**Continuum Mechanics And Transport Phenomena**
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**Lecture - 17**
**Reynolds Transport Theorem: Introduction**

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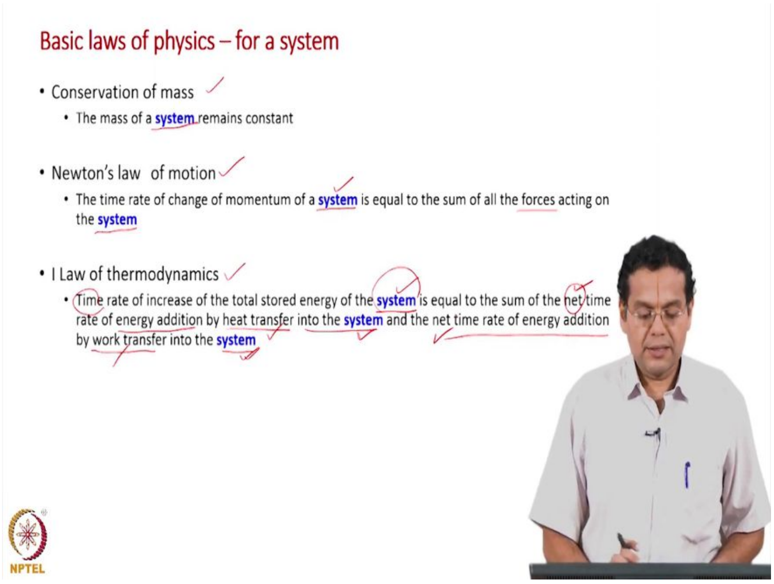


What we have discussed so far is about the system and then control volume. The system is made up of specific fluid particles, that is why I said stationary within brackets, fixed control volume within brackets when we define control volume. It could be moving as well, but in general, they are fixed control volumes. What distinguishes control volume from a system is that the fluid particles which are in the control volume they keep changing with the time that is the distinguishing feature.

Now, having discussed the system and then control volume, we are now going to move on to deriving the Reynolds transport theorem. We said we will do it in two steps, first for the simpler form and then the more general form. Now, what is the need for the Reynolds transport theorem that is what we are going to discuss in this slide? Now, the basic laws of physics conservation of mass, momentum, energy are all stated for a system, let us look at that.

- The mass of a system remains constant, that was very obvious as well the way in which we define the system is at we always focusing on hundred fluid particles. No fluid particle enters, no fluid particle leaves which means that the mass of the system is a constant. So, the conservation of mass is stated for a system, the mass of a system remains constant.

- Next, Newton's law of motion, let us state that the time rate of change of momentum of a system, I have emphasized that is equal to the sum of all the forces acting on the system. When we usually say we say the rate of change of momentum is equal to sum of all the forces, that is a very casual statement. If you want to very specifically say the rate of change of momentum of a system is equal to the sum of all the forces acting on the system. Newton's second law of motion is not for a control volume, Newton's second law of motion is for a system only.

- Now, the first law of thermodynamics, here, many of us should be familiar that the first law of thermodynamics is for a system that goes without saying, maybe for the conservation of mass and motion we are not so familiar. But, because the first law of thermodynamics is discussed in the thermodynamics course, in fact, the first class of thermodynamics was started with the definition of what is a system, what is a surrounding. So, it a very familiar that the first law of thermodynamics is for a system. So, let us state that the time rate of increase of the total stored energy of the system is equal to the sum of the net time rate of energy added by heat transfer into the system and, net time rate of energy addition by work transfer into the system, the first law is known to you what is to be focused is that we refer to the system. So, usually rate of increase refers to the time rate of change, here it is made more specific here time rate of increase of the total stored energy of a system and that is equal to the rate of energy addition; that energy addition could be by heat transfer or it could be by work transfer. What is more important is the total stored energy of the system and heat transfer into the system and work transfer into the system. The whole statement of the first law is for a system, you would have called this as closed systems etcetera, but still, the first law of thermodynamics is for a system. We have the words net here because you can have outflow, inflow etcetera; energy can be added removed etcetera that is why it says net.

Now, the fundamental concept behind this slide or what is that we should take away from this slide is that all the basic laws of physics are for a system.

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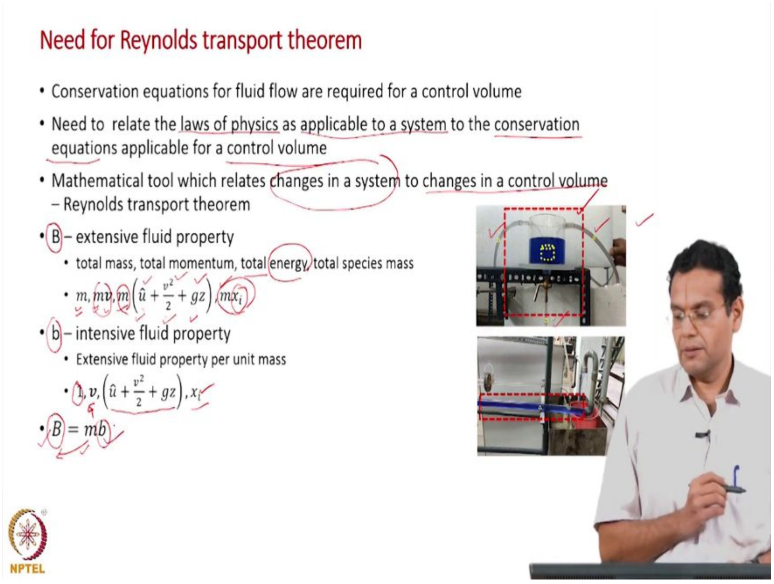


Now, what is a need for the Reynolds transport theorem? If we go back to our first lecture, where we did experiments in this tank and then used a mass balanced equation to predict the flow rate, which means that we require a mass balance. That mass balance we required for a control volume which means that all the conservation equations which we require are for a control volume.

It could be the larger control volume or the smaller control volume, but still, the conservation equations are required for an entire control volume, but now the laws of physics are for a system. So, conservation equations for fluid flow are required for a control volume as we have seen.

Now, the laws of physics as we have seen in the last slide are all stated for a system. So, we need to relate the laws of physics as applicable to a system to the conservation equations applicable for a control volume. Let us emphasize that laws of physics as stated or as applicable to a system to the conservation equations applicable for a control volume, whenever I say conservation equation there all for a control volume. So, I need to go from a system viewpoint to a control volume viewpoint. I should go from the laws of physics to the conservation equations and that is where we require a mathematical tool that relates the changes in a system to changes in a control volume and that is what we call as the Reynolds transport theorem. So, this Reynolds transport theorem helps us to relate the changes in a system to changes in a control volume.

The changes in a system are described with the law of physics, changes in a control volume, or what we require so that for example, we can use in such a case to predict flow rates, to predict velocity profiles etcetera. Before, deriving the Reynolds transport theorem let us introduce some nomenclature. Now, in a thermodynamics course you would have categorized properties as extensive properties and intensive properties; let us do that here as well.

B = Extensive fluid property

What are the extensive fluid properties which are going to come across in this course?

- Total mass,
- Total momentum,
- Total energy, and
- Total species mass.

Let us put them in terms of variables,

- Total mass is denoted by m,
- Total momentum is m x $v$, and then
- The total energy $(\hat{u} + \frac{\hat{v}^2}{2} + gz)$, in fact, I should say total - total energy, one total represents extensive property another total represents the sum of internal energy, kinetic energy, and potential energy. So, this energy really means the sum of all these three forms of energy, these values are per unit mass.
- Mass of the species $(mx_i)$, you would have done species balance equation in processing calculation course. So, this is the mass of one component, for example, let us say there was a stream with sulfuric acid and water this is the mass of the sulfuric acid.

b = Intensive fluid property

Intensive fluid property is the extensive fluid property per unit mass; very simple whatever variables I have explained there just divide by m you will get the intensive fluid property. For example,

- For the case of mass, you divide by mass itself so, it becomes 1 (m/m),

- In the case of momentum divide by mass, you get velocity (mv/m). So now, this velocity I can explain in two different ways; one is velocity which is very well known to us. What is the other way of looking at it? Momentum per unit mass, when you divide this momentum by the mass you will get the velocity so, two different ways of looking at velocity.

So, you should be able to view velocity in two different ways as we go along and that is why I am explaining now so, that you get used to that. First is velocity as such other way is momentum per unit mass and

- Of course, the energy divide by mass. As already explained to you all energy terms represent energies per unit mass internal, kinetic, and then potential energy that should be very familiar to you as well.

- Mass of the species divide by mass is the mass fraction, $x_i$ $(mx_i/m)$, you would have defined mass fraction as the mass of species by total mass exactly same meaning here as well.

So, b can take on values 1, velocity vector $(v)$, the sum of all the energy in terms of mass fraction $(x_i)$. We have seen several examples of intensive property b. They are related as

$$B = mb$$

The extensive property is equal to the mass in intensive property of course, the way in which we started is introducing the extensive property; if we divide by m you will get the intensive property.

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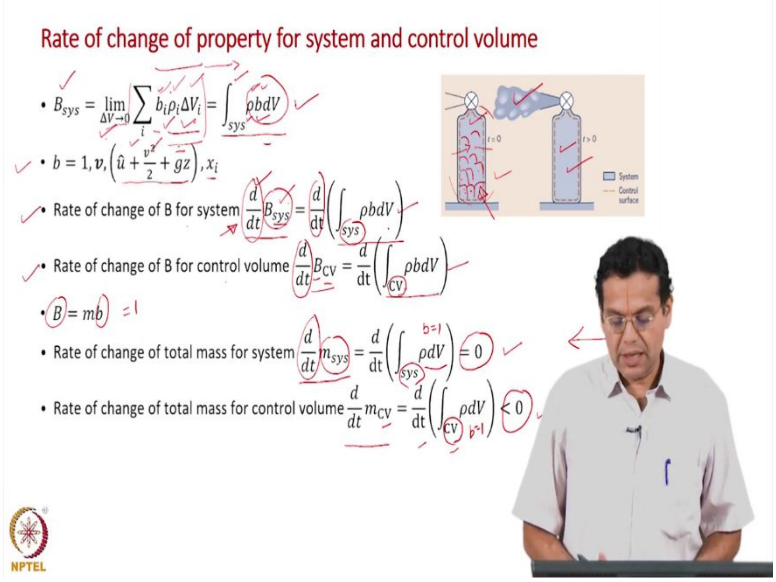


So, having introduced extensive property and then intensive property, let us move onto see how do we express rate of change of property for system and control volume. The object of the Reynolds transport theorem is to relate these two rates of changes, we want to relate the rate of change of property. When I say property, it could be mass, momentum, energy, species mass etcetera. We use the general term property, but for easy understanding, we will always related to mass, becomes easier to understand. So, the rate of change of property for the system, rate of change of property for a control volume. How do we express that and then how are they different for a simple case, that is what we will discuss in this slide.

Now, we introduced property B for a system and let us express that in terms of variables which you would usually measure

$$B_{sys} = \sum_i b_i \rho_i \, \Delta V_i = \int_{sys} \rho \, b \, dV$$

For example, we measure density and so, we will express this B system in terms of the intensive property and the density, let us see how do we do that. Let us take this example of the $CO_2$ gas in the extinguisher, we said the gas inside the cylinder is taken as a system. What I will do now is split into small-small volumes and which I call as $\Delta V_i$. Throughout this course, we are using small v for velocity, capital V for volume that may differ from some fluid mechanics books, the which we are following is small v for velocity which are used several times.

I think first I am coming across volume we are using capital V. So, what does $\Delta V_i$ mean, I split this system into smaller and smaller volumes, now each the density can vary within the system. So, for each volume, there can be associated with density. So, within the system, there can be spatial variation and the density of each volume is $\rho_i$. Then each volume can have its own property value, intensive value because this takes care of the mass and multiply with the property by unit mass. So, I get the value of the total property amount of total property.

The $b_i$ is, let us say if it is energy then energy for could be kinetic, potential etcetera if it is velocity we have seen b could be 1, b could be velocity. So, we allow for variation of the property within the system, we have split the system into smaller volumes and then multiply by the density of each of the volumes. So, that we get the mass of each volume which we have split, and then multiply by the property per unit mass for that particular volume. So, the whole term $(\sum_i b_i \rho_i \, \Delta V_i)$ represents the amount of property for that small volume.

We sum over all the volumes, i goes from 1 to the number of volumes so, that this entire term represents the amount of property for the entire system. We split into smaller volumes and then multiply by density to get the mass of each volume multiply by property per unit mass, get the property value for that small volume summed over all the volumes then that becomes the amount of property for the system.

Now, what do we do? We make this volume smaller and smaller which means that the number of such elements which we consider very large and tending infinity. And so, we take the sum in the limit of these small volumes become smaller and smaller $\Delta V$ which is the volume of each element becoming smaller we consider this limit.

Now, in the limit of $\Delta V \to 0$ the summation can be represented as this integral. Remember there is an assumption behind this representing summation as an integral. What is an assumption? Assumption of the continuum hypothesis, remember we discussed the beginning introduced continuum hypothesis; we said we can define differentiation, we can define integration only when we have assumed the continuum hypothesis. I take here $\Delta V \to 0$ which is valid only when the material completely occupies all the space, then if it is not there I cannot take this $\Delta V \to 0$. So, it may look like writing the summation as an integral, it was an assumption of the continuum hypothesis.

There it was introduced without a particular context, now there is a very good context to explain the use of continuum hypothesis or requirement of the continuum hypothesis. So, the summation has become an integral sign remember it is a volume integral; we are considering the volume of the system and $b_i$ has become b, so now, we allow for continuous variation of rho and b within the volume, that is the distinction between this summation and this integral. Here we allow for discrete variation, when you write us in summation when you write for integral we allow for continuous variation of those properties namely density and b. The moment I say continuous variation, continuum hypothesis invoked we use that to tell that it varies continuously. What is that we have done? $B_{system}$ has been represented in terms of the properties rho and then small b.

As we have seen b could be 1 for total mass, for the momentum it is momentum per unit mass which is velocity and then these energies per unit mass are a mass fraction.

$$b = 1, v, \ \left(\hat{u} + \frac{\hat{v}^2}{2} + gz\right), \ x_i$$

Now, Express rate of change of B for system, that expression is as we write for any rate of change of any value in our calculus

$$\frac{d}{dt}B_{sys} = \frac{d}{dt}\left(\int_{sys} \rho\, b\, dV\right)$$

What is important is we should focus on the subscript that is system; for example, it could be the rate of change of mass for a system; we will take as an example immediately. So, the rate of change of property for the system.

Now, let us write the rate of change of B for the control volume expression is not going to change,

$$\frac{d}{dt}B_{CV} = \frac{d}{dt}\left(\int_{CV} \rho\, b\, dV\right)$$

whatever I have written for the system is applicable for control volume also; only the volume integral is going to change over the control volume defined by the control surface. Now, the subscript now is control volume; what does this meaning of we will see when we take an example.

Why do I do in terms of integral expression? To allow for continuous variation within the volume, in this case, the subscript says control volume. Of course, B = mb. So, all these values the intensive properties, if you multiply by m you will get B.

So, let us take an example of this fire extinguisher case and let us take the example of mass. We will take B = total mass (m) as an example in which case the small b = 1 (m/m). So, the rate of change of total mass for the system

$$\frac{d}{dt}m_{sys} = \frac{d}{dt}\left(\int_{sys} \rho \, dV\right) = 0$$

Now, the way in which we defined system, the mass of the system is a constant, we have seen that as a law of physics also because we are focusing on the same set of fluid particles. And so, the rate of change of mass of the system is equal to 0, because we are considering mass is 0 where other variables are need not be 0. So, in this particular case, because it is mass, the rate of change of the mass of the system is equal to 0.

Now, the rate of change of total mass of the control volume is

$$\frac{d}{dt}m_{CV} = \frac{d}{dt}\left(\int_{CV} \rho \, dV\right) < 0$$

Now, Let us go back to the example, our system at t = 0 is the gas that occupies the cylinder. At some time later our system is composed of whatever gas which is escaped and whatever is remaining, that is why the rate of change of mass of the system is 0. Coming to the control volume, initially let us say I had some 10 kgs of $CO_2$ inside the cylinder, sometime later some gas escaped, and let say it is escaping. So, this mass inside the cylinder inside a control volume will keep decreasing and that is why this value is less than 0. So, while rate of change of mass the system is equal to 0, rate of change of mass of the contents of the control volume. We do not say rate of change of mass of control volume, that is not precisely correct. We say rate of change of mass in the control volume or little more elaborately, we say rate of change of mass of the contents of the control volume. For the system we can say rate of change of mass of the system, for control volume, it is mass in the control volume or mass of contents of the control volume; very precise statements and that value is less than 0.

So, now as I told you the objective of this slide was to express the rate of change of property of a system, rate of change of property for control volume. We also took a simple example to see that these two rates of changes are different, rate of change for a system is different even for the case of the simple example for the case of taking mass an example. The rate of change of mass for the system is 0, rate of change of mass in the control volume is less than 0.

Reynolds transport theorem relates these two rates of changes, that is the objective of Reynolds transport theorem. How is the rate of change of property for the system is related to the rate of change of property for the control volume and that is what is the objective of the Reynolds transport theorem.

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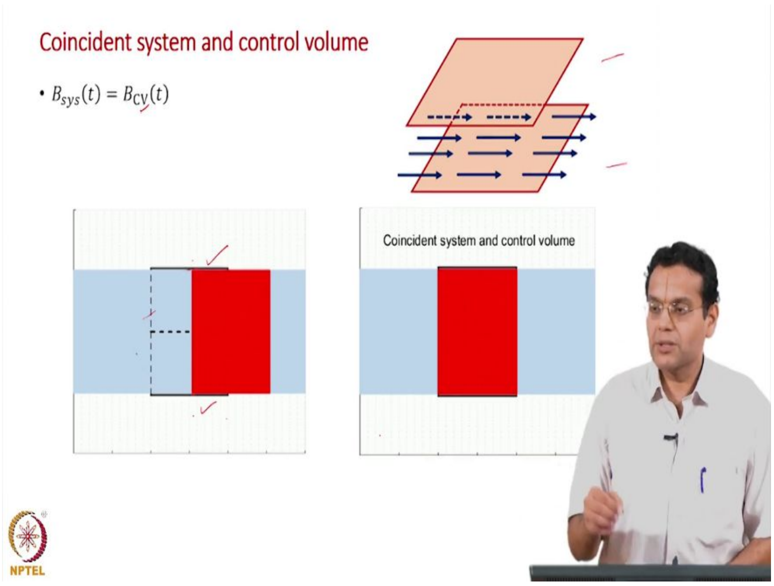


Let us move on, we are slowly learning concepts to start to derive the Reynolds transport theorem. First, we discuss about system and control volume, then we introduced extensive variables, intensive variables. Now, on the previous slide, we discussed about the rate of change of property for system and control volume, now one more concept is required what is called coincident system and control volume. In the first step in the derivation of the Reynolds transport theorem, we will say that we will consider a system and control volume which are coincident with each other.

$$B_{sys}(t) = B_{CV}(t)$$

Symbolically in terms of expression, this is how we write the property of the system, in this case, capital B is equal to property of B in the control volume at a particular time t. We quickly say as B system is equal to B control volume in terms of statements B of the system is equal to B in the control volume at time t. Now, we will take our well-known example: flow between the two parallel plates. Now, I have a two demonstrations here (look at the lecture video), the same demonstrations at different stages; as the fluid flows through between these two parallel plates. Let me explain what are drawn here (above slide picture).

The top plate, the bottom plate, the access and then the control surface through which there is inflow, the control surface through which there is an outflow. This nomenclature is required to understand what I am going to show now; so, the top plate, the bottom plate, and then the control surface with inflow, with outflow. So, which means these four lines, defines the configuration of the control volume.

Now, we want to define our system, now I have different choices for defining my system and what is shown here are different choices for a system. For example I can color a portion of the fluid as it is shown here (red-colored image in the above slide) and call that as my system, let me just go ahead.

Now, what is the most convenient way of defining my system? Several options are there which is more convenient. Remember system was little more imaginary; so, it will be convenient for us if we define a system whose boundaries are well defined. In the example of the fire extinguisher, initially, we had the system as whatever was occupying the tank, sometime later some system escaped etcetera.

So, now system boundary can be anything, but we need to define a system which is more convenient to us and was boundaries are well defined. So now, you just watch the different options of system. The best option is when the system coincides with the control volume. The boundaries and system are also well defined and coincide with that of the control volume for which we need to derive the conservation equation.

So, that connects the boundaries for the system and the boundaries for the control volume that as coincide; so, that we can relate the laws of physics to the conservation equations. You will see a message saying the coincident system and control volume system when that system exactly coincides with the control volume.

So, I can choose system which is just entering which is just coincident, or leaving which is most convenient or which is most well defined is the system which is coincident with the control volume at any instant of time. At every time whatever system is coincident in the control volume, that is defined as my system. This is what I have shown in the next video (see the lecture video), where I have just stopped the animation at the point where the system coincides with control volume.

In case we are not able to see in the earlier case, the coincident system, and control volume. Several possibilities for selecting system are there, but the most convenient, most well-defined possibility is the system which is coinciding with the control volume; remember it is that every instant of time.