Continuum Mechanics And Transport Phenomena
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Lecture - 37
Stress vector - Part 2

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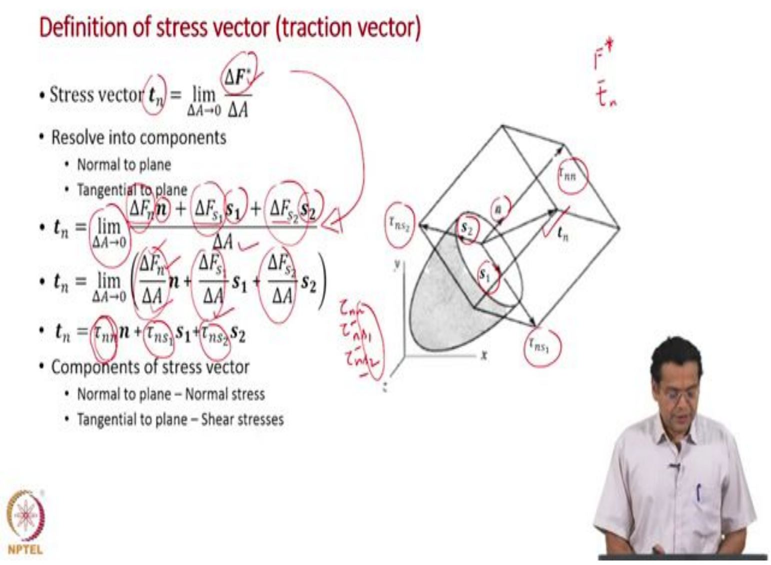

We have defined the Stress Vector as

$$t_n = \frac{\Delta F^*}{\Delta A}$$

Now, what do I do is that, just like we resolved the resultant force along the three directions n and S_1, S_2; I will resolve this ΔF^* incremental force along the three directions which you have chosen, that is what we will do now. So, resolve into components, along our chosen coordinate system n vector, S_1, S_2 vector. So, normal to the plane and then tangential to the plane.

So, now, let us do that, because if you talk about force it has all the information; but what is more useful is if you talk in terms of components ok. If you keep talking about velocity vector that is fine, but you always talk in terms of velocity components v_x, v_y, v_z, fine.

$$t_n = \lim_{\Delta A \to 0} \frac{\left(\Delta F_n n + \Delta F_{S_1} S_1 + \Delta F_{S_2} S_2\right)}{\Delta A}$$

So, the ΔF^* has been resolved along the n direction, denoted as ΔF_n along S_1 ΔF_{S_1}, and along S_2 ΔF_{S_2} divided by area; of course, limit $\Delta A \to 0$. So, this is a component representation of this equation given in the first bullet. That is in terms of incremental vector itself; but now it is in terms of the component. Let us say the F_n is the normal component of ΔF_n represents the incremental normal force and then this is the incremental tangential force and then the incremental tangential force along the respective directions.

$$t_n = \lim_{\Delta A \to 0} \left(\frac{\Delta F_n}{\Delta A} n + \frac{\Delta F_{S_1}}{\Delta A} S_1 + \frac{\Delta F_{S_2}}{\Delta A} S_2 \right)$$

Now, we will define this incremental normal force divided by area, as area tends to 0 as the normal stress. Why is it normal stress? Because this is acting normal to the plane.

$$t_n = \tau_{nn} n + \tau_{nS_1} S_1 + \tau_{nS_2} S_2$$

So, this is definition of stress vector, because we have split into components this term that is incremental normal force or the normal component of the resultant force; we want to be very specific, divide by area as the area tends to 0 is denoted as τ_{nn} which is normal stress.

Now, similarly the second term, that is the incremental tangential force divided by area; tangential force along S_1 direction in the limit of $\Delta A \to 0$, gives you the one of the shear stress component; why is it shears stress? It is acting parallel to the plane or along the plane, tangential to the plane. Similarly, along the other direction you have the other component of shear stress. So, you have normal stress component τ_{nn} and then 2 shear stress components as I told you, because it is on the plane you cannot have one; even if you have one you will not resolve along 2, that is why you have 2 shear stress components τ_{nS_1} and then τ_{nS_2}.

Now, let us understand the meaning of the subscripts. That first subscript represents the direction of the plane and then the second subscript represents the direction of the component. This has to be kept in mind, this is going to play major role as we go along. Just to repeat, the first subscript n represents the direction of the plane, second the direction of the component. Now, all these whatever you have discussed we shown diagrammatically as a figure in the right hand side. what is shown? the stress vector is shown t_n.

And then the normal component, the normal stress is shown and then the shear stress components τ_{nS_1} and τ_{nS_2} are also shown. Normal component is along n and the shear stress components are along S_1 and S_2. What is it we have done? The earlier we resolved F^* the whole resultant force along n, S_1, S_2.

What we have done now is resolved t_n vector along n, S_1, S_2. What is the difference, resultant is over a area, averaged over a area; but now we have resolved a, what is the stress vector acting on a plane at a particular point that has been resolved along normal direction and the tangential direction as well. So, to summarize this slide, we have introduced the normal stress and the shear stress components. Of course, in terms of terminology we have discussed components of stress vector, the normal to the plane is a normal stress and tangential to the plane are the shear stresses. So, 1 component of normal stress, 2 components of a shear stresses.

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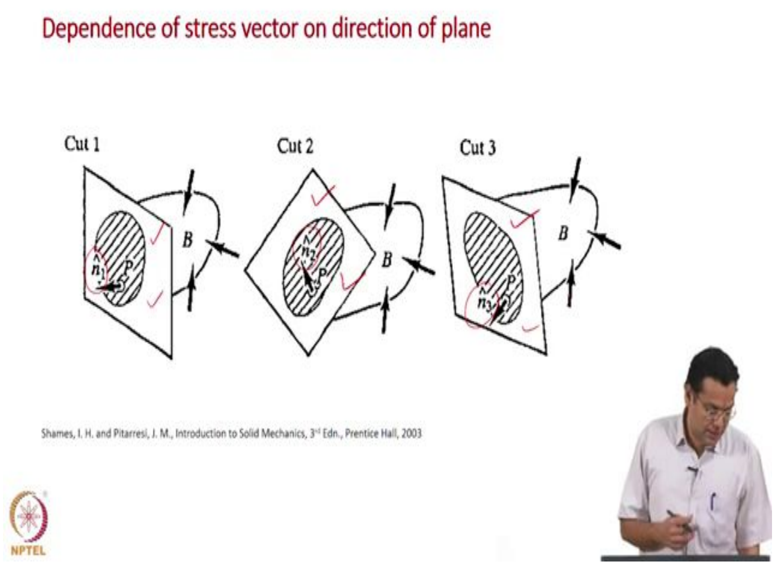

Now, please imagine this to be somewhere inside the object; at a point we have drawn a plane and identified it as n. We also mentioned that, same point you can consider several planes of different n vector. And that is what is shown here, same point; please remember it is a same point, but the orientation of the plane can be different; which means that, the n vector will also change, based on the orientation the n vector will also change.

Now, which becomes little inconvenient for us, because our coordinate system is attached to the plane, it is associated with the plane. So, based on the plane you consider; this will have one n vector S_1 vector, S_2 vector, similarly another one will have another set of n, S_1, S_2.

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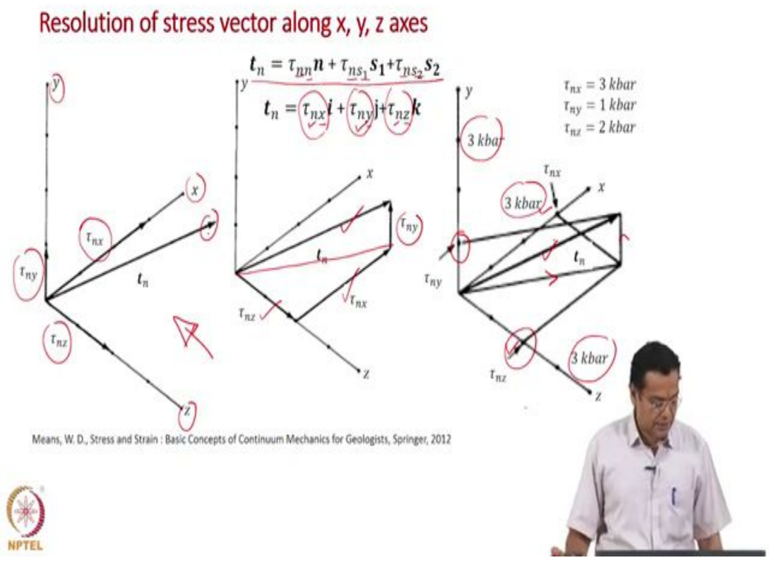

Which becomes very inconvenient for us and that is why we consider the plane, any plane; but now instead of choosing the coordinate axis associated with the plane; we resolve the stress vector along our usual x, y, z axis. Why is that, because in the earlier we want of course, obvious what is that is required for us is the normal stress and the shear stresses. but the way in which we resolved as difficult, we will find out the as we go along, but right now instead of resolving the stress vector along n, S_1, S_2, I resolve the stress vector along the our usual Cartesian coordinate systems directions; namely x, y and then z.

Like to mention the books of course, you would have observed, the previous figures are all from Raymond Parnes; then this particular and few other figures are from the book Stress and Strain by W D Means. I did not mention about this in the introduction, very good book to understand about stress, split into very small chapters, looks handy like a story book as well. So, strongly recommended for understanding fundamentally about stress relatively a difficult concept put nicely very sequentially systematically in terms of slowly building on the concepts in this particular book.

The stress vector has been resolved, now what is the nomenclature you will give. Remember the stress vector t_n was resolved along n, S_1, S_2. So, notation was that, the first subscript represents the direction of the plane; that is not going to change. We are still considering any plane, so the first index there is no change, the first subscript there is no change.

Now, remember the second subscript represented the direction of the component. Now, we are resolving the stress vector along the x, y, z coordinate; that is why second subscript has been now changed to x, y, z.

$$t_n = \tau_{nx} i + \tau_{ny} j + \tau_{nz} k$$

So, n represents the direction of the normal to the plane; and then the second subscript represents the direction of the component of the stress vector.

Now, the same thing has been shown in two other different ways nicely in this book. Look at this first figure shows the component of stress vector along x, y, z axis $\tau_{nx},\ \tau_{ny},\ and\ \tau_{nz}$. Of course, the coordinate axis has different orientation; but as usual if you look at in this direction this is our usual x, y, z coordinate system ok.

The figure in the middle shows the same information in slightly different way; the way in which you construct the stress vector. Let us say τ_{nx} vector is drawn parallel here and then you have τ_{nz}; let us say you add this, you will get one vector and then to that you add τ_{ny}, you will get the stress vector. So, the construction of stress vector is nicely shown, that is why this book is very good in terms of understanding.

Once again example is also shown here, for example what is shown is, we have chosen a value of τ_{nx} = 3 kilo bar; of course, all stress value stress units are force per area. So, that is why the unit is kilo bar, something like our pressure. And then τ_{ny} = 1 kilo bar and τ_{nz} = 2 kilo bar.

So, now, once again the same construction is shown here let us say you take this 3 kilo bar and add the 2 kilo bar; you get one vector, to that you add the 1 kilo bar and you get this stress vector. So, very good representation of the stress vector resolved, remember the plane is still any plane; I repeat this because will relax that as well as we go along on any plane, the stress vector acting. Remember all these are point values, already we have taken limit; you have a plane at a point, you are resolving this stress vector along x, y, z axis and they are called as $\tau_{nx},\ \tau_{ny},\ and\ \tau_{nz}$.

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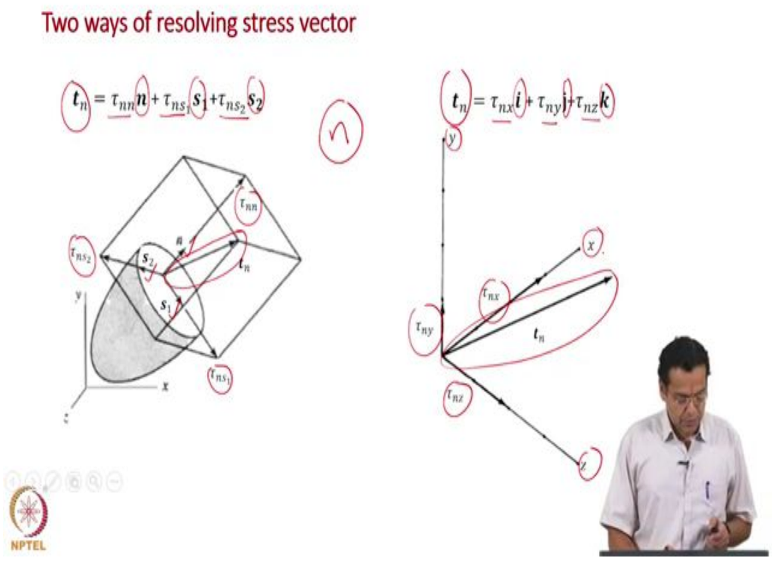

Now, let us proceed. We have resolved the stress vector in two different ways ok, that is what this slide is about; stress vector is same, remember t_n is same, the left hand side that is same. The way in which you have resolved only is different;

$$t_n = \tau_{nn} n + \tau_{nS_1} S_1 + \tau_{nS_2} S_2$$

In the first case we resolved along coordinate axis associated with the plane. We resolved along n resolved along S_1, S_2 and hence represented as τ_{nn}, τ_{nS_1}, τ_{nS_2}, and the unit vectors where n vector, S_1 vector, S_2 vector.

$$t_n = \tau_{nx} i + \tau_{ny} j + \tau_{nz} k$$

Same stress vector only, has now been resolved along our usual x, y, z Cartesian coordinates and that is why the unit vectors are now i, j and then k; and then now the components become τ_{nx}, τ_{ny}, τ_{nz}. Please note the first subscript is still n, the same plane; only the second subscript has changed, because the coordinate directions along which you are resolving is different in both the cases, stress vector is still same.

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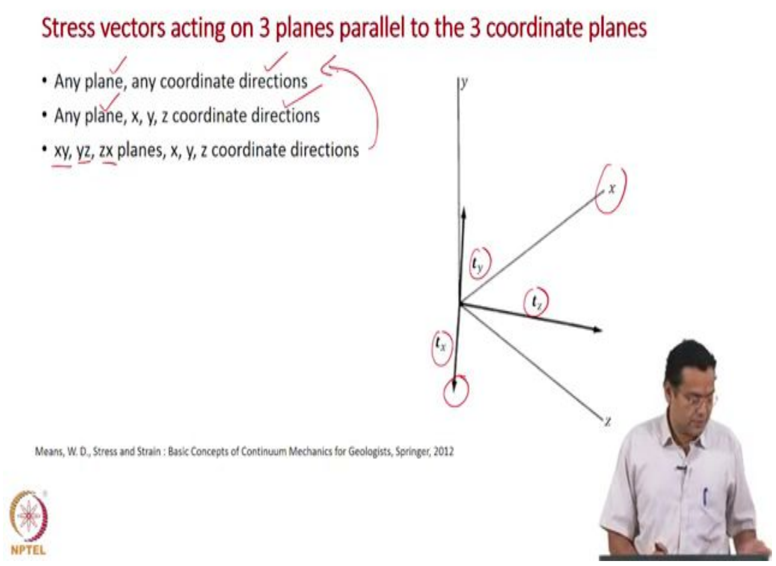

Now, as I was telling you, we have considered any arbitrary plane at given by n vector. If you look in terms of Hierarchy,

- First we said any plane and then we said any coordinate direction; because the coordinate direction is associated with the plane, which means any plane any coordinate directions. That is very generic, very difficult to handle, we want some form of uniformity, some form of generalization, some way of representing in universal way;

- Secondly we said we will consider any plane, but we will resolve along x, y, z coordinate direction becomes little bit simpler to handle. But even this is difficult because, though we are interested in stress acting on any plane, we will make it little more easier to handle.

- Instead of considering this any plane, I consider the planes which are parallel to our coordinate axis. That is why I have shown here, remember all these planes are at the same point; you have understand that as I show further figures, at the same point. When I say a plane, you should imagine a very small plane at the same point; instead of considering plane of normal n, you are considering our usual planes which are parallel to the coordinate axis. What are the planes; xy plane, yz plane, and zx plane; that is what is shown here.

What is now shown here? Stress vector not acting on the n plane it is stress vectors acting on the xy, yz, zx plane. We do not call as xy, yz, zx plane. We call in terms of the normal; that is why we denoted it as stress vector acting on x plane. What is x plane? x plane is one for which x axis is normal. So, in this case it is something like this. So, t_x, t_y, t_z of course, I will discuss more on this in future slides; as of now we have moved one step I would say closer to in terms of understanding easier to represent, that is more important.

So, from any plane, any coordinate directions to any plane, but x, y, z coordinate directions; but now see look at the organization becomes more and more organized; now easy to understand these planes are very well known to us and coordinate axis are also very familiar to us. Of course, finally, we have to go back to this, because our interest is on any plane what is the normal force acting, what is the shear force acting, we will have to come back to that; but now we are breaking down the problem into simpler and simpler conditions. Now, let us discuss more about this in the next slides.

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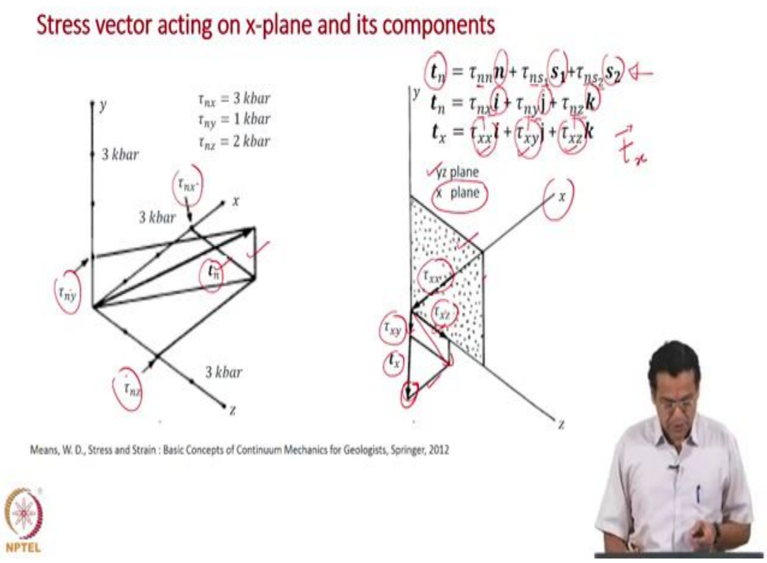

Now, what is shown here, earlier; in the slide prior to that, we have resolved the stress vector on any plane along x axis, y axis, z axis. Now, instead of that any plane, I have a well defined plane. As I told you, this is the plane which you call as x plane or actually the plane is, is in the plane of y-z axis. So, you can call either as yz plane; but we have denoted the plane by the normal to the plane. So, according our nomenclature it is x plane, the x axis is normal to this plane hence it is called x plane. So, we will use this nomenclature in further discussion; when

I say x plane usually immediately let us say this is your x axis, you should imagine this also x plane.

The plane to which x axis is normal, that is such big sentence has become x plane. Now, t_n is a stress vector acting on any plane; now what is the notation, should be t_x, because our x is a normal and that vector is shown here t_x.

$$t_x = \tau_{xx} i + \tau_{xy} j + \tau_{xz} k$$

Now, just like τ_{nx}, τ_{ny}, τ_{nz} were components of t_n, we will introduce τ_{xx}, τ_{xy}, τ_{xz} as the components of stress vector acting on x plane. How do you interpret that how do I understand that, let us look at this first representation of stress vector; stress vector acting on any plane. So, that is why the first subscript was n, we resolved along coordinate axis associated with the plane. So, second subscript is n, S_1 and then S_2 and the co-ordinate directions where n S_1 and S_2.

Next step what did we do? We resolved along x, y, z axis. So, unit vectors became i, j, k and the components still had n; look at this, the first subscript is still n, because it is any plane. But now, because we resolved along x, y, z axis, the second subscript became x, y, z. Now, we are made one more simplification, the plane is not any plane, it is x plane. So, I am replacing the n with x; hence I get τ_{xx}, τ_{xy}, and τ_{xz} as the components of the stress vector acting on x plane.

So, now, τ_{xx} represents the normal stress and τ_{xy}, τ_{xz} represents the shears stress on the x plane, normal stress acting on the x plane and then the shear stresses acting on the x plane. And that is shown here, also nicely here look at this, the τ_{xx} is a component of course; you can have any component it is now along the negative x axis and you have τ_{xy} here and then τ_{xz} here. So, what we have done here is moved from any plane to the x plane and resolved the stress vector, stress vector acting on the x plane into it is components along x, y, z axis.

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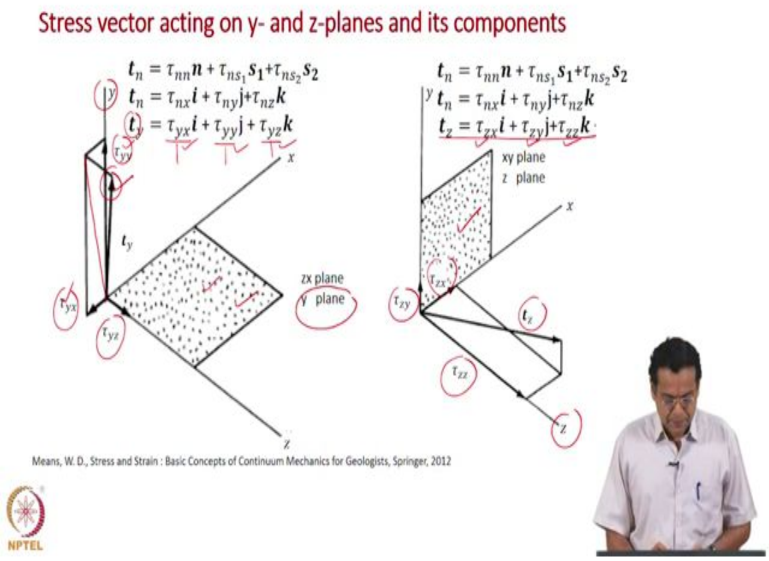

Now, just quickly we will do for the other coordinate planes as well. Remember, once again I want to repeat, at the same point instead of our general plane we are considering 3 planes which are parallel to our coordinate axis planes. All these planes are at the same point not a different points. Now, this is the y plane of course, this picture is much more clearer. What is a y plane? This is y plane, because y axis normal to the plane ok. Now the stress vector acting on the y plane is, t_y.

$$t_y = \tau_{yx}i + \tau_{yy}j + \tau_{yz}k$$

So, as we have done earlier, the components are denoted as τ_{yx}, τ_{yy}, τ_{yz}. Why is that, the first subscript represent the direction of the plane it is y; second subscript the direction of the component, so x, y, z. So, these τ_{yx}, τ_{yy}, τ_{yz} are the components of stress vector acting on a plane whose normal is the y axis. And once again the construction is shown nicely in the figure.

$$t_z = \tau_{zx}i + \tau_{zy}j + \tau_{zz}k$$

Now analogously, we can discuss that plane very clear picture here of course; this is the z plane, because z axis is normal to the plane. And the stress vector acting on this plane is t_z resolved along x axis as τ_{zx} and then τ_{zy} along y axis, τ_{zz} along z axis. And this stress vector acting on the z plane is represented as τ_{zx}, τ_{zy}, τ_{zz}; take away from all this as, the first index represents the direction of the normal to the plane; the second subscript represents the direction of the component of the stress vector acting on the respective planes.

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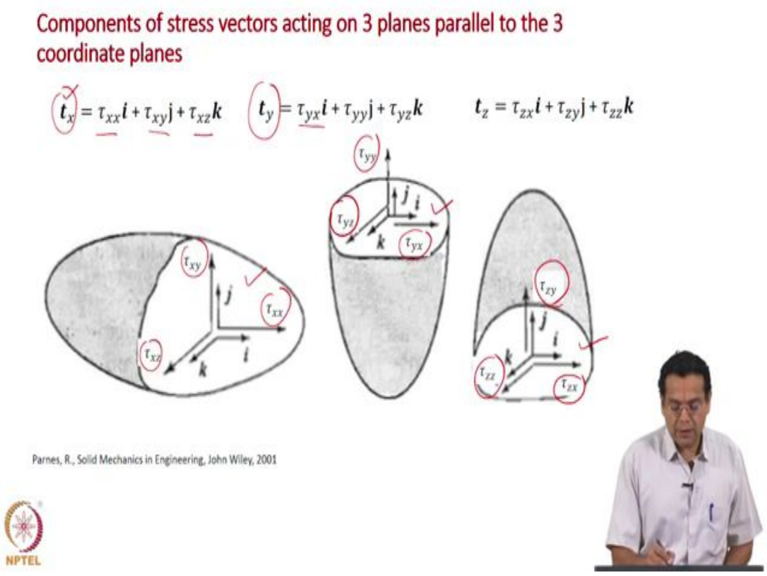

This summarizes whatever we are discussed. So, what is shown here, components of stress vectors acting on 3 planes parallel to the 3 coordinate planes.

$$t_x = \tau_{xx} i + \tau_{xy} j + \tau_{xz} k$$

$$t_y = \tau_{yx} i + \tau_{yy} j + \tau_{yz} k$$

And,

$$t_z = \tau_{zx} i + \tau_{zy} j + \tau_{zz} k$$

So, the x plane, the y plane and the z plane and all the components, so we have 9 components now, τ_{xx} , τ_{xy} , τ_{xz} similarly τ_{yx} , τ_{yy} , τ_{yz} and τ_{zx} , τ_{zy} , and τ_{zz} .

So represented vectorially, components of a stress vector on x plane and then the components of they of that stress vector; in this diagram of course, stress vector is not shown only the components are shown; stress vector on y plane and the components stress vector on z plane and the components.

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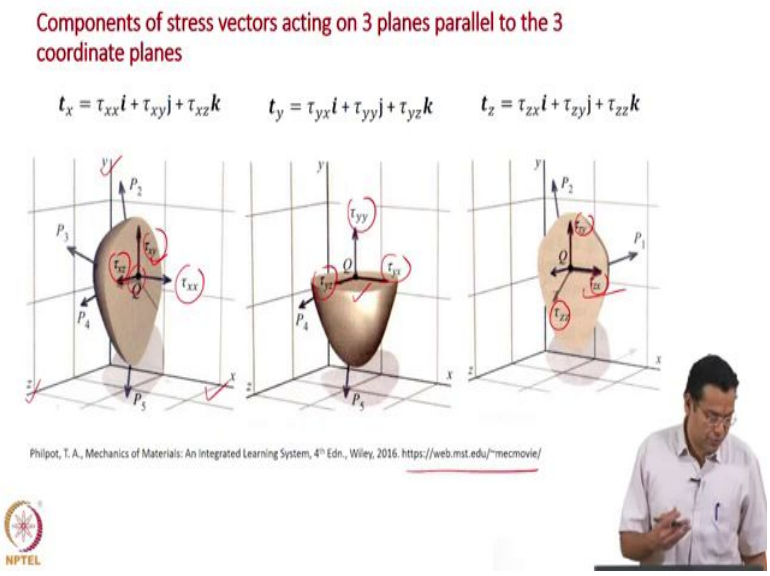

The same representation little more colourful from a recent book, I think I should mention about this book and the website. Please look at the title of the book it says Mechanics of Materials: An Integrated Learning System. Extremely well presented book and the website is more of animations are there, the entire subject of Mechanics of Materials are represented, it has represented in terms of animations and movies and very interactive and easy to learn also.

So, and even look at this picture the way in which they are represented, shows like a 3 dimensional view; you have x axis here, y axis here, z axis here, we have shown some wall like and a floor like surface becomes very clear. And this also shows very clearly, that we are at the same point Q and then now we are considering the body is cut along using a plane which is perpendicular to the x axis here.

Same point same that is important, same point the plane is now perpendicular to y axis; another plane is perpendicular to the z axis and of course, the components are shown here τ_{xx}, τ_{xy}, τ_{xz} similarly τ_{yx}, τ_{yy}, τ_{yz} and τ_{zx}, τ_{zy}, and τ_{zz} little more colourful representation of what we have been discussing so far, that is from Raymond Parnes and this is from this book by Philpot.

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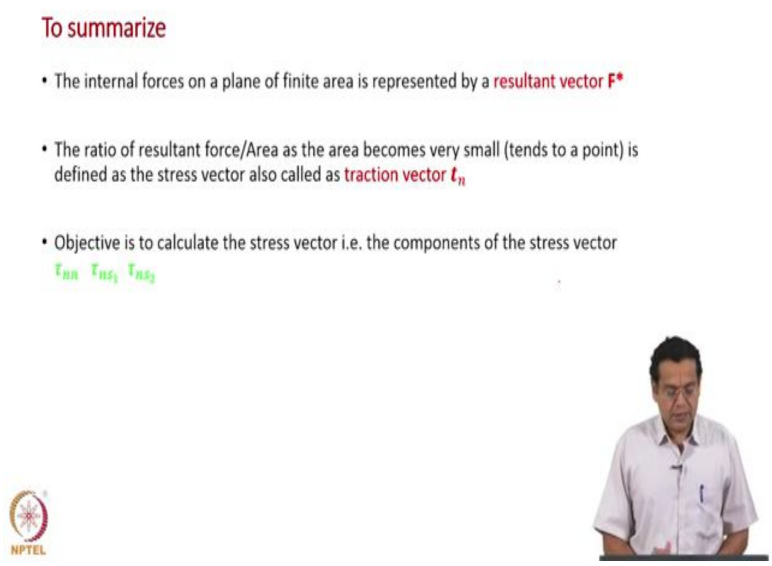

To summarize what all we have discussed so far, just putting in sentences, just three bullets.

- The internal forces on a plane of finite area is represented by a resultant vector F^*; F^* was sort of finite area.

- The ratio of resultant force per area as the area becomes very small, tends to a point is defined as a stress vector also called as traction vector, that is why it is called t_n.

- And, objective is to calculate the stress vector that is our objective remember always, only to simplify things we have moved to from any plane to any plane x, y, z coordinate axis and then to three well defined planes. Objective is to calculate the stress vector that is a components of a stress vector; τ_{nn}, τ_{nS_1} , and τ_{nS_2} that is objective.