Continuum Mechanics And Transport Phenomena
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Lecture - 36
Stress vector - Part 1

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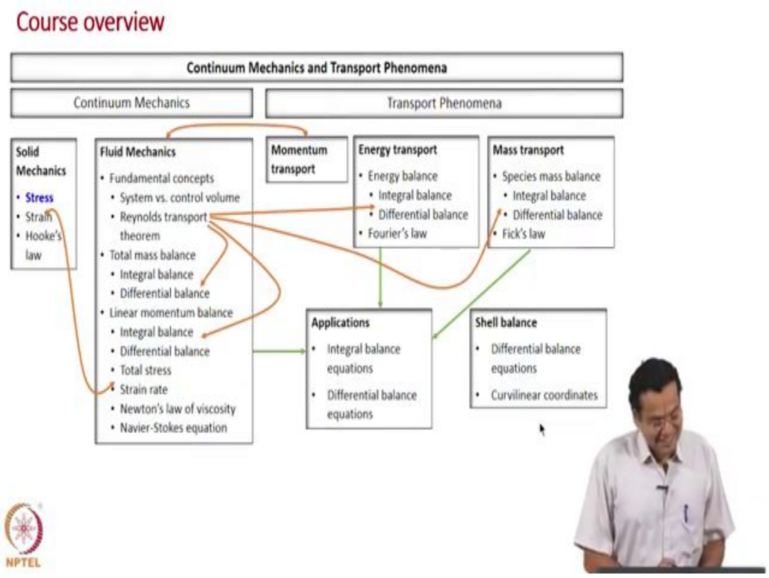

We will get started. We are into linear momentum balance, we are into the differential form of linear momentum balance. We derived the left hand side, the right hand side looked at the gravity the body force and for understanding the surface forces in the right hand side we are taken a divers into solid mechanics. That is main reason is to understand better and we will realise as we go along and also to look at the analogy between solid fluid mechanics. So now, we are the first time using this arrow mark, we are taking a diversion going to solid mechanics. In that we are going to discuss about Stress that is way that is highlighted.

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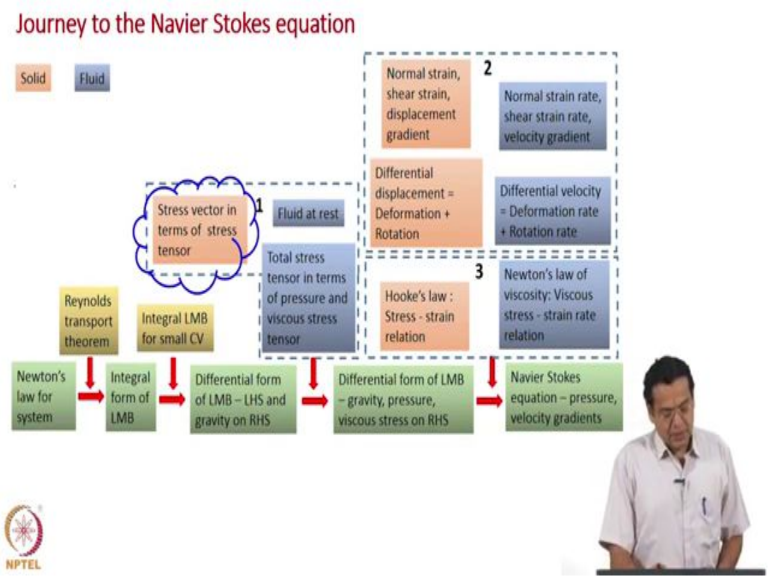

So, in terms of our journey to the Navier Stokes, we started with the Newton's Second Law of Motion, after the Reynolds Transport Theorem and derived the integral form, applied that integral linear momentum balance to a small control volume. Derive the differential form of linear momentum balance not entirety, but the left hand side and also the gravity on the right hand side.

Now we are surface forces to be included and that is why we are entering this domain, this dashed box named as 1, in that we are going look at the particular section. Just want to tell you that orange represent solid that is the legend used here and blue represents fluid.

So now, if you look at each section 1, 2 and 3 each has both solid and fluid. That is why we say we are going to study both parallelly. So now, we are going to look at the orange colour box which tells about a stress vector and in terms of stress tensor; what is that we will understand as we go along.

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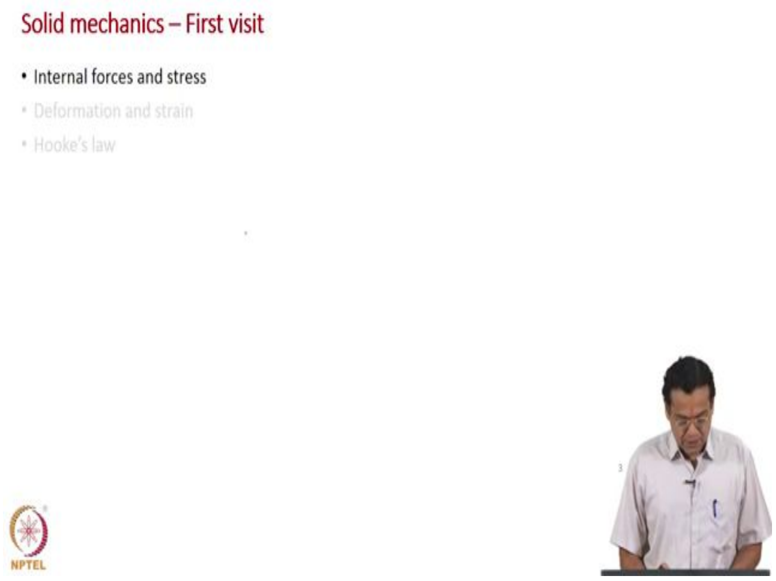

So, solid mechanics the first visit, and the first highlighted one shows the title of the first visit internal forces and stress and whatever is not highlighted are the topics for the subsequent visits. So, we are going to discuss about internal forces and stress.

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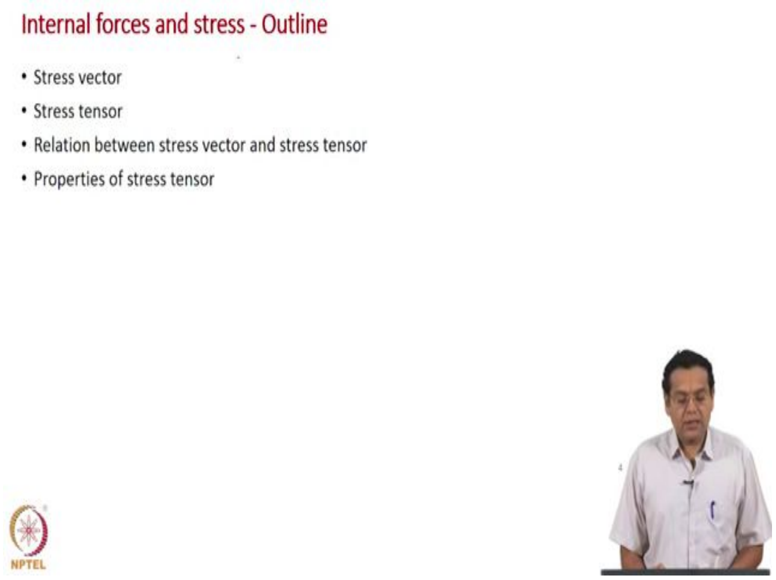

And now this is an outline for internal forces and stress. So, I think should have very clear idea where we are in, we are somewhere deep inside we are deriving linear momentum balance, in that differential form, in that the right hand side the surfaces force and to understand that we are come to solid mechanics. In that there are three sections and this outline for first section; internal forces and stress that is the title.

We are going to discuss about stress as a vector, stress as a tensor, and what is the relationship between the stress vector and stress tensor, and properties of stress tensor. As we go along we will understand the these terminologies.

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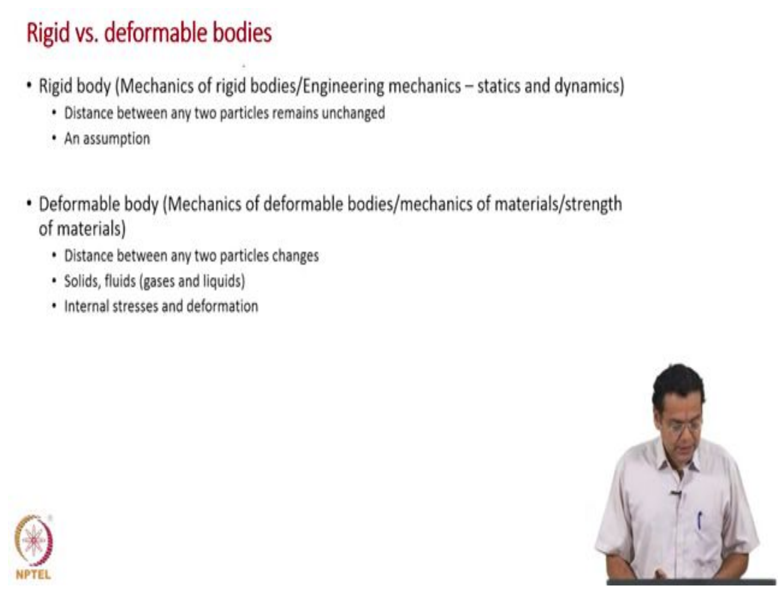

Now, we start distinguishing between rigid versus deformable bodies. Mechanics of rigid bodies if you have taken a first year course engineering mechanics; you would have studied mechanics of rigid bodies under static condition and dynamic condition. Usually the title of the book or the course is engineering mechanics.

Now, as a name suggests rigid body the assumption in this course throughout the course in fact throughout the course is that, the body which are considering is rigid. What does it mean; we will take an example like this an object and then how much ever force you apply; of course, you can divide by area expressed as normalised force. How much ever force you apply there is no change in shape there is no change in volume, no change in size as well. So, this means an assumption.

How do you more formally say: if you mark two points in the object there is no change in the distance between the two particles. Now I should emphasize that when I say particles these are solid particles; we have come across several times fluid particles now moved to solid mechanics. So, these are solid particles.

So, the entire course is based an assumption that the body is rigid which means that if you apply whatever amount of force absolutely there is no change in shape or size or volume whatever happen.

Now, of course in reality that is not going to happen ah, so that is why it is an assumption ok. Now, what is that we should discuss? We should discuss mechanics of deformable bodies ok and what are the titles of the books which deal with the mechanics of deformable bodies either they are called as mechanics of materials or strength of materials. Accordingly the course were also been named as mechanics of material and strength of materials.

Now, why should we discuss deformable bodies? Now remember we have come to solid mechanics not really understand solid mechanics as a whole subject, the whole ideas to understand fluid mechanics and take some part of solid mechanics to understand fluid mechanics. So, if you are restricting our self to rigid body mechanics that is not going to help us, because fluids are deformable. And that is why we discuss solid mechanics deformable solid mechanics and of course that is the reality. what does it What does it mean?

If you apply a very large force there is some small change in dimension, some change in let say shape or volume and, but of course that is not visible to your naked eye. You will see later on that those changes are order of 10^{-6} even if you have apply mega Pascal of let us say force per area, the change in dimension would hardly be something like 10^{6} meter. So, you not you may not be able to visually see but they do undergo a change in shape, change in size or volume etcetera.

So, that is a reality and that is why we discuss mechanics of deformable bodies ah. So now, in terms of a more formal definition if you take mark two points, the distance between them changes. Of course, we will discuss more about this in detail; what do you mean by distance between these two particle changes etcetera, but right now as a very good introduction.

If you mark two points subject to this to very large force per unit area then there is change in the distance between them. If there is no change in distance between them its behaving as a rigid body ok; but there will be a very small change, it is for us to consider that or not; you are not consider that when you discuss mechanics of rigid bodies.

Solids: when I say solids deformable solids and fluids are deformable; that is why we are discussing ok. Now when I say fluids of course gases and liquids are both included and now we are going to discuss about internal stresses and deformation.

When you discuss rigid body you need not discuss about internal stresses and deformation but when we are discussing deformable bodies internal stresses are set up the body undergo deformation and once again later on we will discuss formally what is deformation right now let understand in usual way. So, we discuss about internal stress and deformation and this of course the first part of solid mechanics we discuss internal stresses. That is the scope of what we are discussing.

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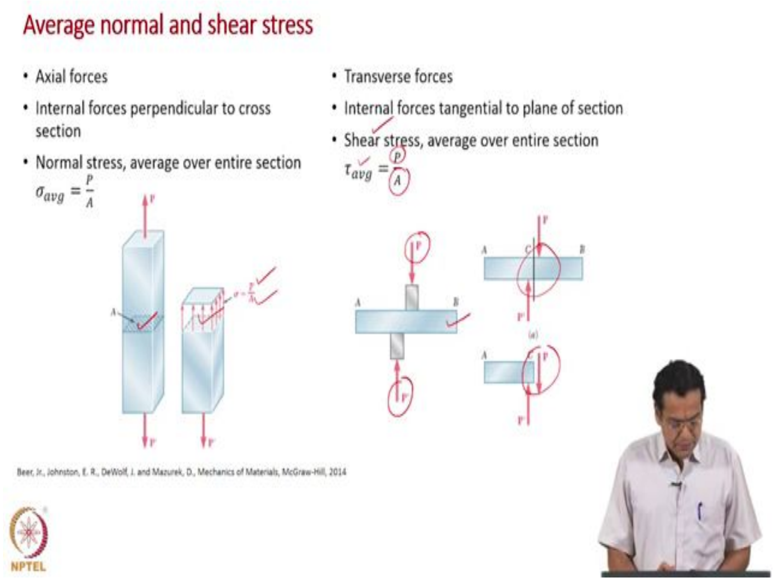

Now, if you had introduction to deformable solid mechanics, this kind of diagram should be kind of familiar and introductive part. What is shown here is a square rod subjected to axial loads; axial loads are something perpendicular to the cross section.

Now, if you make a cross section as it is shown here and then of course internal force will be internal forces are setup in response to the external load what you apply and if you take a cross section internal forces will be perpendicular to the cross section and hence we can evaluate a normal stress.

$$\sigma_{avg} = \frac{P}{A}$$

So, let us say you take the resultant of the force is distributed across the cross section you take a resultant take the magnitude, let say that is P divide the area or its acts you get the normal stress and these are just average values over the entire section.

What we are interest is more than this we are interested in the variation of stress point by point and of course respect to angel as well but these are very introductory definition of introductory way of looking at the normal stress by dividing the magnitude of the resultant force divide by the area and because the forces are perpendicular to the cross section they are called normal stress. You have a plane and the force are perpendicular to the cross section and hence called as normal stress.

$$\tau_{avg} = \frac{P}{A}$$

Now, let us look at another configuration shown on the right hand side. So, here again you have a let us say square rod, but now the you are applying transfers forces as shown and once again if you make a section between the two points over it is the forces are applied the loads are applied then you will have a section over which there is a once again a distribution of forces and then you take a resultant and then if you take the average of that force and now that force acts tangentially on the surface. So, let us take this one and then that force acts tangential to the surface and hence it is named as shear stress.

So, how do you once again evaluate that? You take the magnitude of the resultant force, if you denote it as P and then area now area become the tangential area and if you divide you get the average shear stress. Once again this definition of average shear stress we are interested in more detailed description of stress in terms of at every location, in terms of with respect to different angles of the plane etcetera.

These are well defined plans one is let say horizontal, one is vertical, but now we are more interested in stress on any plane etcetera. So, this is just to give an introduction to what you would have come across as normal stress and then shear stress; normal stress acts perpendicular to the plane, shear stress acts tangential to the plane.

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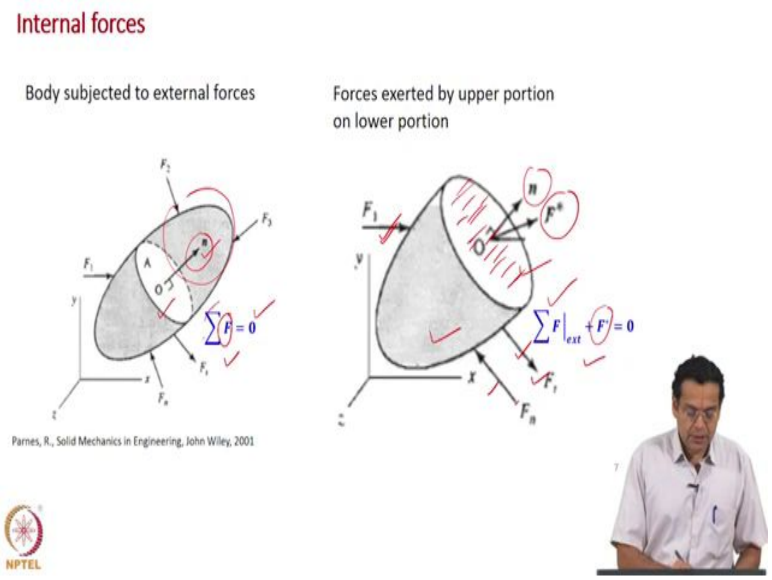

Now, so let us move onto more detailed description of stress. Now we are going to discuss about an internal forces.

What is shown on the left hand side is a body subjected to external forces. Let us say a body and subject to let us say some we has look at reaction forces etcetera, let us say it is subject to external forces and then since we are interested in internal forces we make a section at an particular angle, at a particular point inside the object we make a plane and now because there are different angles same point, but different angles of the planes are possible which mean that you will have to identify that plane. So, as usual we will use the unit normal drawn outside our region of interest to help identify the plane; or the angle of the plane.

That is what we have done earlier also for control volume we have taken an outward normal to identify the direction; for example this is the surface then we take this is outward normal. So, this helps identify the direction of the surface. But now what we have is, we have taken an object and made a section and that we have a point at that point I pass a plane and that plane the orientation of the plane is identified using the outward drawn normal vector.

Now, I section the solid and we are now going to discuss the bottom portion of the solid. Now this is one reason why we are come to solid mechanics. If I say that imagine a fluid or you can make a section imagine a plane there little more difficult there, but now it becomes very easier I can even show you that I have a solid I make a section that is one reason we are come to solid mechanics to understand about internal stresses. In terms of principles analogously most of the concept can be carried out of fluid mechanics that we will discuss later.

Now, we will say the entire object is under equilibrium; the left hand side entire object. If you want represent in terms of vectorial notation this $\sum F$; F represents all the external forces acting on the object is equal to vectorial 0,

$$\sum F = 0$$

So, if we sum all the forces it should be 0 because we are taking the body to under equilibrium. Now, in the whole body is under equilibrium every point in the object is also under equilibrium.

So in the right hand side, when you take a make a section and analyse let say the lower part that is also an equilibrium. But only difference is that, in addition to the external forces you have a distribution of internal forces why is that this top portion whatever it exerted on the bottom portion those forces are distributed over the entire area. So, you have to take that forces which are the internal forces, you also into account in right in the equilibrium relationship.

Now, we are not going to consider the distribution we are going to just now consider the represent that as a resultant force which is denoted as F^*. Of course, the normal to the plane is shown here and F^* can be at any angle to the normal to the plane. So now, if you want to express the condition for equilibrium sum of all the external forces plus this resultant force should be equal to 0.

$$\sum F_{ext} + F^* = 0$$

This is the equilibrium condition for this lower portion of the object. Difference between the two expressions are that because we are writing equilibrium relationship for the entire object you do not have any internal forces or any this case internal forces because we analysing a part of it we are accounting for the forces exhorted by the upper portion on the lower portion, and representing it as though it is distributed over the entire area we are representing it as a one resultant force F^*.

I like to mention that in the analysis what we are discussing even disused, we are not going discuss about moments. Few of very formally discussing we will have to discuss moments as well, but in terms of the concepts which we want to take over and understand discussion of moments is not required. Since that will add to a little more difficulty we are not discussing about moments but otherwise formula I should write moment balance etcetera, in hence forth also we will not discuss about moment or moment balance.

Now, in terms of a definition of a stress vector etcetera it is not required. So, let us proceed we are consider a body; equilibrium for the entire body; made a section with a particular point of a plane at a particular angle represented by n vector, and consider that also under equilibrium represented by this relationship.

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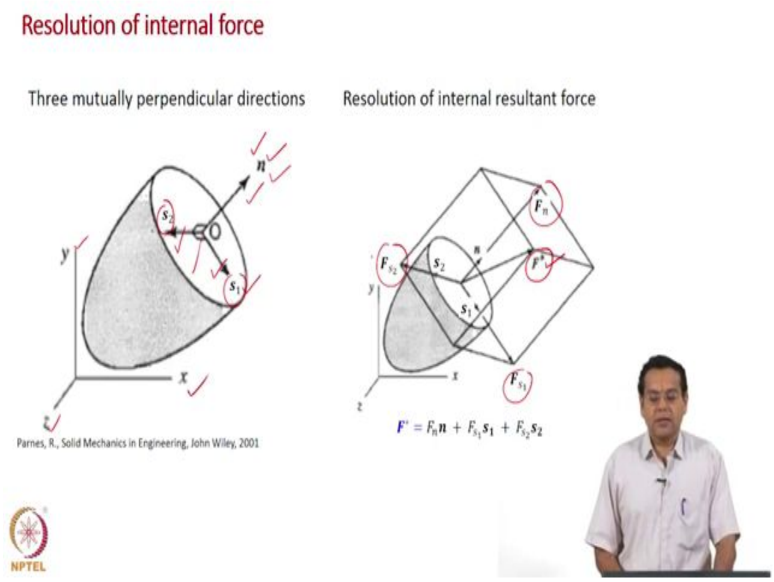

Now this diagram is same as same as what process shown in the right hand side. Now, if you are considering the let say we have the body, we consider the lower portion. Now if we are considering the top portion what happens the lower portion exerts the force on the surface on the top portion. So now, because at the same point we are on either side of the same surface.

So, based on Newton's Third Law both should be same that is what we shown here. What we shown is this the lower portion the same as what is shown in other image and then the top portion. What we consider is, this resultant force which is effect of the top portion on the bottom portion.

Now, the $-F^*$ is the resultant force because of the bottom portion on the top portion and remember they are the same surface one is on this side one is on the other side that is also clearly indicated here by using the normal this n vector, this is minus n vector so

$$F^* =- F^*$$

If you look at the bottom portion it is F^* which is the first because the top portion, look at the top portion $-F^*$ is because of the bottom portion. Just to tell you that on either side the forces are of equal magnitude opposite direction.

So now, let us proceed further. What we will do now is we got resultant force, we will resolve that internal resultant force. Now if you want to resolve then you should choose a coordinate axes you should choose directions. Now we will choose a set of three directions which are attached to the surface; what do you mean by attached to the surface.

I take one direction which is normal to the plane; I have a plane and take a directions which is normal to the plane. Let us assume this a internal surface though it is an external this is a plane I take a directions which is normal to the plane and then I take two directions which are on the surface and tangential to the plane. Now the question arises why should I take two directions on the surface? The reason is that let us say you resolve one normal to the plane and one on the surface that once again can be resolved to two further components. That is why we take two directions on the surface of the plane and then of course one normal to the plane.

So, we are chosen three mutually perpendicular directions, because we are going to resolve the resultant force along these three mutually perpendicular directions. Because they are normal to the plane and two on the surface of plane that is why I say this coordinate axes is it goes along with the plane. It is not our usual x, y, z coordinate axes we will came to that shortly, but now as of now this coordinate system is associated with the surface.

Now, we will resolve our resultant force along these three mutually perpendicular directions. That is what we shown in the right hand side the force F^* we are resolving. So, how should we look at it, we are very familiar with the velocity vector; we know v_x, v_y, v_z which means that velocity has been resolved along x, y, z axis; instead of velocity vector we have F^* vector, instead of x, y, z here we have denoted the coordinate axes n and because there on the surface it is denoted as S_1 and S_2. So, instead of x, y, z we have n vector, S_1 vector, S_2 vector.

So now, we are resolving this F^* along n vector which I denote as F_n along S_1 which is F_{S_1} and then similarly along S_2 which is F_{S_2}. The F^* vector has been resolved along n direction S_1 direction S_2 direction and the components are F_n, F_{S_1}, F_{S_2}, only steps which are done here.

Two things we are done in this slide: first is introduce a introduce a coordinate system associated with the plane, one is normal to the plane and two along the surface of the plane, a normal direction, tangential directions and then the resultant force has been resolve along the normal direction and the two tangential directions; we have given notations for them as well.

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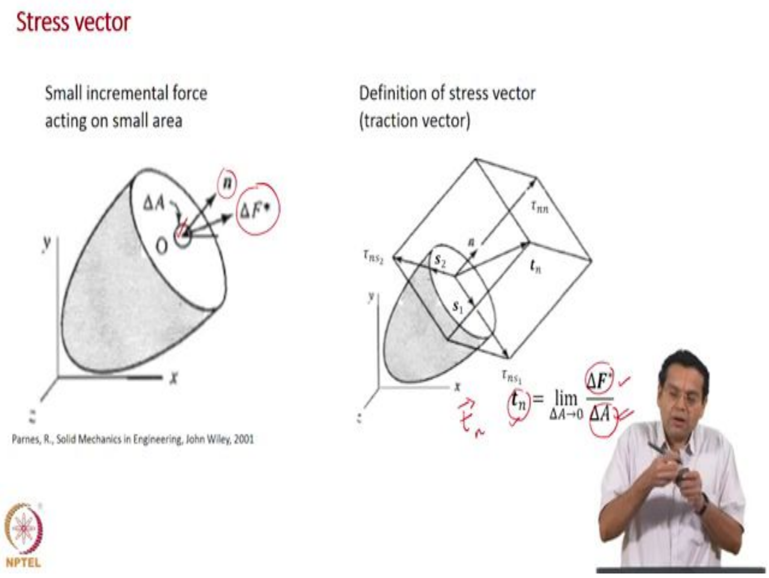

So, now let us proceed further. We said we are interested in the stress etcetera at a point but now we have taken a plane and then defined our resulted vector over entire plane. So, as we have done for remember for control volume we took a control volume and then shrink to a point so that we get the point form of differential balance of mass and the left hand side of momentum balance etcetera.

Similarly what we will do now here, we take a small area we are not taking a volume very obvious reason because these are surface process. So, we are interested in a small area and look at what is happening that area and shrink it to a point so that we can look at the variation of the surface force or the stress over the entire area. So, we take a small incremental area and the direction of that is n vector. So obviously, it can vary from point to point, taking a small area Δ A, and then the normal to that area is Δ n.

So now, because the area is incremental the forces also denoted by incremental ΔF^*. What is that we are taking? We have F^* which is an average over the entire area taking a small area, so the incremental of the resultant force is denoted as ΔF^*. Now this is where we are going to find what is a stress vector, also called as traction vector.

$$t_n = \frac{\Delta F^*}{\Delta A}$$

The physical significance as we have done exactly earlier; writing the balance for a small volume and then syncing to the volume to the point.

So, remember all the terms in our equation should be point values. Remember we are going to go back to the fluid mechanics linear momentum balance equations, differential form on the right hand side the point value of surface force, which means that my definition should be applicable at a point and should be valued at a point. That is why I make this $\Delta A \rightarrow 0$, so that I get stress vector why is a vector because incremental force which is a vector divide by area gives you a stress vector acting at a point on a plane whose direction is n.

The t_n is a stress vector denoted by t vector and then the subscript n tells you that this stress vector acts on a plane whose normal this n vector. So, when you imagine this you should imagine a very small plane just like we said you imagine a small control volume, here again you should imagine a very small area because that can vary from point to point. Imagine a small area very small area because it tends to 0, and what is the stress vector acting on that plane we say plane is denoted by the normal and that is small point.

So, the first notation introduce now in terms of stress vector we will use it very frequently as we go along also called as traction vector. So, you come across both these terminologies. We will mostly use terminology stress vector. Maybe stress vector we are going to discuss later on stress tensor.

So, if you just tell stress that does not give the complete information. You should clearly mention whether a stress vector or stress tensor but we mention just traction it is all automatically indicated that it is a vector.

That is a small distinction between using stress vector as a terminology and traction vector as a terminology. We just leave it as stress it is incomplete; it could vector, it could be tensor but if you say traction invariably it is a vector; traction vector, but we will use the terminology stress vector. This right hand side, we will come across in a next slide as well ok. So, I will discuss then. So, we have defined the stress vector and that is what is shown here.