again we have a negative stress component.

So, if both are positive or both are negative you have positive any other combination other combination gives you the negative value of stress component. I have an example to show you will understand better. Now, the question arises why eighteen are shown why is it shown as a something like a closed box.

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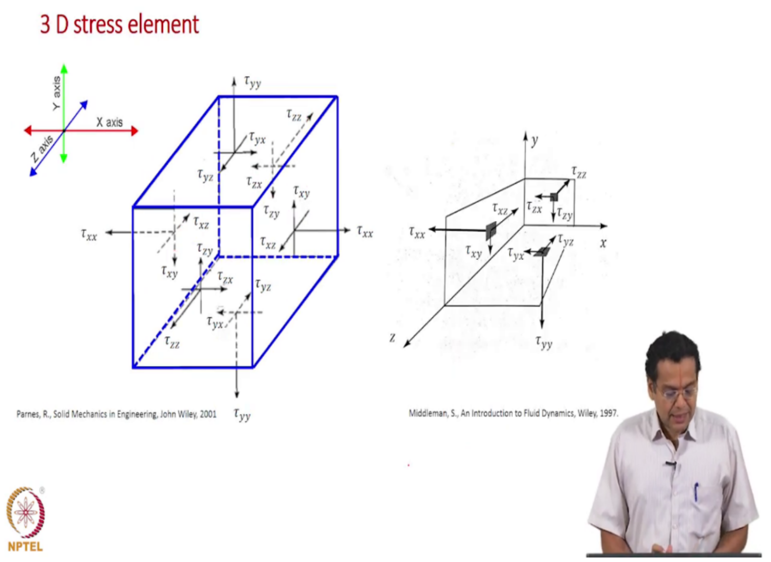

Explain that here this diagram is same as our last slide, what is shown in the right hand side is the very good representation from this book by Stanley middleman on introduction to fluid dynamics. Where only 9 components are shown that is what we want and all the left side face, the bottom face and the rear faces are shown and all the forces are along the negative direction.

Now, when we study, when we represent we always like to represent or easy for understanding imam imagination if all of them are along the positive axis, that is why this representation is shown where all the negative direction of forces and the positive direction forces are shown, we require only either one of them either positive or negative, this representation is sufficient very much, but we are more comfortable looking at forces along the positive axis. That is why this representation is on the left hand side is shown, but the right hand side representation is very much sufficient.