**Continuum Mechanics And Transport Phenomena**
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**Lecture – 34**
**Differential linear momentum balance:**
**Introduction**

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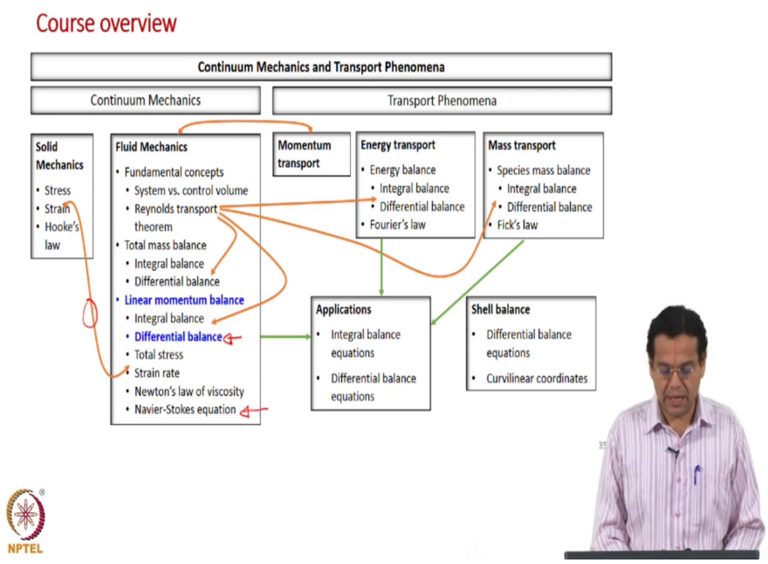


We are discussing the linear momentum balance part in fluid mechanics. In that, we have derived the integral form of the momentum balance starting from the Reynolds transport theorem and you also looked at applications of the integral linear momentum balance. Now, we are moving towards the derivation of the differential form of linear momentum balance, and that you are going to in a series of lectures our different. Now, this is a major part of the course I would say and in this part only we study solid and fluid mechanics parallelly.

Also, we are going to go back and forth between fluid and solid mechanics and, hence understand the commonalities between them and that is why we have the term Continuum Mechanics in the title of the course. So, this exactly the part of the course with justifies why we have continuum mechanics in the title of the course and after several lectures, we would have derived the differential form of the momentum balance which is called the Navier-Stokes equation based on the researches would derive it.

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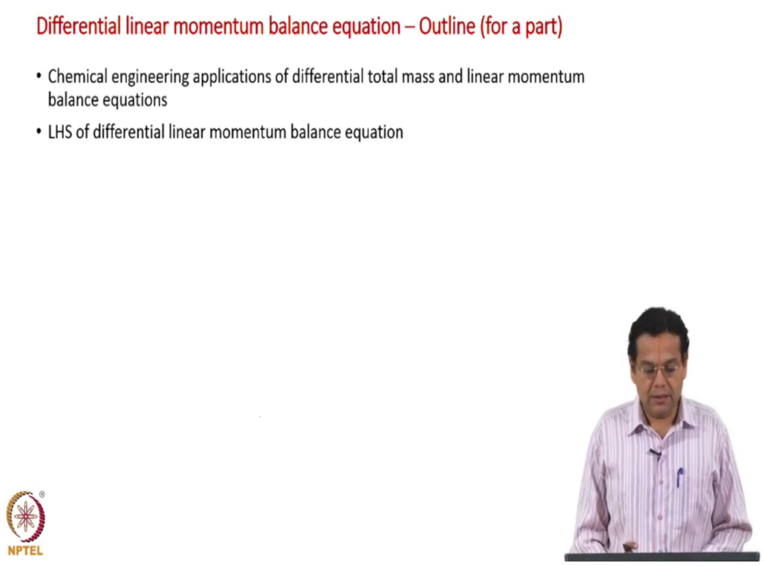


So, this is the outline for the differential linear momentum balance equation, as I told you it is over several parts. So, outline for a very small part of it, the first part of it

- First, we will look at the examples for mostly if I am from the chemical engineering field where these linear momentum balance equations are used, the differential form is used and usually, we solve the differential linear momentum balance equation along with the differential form of total mass balance equation. Obviously, the mass balance has to be valid. So, we first look at chemical engineering applications of differential total mass balance and differential linear momentum balance equation that gives us a kind of motivation why we are deriving and why this equation is so important and,
- Then we will derive only a part of the equation here being involved we will derive only the left-hand side of the differential momentum balance equation which we would see which almost be same as our mass balance equation. So, this is the outline for a very small part of this portion on the differential linear momentum balance equation.

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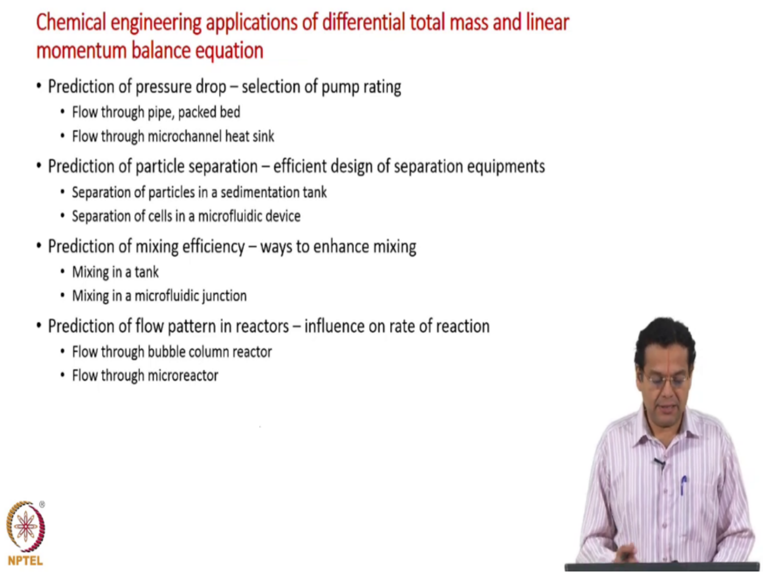


So, let us starts with some chemical engineering applications. Now, I split this into four categories or four types of applications that can be many more, this I will give a just overview.

- The first and foremost application of the differential momentum balance is the prediction of pressure drop. If you have a flow-through any pipe or packed bed that is the first example which we are going to take, then this pressure drop determines the rating of my pump. You would remember in a household pump water from a sump to over a tank you desired on the rating of the pump you would use a half HP or 1 HP pump. So, pressure drop determines the selection of pump rating. We look at two examples in that; one flows through a pipe or packed bed, the second example flows through a microchannel heat sink. In microelectronic devices, heat is generated and it will remove that. So, microchannels are used to remove that generated heat and that is why it is called microchannel heat sink. And, we see the application from a pressure drop prediction of point of view in these two cases.
- The second type of application is for particle separation. What is the advantage? You can design efficiently the separation equipment. Here again, we look at two examples separation of particles in a sedimentation tank which is a unit operation usually fine found in wastewater treatment, water treatment etcetera and, the second application that we see is the separation of cells in a microfluidic device. We see how to separate pathogenic bacteria from cells from red blood cells.

- The third type of application is for predicting mixing efficiency. Why do we do that? So that we can find out ways of how to enhance the mixing all, we also always like to have maximum efficiency in mixing. We look at two examples in that mixing in a tank where we like to mix let us say two miscible liquids, immiscible liquids etcetera, and, then mixing in a microfluidic junction we have the small microfluidic device and then a junction there is mixing how do you predict that.
- The fourth type of examples are the prediction of flow pattern in reactors because these flow patterns influence the rate of reaction. So, we look at two examples that flow through a bubble column reactor; a bubble column reactor is just a column full of liquid where you spurge the gas or air, second is a small reactor namely a microreactor flow-through microreactor.

Selecting these examples if we look at the examples the first example in all category are all large scale equipment. The second example in each category are all small scale equipment. So, the reason for selecting this combination is that research in chemical engineering has moved in terms of scale. Earlier we are working in terms of equipment which are the size of meters and centimeters, and now we are moved to equipment or devices which are millimeter, micrometer, and nanometer sizes to reflect this changing trend or I would say change the trend these examples are chosen.

One set of examples to reflect the established and conventional equipment of large scale, another set of examples which are a small scale which reflects the relatively modern trend.

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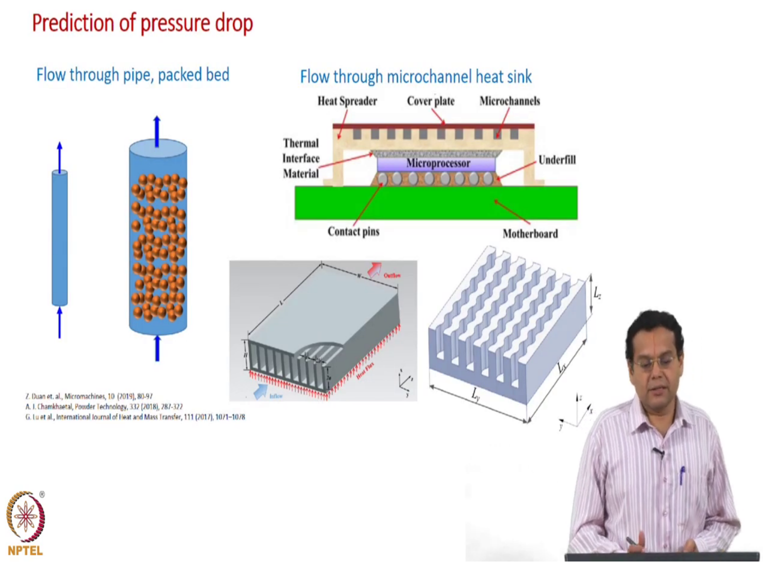


So, let us take the first example first category of example namely prediction of pressure drop and that is the foremost application of solving the momentum balance and mass balance equation is differential form. So, first would be to consider a case of flow through a pipe; throughout the chemical engineering plant you have a lot of pipes connecting the equipment, fluid flows through that and we need some power to make the fluid flow through that. You need to decide the capacity of the compressor or pump and for that to predict the pressure drop through the pipe we solve the differential linear momentum balance equation, along the mass balance equation get the velocity profile, and then predict the pressure drop.

Now, instead of the pipe, we could have a packed bed. What is the packed bed? You have a pipe pack with some packing's could be spheres or some other shapes, it could be well-structured packing as well and use this packed beds for carrying out absorption, distillation and you have catalytic packed bed reactors. In fact, packed beds I have considered as a workhouse of chemical industries because you do not have any moving parts. So, that way it is simple. So, to predict the pressure drop of fluid flow through the packed bed once again you use the differential linear momentum balance equation. So, that is more a traditional application.

So, now let us look at a relatively novel application where we have flowed through a micro-channel heat sink. An inside of a microelectronic device is shown (right bottom image in the above slide), our focus on the microprocessor where heat is generated and then we wish to dissipate this heat or remove this heat as quickly as possible. What we do is bring this microprocessor in contact with the micro-channel. Of course, in particular geometry, and one such geometry is shown in the middle another geometry is shown in the left.

So, we have to send the fluid through these channels it could be air, it could be liquid even nano-fluids are used because of the air; very good heat transfer characteristics, and bypassing this fluid through the channels we room the heat generator. Now, the question arises that you will have to remove the heat. So, if you use this kind of simple geometries, the pressure drop is less that is our focus here pressure drop is less, but at the same time, the heat removal is also less.

So, they tried such kind of geometries corrugated wall or different other geometries, now this enhances the rate of heat transfer, but at the same time enhances pressure drop. So, we like to design a micro-channel heat sink where the pressure drop is less and we have very good heat transfer characteristics. So, which means if we want to predict the pressure drop characteristics, we solve the differential linear momentum balance equation, mass balance equation along with these geometries, and then find out what is the pressure drop so that we can design a very efficient micro-channel heat sink that is the idea.

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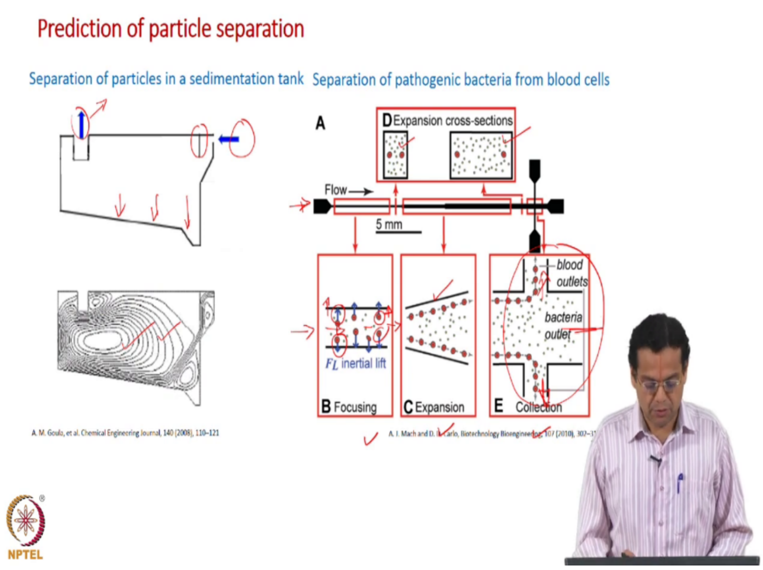


The second category of examples deal with the separation of the particle at two different size rangers; so, left-hand side shows the traditional sedimentation tank which are used in wastewater treatment or portable water treatment almost the first stage there. In a sedimentation tank, you have inflow with water with solid particles. It enters the sedimentation tank, the solid particles settle down, and then relatively clear liquid leaves out of the sedimentation tank.

Now, we want to have a very good separation of the particles from the water and that depends on the design of the sedimentation tank, that depends on the flow pattern inside the sedimentation tank, and for predicting the flow pattern we solve the differential linear momentum balance equation, mass balance equation and what a shown here are streamlines in such a sedimentation tank. Remember, we discuss streamlines when we discuss flow visualization we also mention that results of a simulation are expressed in terms of streamlines.

It is a good example where the equations are solved on the velocity profile or velocity field and then expressed in terms of streamlines that help us know where there is circulation, no circulation, and more circulation etcetera. In this particular study, they have studied what is the influence of adding this baffle and they found out that by adding this baffle better separation equation this possible. So, in short designing, an efficient sedimentation tank requires a solution of the governing equations namely the differential balance of momentum and mass.

Now, let us look at a relatively modern application and novel application as well. In the right-hand side equipment also need to separate particles, but our particles are not solid particles like this. Here our particles are bacteria and blood cells etcetera. So, the application is on separating pathogenic bacteria from blood cells. This of course application in medical diagnostics is very obvious here.

What we do is what we have shown here is first of all these three diagrams show the top view of the device ok. No longer we say equipped and they are very small. So, we call as device we send in a sample of blood, it is diluted blood which as pathogenic bacteria and red blood cells. And now first the property which is being used is that the bacteria are much smaller; they are about 1 to 3 microns. The red blood cells are relatively larger their 5 to 12 microns.

Now, the force acting on these particles so called particles depends on their size. When I say force in the direction perpendicular to the flow. So, that force depends on the particle size. So, because of the difference in size, there is a separation of the particles. The larger particles move towards the wall and the smaller particles are in the center, which means that the red blood cells move towards the wall, the pathogenic bacteria are relatively in the center of the channel. So, we achieved separation.

What they have studied in this study is that they have provided an expanding channel after that so that the separation is further enhanced that is what is shown. This is at the beginning of the expansion section and the end of the expansion section. So, you enhanced the separation between the red blood cells and the bacteria, so that if you provide an outlet section of this geometry it is a trifurcating section where you have 2 – 3 outlets and because the red blood cells were focusing near the walls they go out through this outlets and then, of course, a pathogenic bacteria go out through the central outlet. So, in that way we have achieved separation of the pathogenic bacteria from blood cells.

Now, the fundamental principle is a difference in the force acting on the red blood cells and the pathogenic bacteria. This force depends on the velocity field in this device and for predicting this velocity field we solve the differential form of momentum balance and mass balance. So, that is where we bring in the differential balance equation. So, by this, we can design very efficient separation devices, be it pathogenic bacteria and blood cells or particle solid particles from water.

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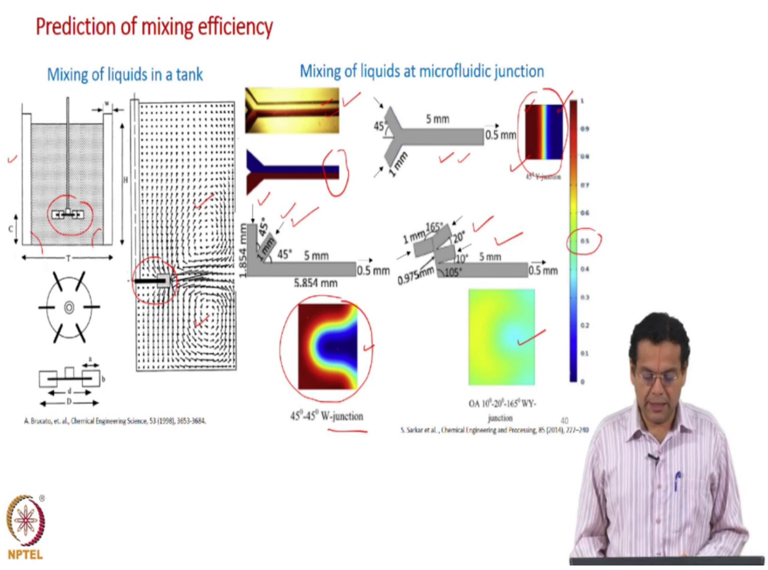


The third category of example is for predicting the efficiency of mixing. Once again a traditional example and novel example are shown. The traditional example would involve mixing of two miscible liquids could be immiscible liquids, so that you maintained dispersion or it could be mixing of solid particles you maintain a suspension of solid particles and these are could be a react carrying out reactions; so, those some of the applications.

Now, efficient mixing depends on the geometry of the vessel and the geometry of the impeller ok. Now, to design efficiently the tank and the impeller design the impeller the mixing obviously, depends on the flow pattern inside the vessel. So, we solve the differential balance of momentum and mass, get the velocity field and that is what we have shown here in terms of a vector plot. So, that you take this velocity field generated by the impeller in arriving at a good design for the mixing tank. What is good design? You get a uniform mixture, there are no dead zones etcetera where there is no mixing, and mixing takes place very quickly as well.

Now, let us look at a relatively modern application. We have a microfluidic device. Now, why do we use microfluidic devices? They have several advantages. The first and foremost advantages that their surface area per unit volume is very large, you have a very long length the area is very small and so, the surface area to volume ratio is very high.

And, you can if you are having a two-phase system, the interfacial areas very large and then the volume what you use here a very less so, we can try out some hazardous exam reactions as well without causing many safety issues. And, then this being very small you can carry take to a place where you want to produce the chemicals, but these microfluidic devices. Because the channel cross-section is very small suffer from a serious limitation of very less fluid mixing of fluids.

And, suppose if we are one to enhance the mixing you try out different configurations that is what is being tried here. What is shown here is an experimental image of the experimental flow of water and then colored water and this is from the simulation. This figure is from simulation; the figure is shown is from simulation and the figure at the top is from experiment. How do you simulate? This mixing of the fluids depends on the flow pattern and that flow pattern is obtained by solving the differential linear momentum balance and mass balance equation.

Now, we need to arrive at the most efficient geometry where this mixing takes place very well. So, one option is to try out different geometry, do experiments and then arrive at the optimal configuration which is time-consuming; alternatively, we try different geometries and simulate solve the momentum balance equation and then of course, along with mass balance equations to arrive at the mixing patterns .

A few geometries are also shown there. A simple Y junction is shown here and if you look at the diagram shown here you have hardly any mixing here. The colorless the pure water is here, the dyed water is here, almost you do not see any mixing at all which means not an efficient design for mixing of liquids.

Now, the alternate configuration is shown which is called a W junction based on the geometry. So, you allow the one liquid here, another liquid here. Now, if you look at this contour map of concentration we can obviously, see a better mixing in this configuration, in this geometry compare to this 45 degrees Y junction. You have more mixing of this pure liquid along with the liquid dyed water.

Now, to improve the mixing further another geometry has been tried where you combine both the W and Y junction, and once again all these cases you simulate the velocity profile, its influence on mixing now if you look at the contour plot here. Of course, almost uniform mixing is achieved meaning if you get 0.5 it indicates uniform mixing. So, in this way you can arrive at the best geometry of the device which will result in very good mixing.

You also should keep in mind that different geometries will result in different pressure drops. So, for example, this geometry may result in higher pressure drops which also has to be taken into account. So, eventually, all this requires the solution of the linear momentum balance is the differential form of it.

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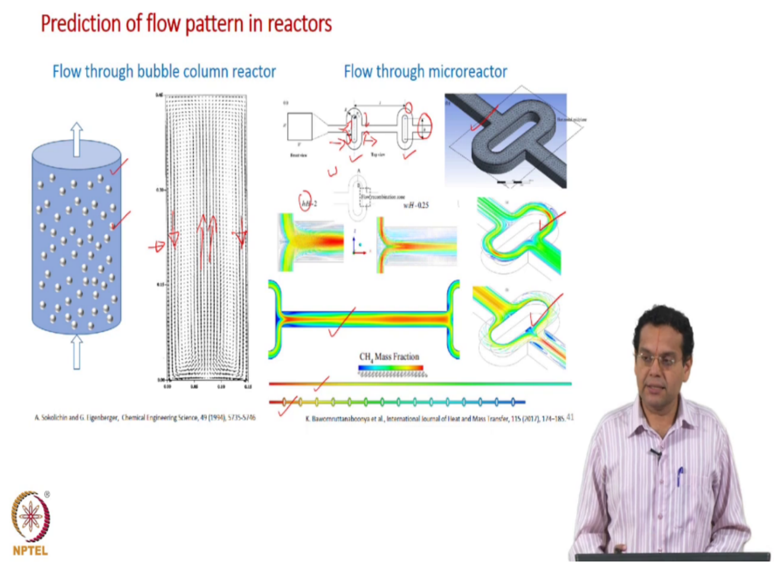


The last category of examples is on predicting the flow patterns in reactors. First, we look at bubble column reactors, a very traditional reactor. Bubble column reactor corns of a column of a liquid wherein new bubble, the reactant gas or simply can imagine air to be a bubble. A fish tank is a quick way to imagine a bubble column. Now, where are they used? This bubble column is used for several unit processes like chlorination, oxidation and it is used for hydrogenation of oil with a nickel catalyst, finely dispersed and also use it in fermentation, used in wastewater treatment extract. So, a lot of applications are there.

Now, the performance of this bubble column reactor in terms for a reaction, in terms of heat transfer, mass transfer characteristics depends on the flow pattern inside the reactor. Of course, for predicting the flow pattern we solve the differential linear momentum balance equation and such a flow pattern is shown in this figure. In case of an interesting flow pattern, the liquid rises in near the axes of the column and then flows down near the walls of the column ok; some circulation pattern can be seen near. So, this process and this kind of flow pattern influence reaction, heat, and mass transfer characteristics. So, if you want to predict the performance you will have to solve the differential form of linear momentum balance mass balance equations to take this flow pattern to enhance it influence on reaction heat and mass transfer ok.

Now, let us look at a microreactor. The example which is considered here is the partial oxidation of methane. We do not one complete oxidation because all result in CO 2, we want carbon monoxide and it is a heterogeneous reaction. What we do is catal code the catalyst on the walls of the reactor and being a microreactor it has all the advantages mentioned previously.

Similarly, it also has a disadvantage of lack of mixing and so, need to improve mixing and also we need to improve the rate of heat transfer and also we need to improve the rate of transport of the species to the surface of the wall the catalyst is a surface. So, the reaction happens at the catalyst surface and the species are going in the bulk of the channel. So, species were asked to get transport to the surface. So, an efficient micro reactor should have higher mixing, efficient mixing, and also better heat and mass transfer characteristics.

In this study what they considered is a split and a recombination design. The fluid comes here, it gets split and then recombines here and then this element can be repeated several times. One element is shown here; another element is shown here 3-dimensional views shown here. Now, the performance of such a kind of split and recombination reactor depends on the flow field, once again for which we need to solve the differential linear momentum mass balance equation.

What is being studied in this work is what is the effect of geometry; what is the effect of this height h, what is the effect of this w width, the width of the element. And, then what is the effect of velocity? This is a flow pattern with low velocities, this is the flow pattern at high velocities, and what is the effect of the length of the connecting channel. So, all these effects can be simulated can be predictor by solving the differential mass balance total mass balance equations and the momentum balance equation, of course, the species balance are also solved here.

Now, based on and then you can arrive at the optimal design in terms of height, width, velocity etcetera. And, then the authors of shown that if you use the split and recombination reactor you get higher methane conversion compared to a straight channel microreactor. So, in this way, the solution of these equations helps us to design optimal microreactors which are very good mixing characteristics and heat and mass transfer characteristics as well ok.

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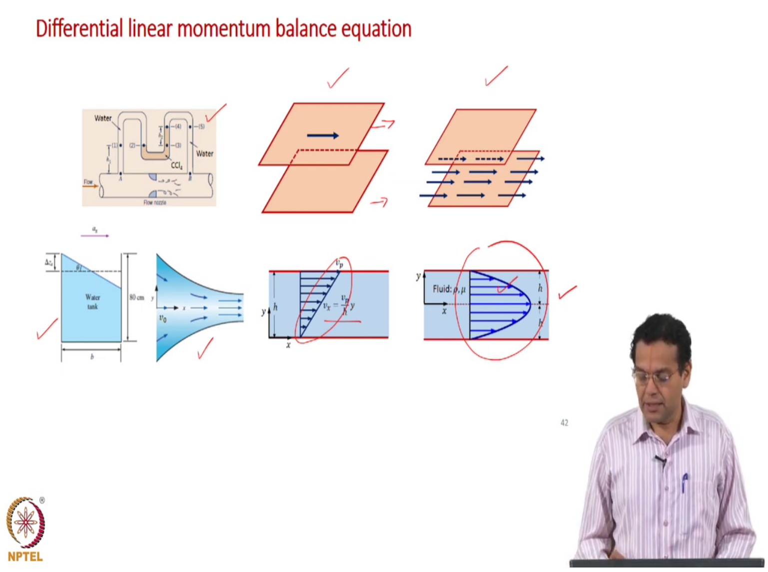


So, those are all very sophisticated examples. To solve that you will have to take a very advanced course either at your post-graduate level or the very final year of your B.Tech course what is that we are going to do. This being an introductory course, our applications will be very simple extremely simple. So, let us look at the examples which we are going to solve after deriving the differential linear momentum balance equation. They are very simple examples.

- The first example would be how to use this equation to calculate the pressure drop for flow through a pipe. We take reading from manometer, from that how do you find out the pressure drop in the pipe.
- You have a tank filled with water and the tank is being accelerated, it moves. What is the profile of the liquid surface? Will it spill or will it not spill?
- Next example, through a converging nozzle, what is the pressure field in this converging nozzle under certain assumptions. And,
- Then the geometry in the middle of the slide something known to us we have come across this. Fluid motion induced by the moment of a plate. So, fluid motion takes place between the moving plate and the fixed plate. And, in an earlier lecture, we have seen that the velocity varies linearly from 0 to the velocity of the plate. We assumed it or I said let us take that the velocity varies linearly. So we will solve really the differential linear momentum balance equation and show that the velocity profile is indeed linear without assuming it.

- Of course, the last geometry is very familiar to us flow between two parallel plates we come across several times and we also looked at this parabolic profile several times, but we have taken assets we have assumed it. Now, at the end of this derivation of differential linear momentum balance equation we will really solve that for this particular example and then arrive at a parabolic velocity profile and show that the profile is in the parabolic.