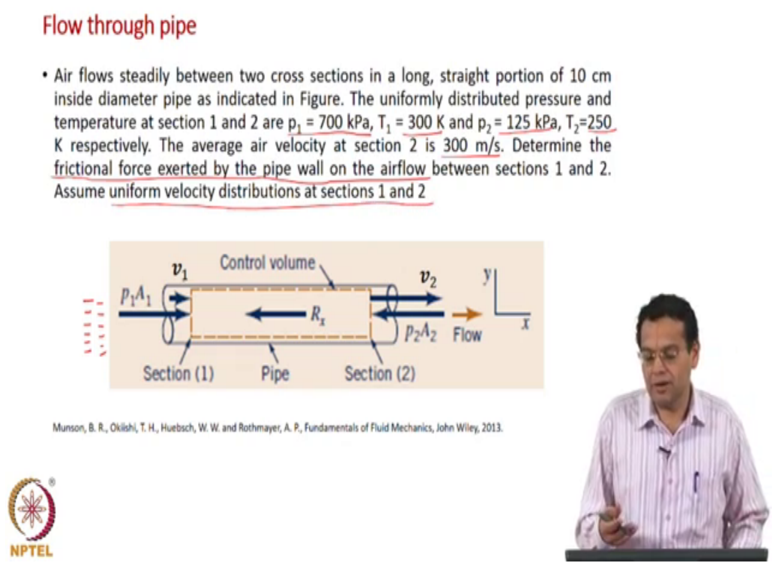
Continuum Mechanics And Transport Phenomena
Prof. T. Renganathan
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture – 33
Integral linear momentum balance: Examples – Part 3

Example: (Refer Slide Time: 00:13)

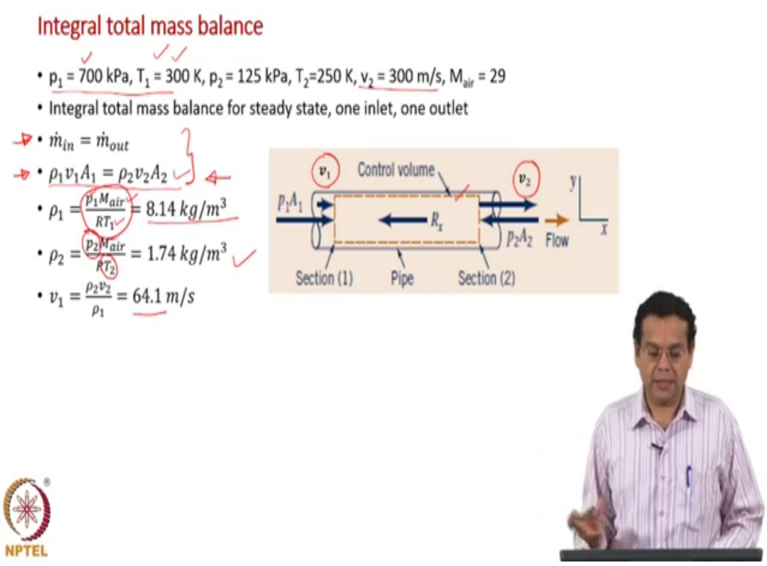


In this example, we are going to look at another form of surface force, namely frictional force that is the main object of this example. In addition to that, we are going to consider the example, where they have compressible flow.

Let us read the example. Air flows steadily between two cross-sections in a long straight portion of 10 centimeters inside diameter pipe as indicated. Simple pipe, no uniform cross-section no change in direction nothing. We want to focus air only on the frictional force. So, geometry has been kept very simple, just a straight pipe, and then the diameter is given to us. The uniformly distributed pressure and temperature; At the inlet, the pressure distribution is uniform. Temperatures also uniform along the cross-section. So, uniformly distributed pressure and temperature at section 1, similarly at the outlet also. And section 1 and 2 are given p_1= 700 kilopascals, roughly about 7 times or above atmospheric pressure and temperature roughly about ambient 300 Kelvin. At the exit, you have roughly about atmospheric pressure, p_2 = 125 kilopascals and a lower temperature less than 0 degree centigrade (250 Kelvin).

Now, the average velocity at section 2 is 300 m/s and we are asked to find out the frictional force exerted by the pipe wall on the airflow between these 2 sections, as I told you that subject of this example. And then, we are also asked to assume uniform velocity distributions at section 1 and 2; in fact, that we have been assuming our all the examples.

Solution: (Refer Slide Time: 02:14)



Let us say proceed with the control volume. So, the control volume is shown along with the control surface.

Given data are:

$p_1 = 700\ kPa;\ T_1 = 300\ K;\ p_2 = 125$ kPa; $T_2 = 250$ K; $v_2 = 300$ m/s; $M_{air} = 29$

Now, let us write the integral total mass balance, to begin with, because we are given the exit velocity. We need to find out the inlet velocity, something similar to what I have done in the previous example.

$$\dot{m}_{in} = \dot{m}_{out}$$

There is only 1 inlet 1 outlet and we have seen that this expression of no use. It has to be expressed in terms of density velocity and area.

$$\rho v_1 A_1 = \rho v_2 A_2$$

I think previously I have discussed the utility of these 2 expressions. One in terms of mass flow rate straightaway; the other in terms of density velocity and area. This is a nice example, where the real utility of the second expressions becomes very obedient. Though both tell us the conservation of mass, the second expression is more useful because the beauty of this example is that these are all measurable variables. We have a pressure sensor to measure pressure, let say thermocouple to measure temperature or thermometer. So, these are all measured values. Similarly, measure velocity, and the variables are given straight away in terms of what you would practically measure.

So, now let us evaluate the density, we assume air to behave as an ideal gas

$$\rho_1 = \frac{p_1 M_{air}}{RT_1} = 8.14 \frac{kg}{m^3}$$

We substituted the pressure and the temperature at the inlet, the molecular weight of air and got a density.

$$\rho_2 = \frac{p_2 M_{air}}{RT_2} = 1.74 \frac{kg}{m^3}$$

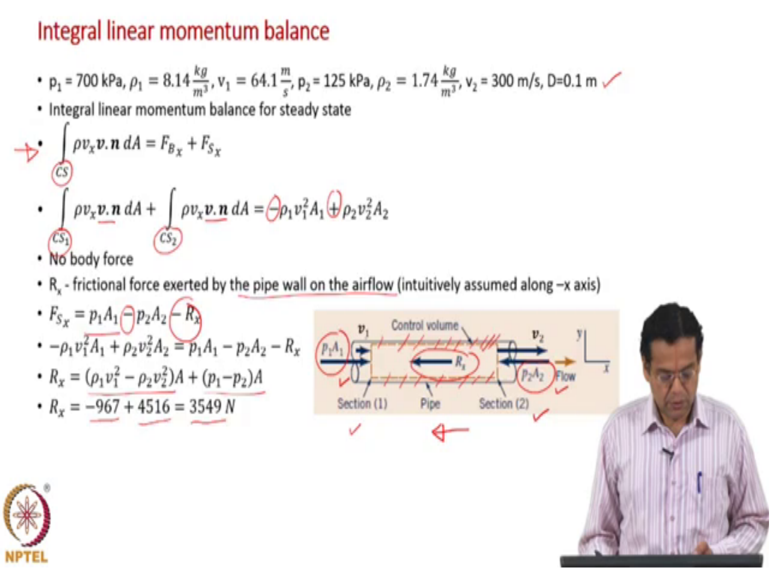
Similarly, we evaluate the density of air at the exit, use the pressure at the exit remember we should use absolute pressure, those pressures are given not the gauge pressure. The temperature and the density slightly above that of atmospheric air.

Now, we use the integral mass balance and evaluate the inlet velocity

$$v_1 = \frac{\rho_2 v_2}{\rho_1} = 64.1 \frac{m}{s}$$

So, there is a huge acceleration happening over the section accelerating from 64 m/s to 300 m/s. You will see why I mentioned that shortly know.

(Refer Slide Time: 05:20)



So, now we can proceed further, whatever values be calculated or brought in here, now whatever is required for the calculation of the frictional force.

$p_1 = 700\ Pa$; $p_2 = 125$ kPa; $v_2 = 300$ m/s; $v_1 = 64.1$ m/s; $\rho_1 = 8.14$ kg/m^3; $\rho_2 = 1.74$ kg/m^3; $D = 0.1$m

So, let us write the integral momentum balance along the x-direction with the convicted term on the left-hand side body in surface force on the right-hand side.

$$\int_{CS} \rho v_x\, v.n\, dA = F_{B_x} + F_{S_x}$$

Now, along the x-direction, there is a contribution to the convective momentum both at the inlet and at the outlet. So, CS has been split into CS_1 and CS_2; control surface 1 and control surface 2

$$\int_{CS} \rho v_x\, v.n\, dA = \int_{CS_1} \rho v_x\, v.n\, dA + \int_{CS_2} \rho v_x\, v.n\, dA$$

$$\int_{CS} \rho v_x\, v.n\, dA = -\rho v_1^2 A_1 + \rho v_2^2 A_2$$

Now, there is no body force along the x-direction. Coming to surface force, so far we have been taking the reaction force as positive along the x-axis or y-axis. In this case, we are asked to find out what is the force exerted by the pipe wall on the fluid. Intuitively, we can feel that fluid is flowing along the positive x-axis. So, the frictional force exerted by the pipe wall on the fluid flow will be towards the negative x-axis.

So, to begin with, we have taken along the negative x-axis, nothing wrong in taking along the positive x-axis and concluding it as an opposite direction as we have done. But just to get practice to do another way also I have done this; another reason is the book from which I am taken Munson et al follow this. So, that will be easy for you to follow the book ok, that is why I have taken Rx along negative x-axes.

Now, also you should know that the pipe wall shown has contact between the fluid and the surrounding pipe so that this reaction force in this case frictional force represents the friction between the fluid surface and the control volume. So, Rx is the frictional force exerted by the pipe wall on the airflow and intuitively assumed along the negative x-axis.

$$F_{S_x} = p_1 A_1 - p_2 A_2 - R_x$$

Now, the surface force we have a contribution from pressure also because the pressure at inlet and outlet are different.

So, now let us substitute in the integral balance, the convective momentum terms on the left-hand side and the surface forces on the right-hand side.

$$\int_{CS} \rho v_x \, v.n \, dA = F_{B_x} + F_{S_x}$$

$$-\rho v_1^2 A_1 + \rho v_2^2 A_2 = p_1 A_1 - p_2 A_2 - R_x$$

Rearrange for the frictional force Rx

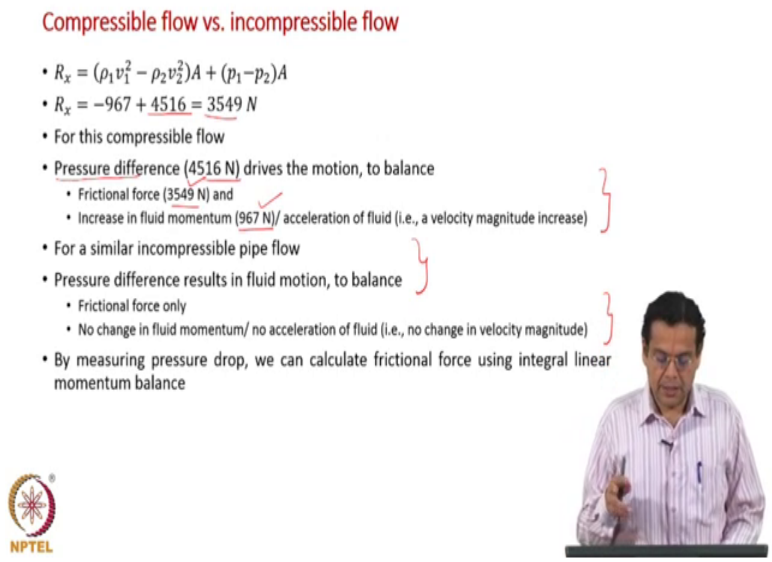
$$R_x = \left(\rho v_1^2 + \rho v_2^2\right) A + (p_1 - p_2) A$$

Now, this frictional force has two components; one because of the convective momentum and one because of the pressure change. So, we can work in terms of either the total pressure or the gauge pressure, the atmospheric pressure cancels out.

So, if we substitute all the values, we get the frictional force as

$$R_x = -\ 967\ +\ 4516\ =\ 3549\ N$$

(Refer Slide Time: 10:11)



Now, let us interpret this and also compare compressible flow versus incompressible flow. So, let us rewrite what arrived in the previous slide.

$$R_x = \left(\rho v_1^2 + \rho v\ _2^2\right)A\ +\ (p_1 - p_2)A$$

$$R_x = -\ 967\ +\ 4516\ =\ 3549\ N$$

Rx has two components; one because of convective momentum and one because of the change in pressure. Now, how do you interpret this?

Now, here we had compressible flow. Now, let us interpret it differently. The pressure difference is roughly about 4500 which has to be supplied by a compressor or any driving equipment. This pressure difference is balance by two components, one is the frictional force which is the major component here which is about 3500, and then, another component is the increase in fluid momentum that is why we are emphasizing that there is huge acceleration in this example; accelerating from about 60 to 300 m/s. So, the pressure drop, which we have to supply the pressure difference we have to supply has to overcome this frictional force and also the increase in fluid momentum. That is about 3500; that is about 1000.

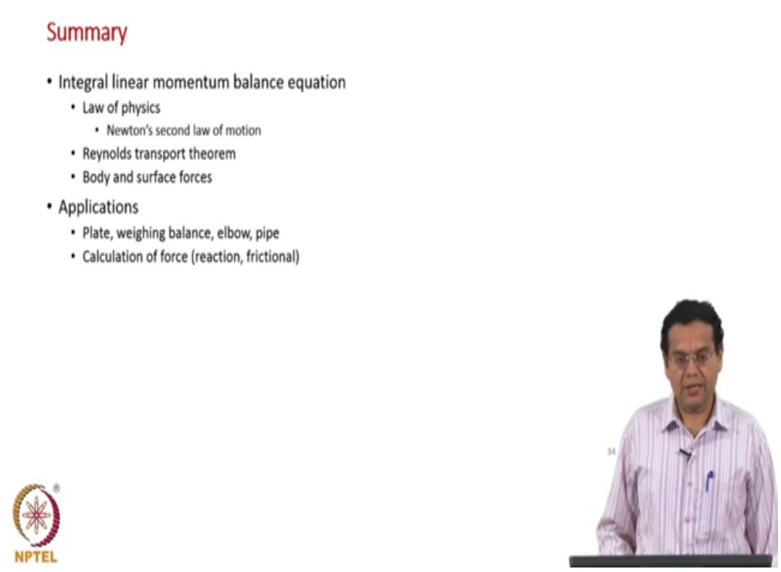
Now, suppose if this flow were an incompressible flow, for example, just water is flowing then the density would hardly change, velocity would hardly change in which case what will happen the pressure drops we have to supply will have to overcome only the frictional force alone. And no change in fluid momentum or no acceleration because the density all almost remains the same.

So, this is a good example. We have not worked out numerically this case, but qualitatively also. We are discussing how the pressure drop that we supply gets split into its components based on whether the flow is compressible or incompressible.

So, what we are concluding here is the apportionment of the pressure difference depends on whether the flow is compressible or incompressible. If it is compressible, two components contribute to the frictional force and the acceleration of the fluid. If it is incompressible, only the frictional force has to be overcome.

One more conclusion is that remember that we are measuring the pressure so by measuring the pressure drop, we are able to calculate the frictional force using the integral linear momentum balance. Our measurement is on the pressure on pressure drop, we use a balanced equation and we estimated the frictional force.

(Refer Slide Time: 13:15)



So, let us summarize this part on integral linear momentum balance,

- we first derive the integral linear momentum balance equation, the starting point was the Law of physics. This is Newton's second law of motion and then, we apply Reynolds's transport theorem to arrive at the integral linear momentum balance and then, on the right-hand side, we have on the forces acting on the contents of the control volume. We classified and discussed the two types of forces namely body and surface forces. Looked at their characteristics also examples. For example, gravitational force or the body force, pressure, surface force.
- And we looked at the application of the integral linear momentum balance equation. The applications were on the plate, the weighing balance elbow and pipe different geometries, different conditions. Now, in all the examples we were interested in calculating the force that is the prime country of interest, but different kinds of forces namely reaction force in all three cases and frictional force is the last case.