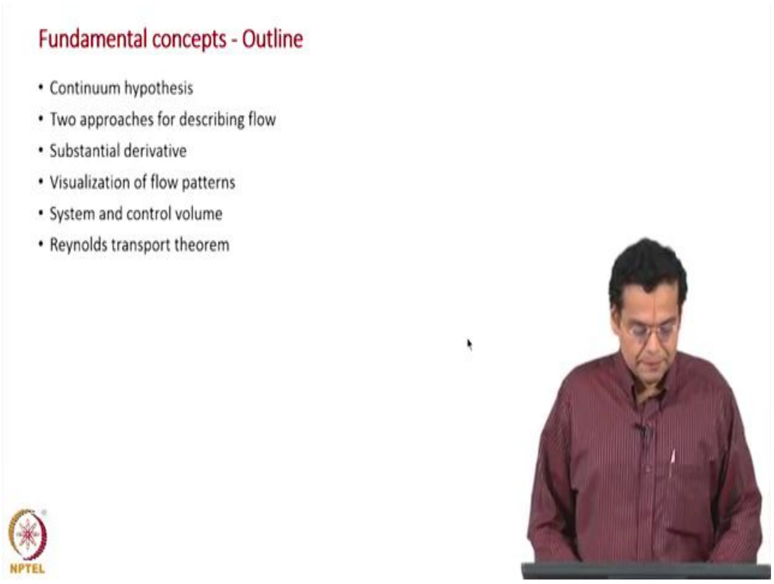
Continuum Mechanics And Transport Phenomena
Prof. T. Renganathan
Department of Chemical Engineering
Indian Institute of Technology, Madras

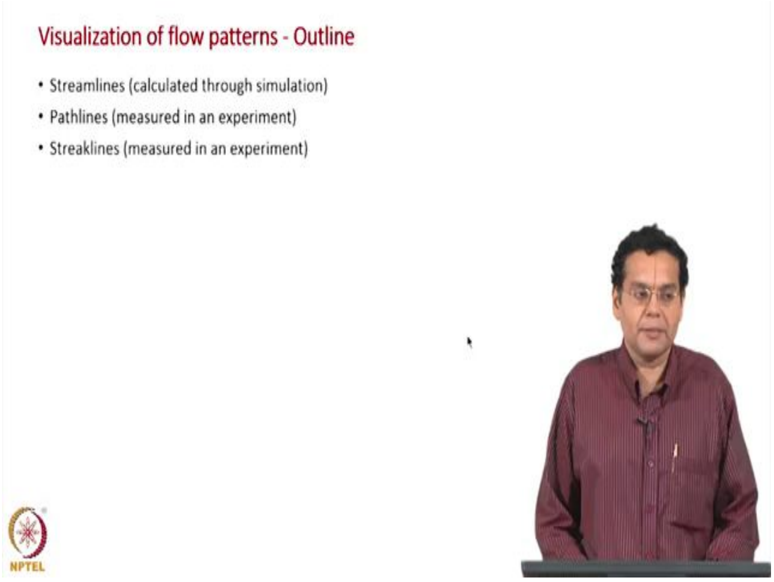
Lecture – 12
Visualization of Flow Patterns: Streamline, Pathline

(Refer Slide Time: 00:14)



We are discussing the fundamental concepts wherein we have discussed the continuum hypothesis. There are two approaches for describing fluid flow namely Eulerian, and Lagrangian. And, the last lecture, we discussed about the substantial derivative, the derivative characteristic of continuum mechanics and transport phenomena, and also looked at in detail the say the physical significance of that the derivatives which are following the fluid motion. In this lecture, we are going to discuss about Visualization of Flow Patterns, before looking at two other topics as part of fundamental concepts.

(Refer Slide Time: 01:00)



We have been telling about flow through some geometry, flow through some pipe etcetera then, but how do you represent that either experimentally or in terms of simulation that is what we mean by visualization of flow patterns. We have some equipment and there is some flow through that and you would not describe that in terms of some let us say a representation of figure and how do you do that.

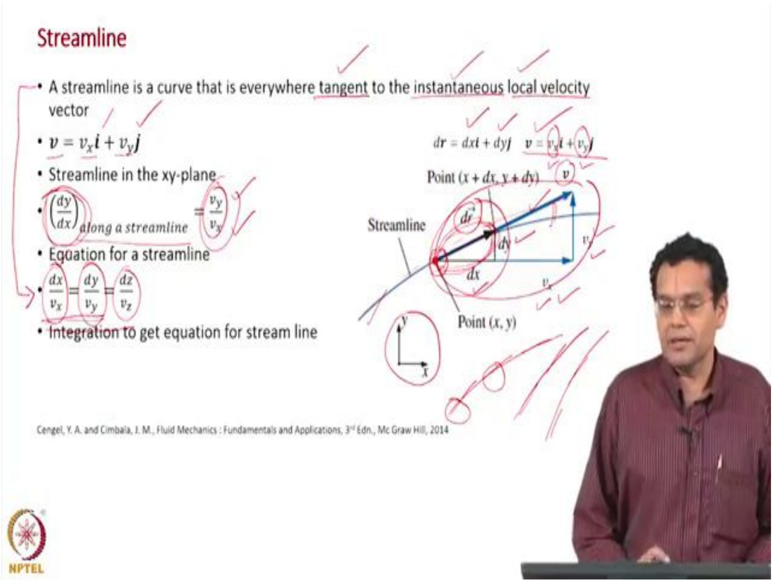
So there are three different ways of representing a flow pattern. First is a method by streamlines, and streamlines are a representation of a result of a simulation. You do not measure streamlines if you have calculated the flow field then we can represent the flow field in terms of streamlines. That is why it says calculated through simulation.

Now, the second way of visualizing flow patterns is by pathlines, and path lines are measured in an experiment; you can measure path lines. You can also represent simulation through path lines, but the moment you say path line we usually relate to experimental measurement.

The third method is through streak lines. Once again it is measured in an experiment. We do not usually represent the result of a simulation through streak lines. Usually, we represent mostly based streamlines may be pathlines also, streaklines rarely.

That is why I separated into simulation and experiment; streamlines through simulation, path lines through an experiment which means measure path lines in an experiment, also represent simulation results through path lines, streak lines are predominantly measured experimentally.

(Refer Slide Time: 03:06)



Now, let us start with this definition of a streamline. The whole idea of this lecture is to define all these lines and how are you going to calculate these three lines and then represent pictorially. So, let us look at the definition of a streamline. A streamline is a curve that is everywhere tangent to the instantaneous local velocity vector. Now, first, the keywords here are; it is a tangent, the next keyword is instantaneous, the next keyword is local velocity. So, three aspects are there in this definition.

First, it says that it is a curve that is everywhere tangent to the velocity vector. Looking at the other way, if you have a streamline and if you draw a tangent, that would represent the velocity. Now, why is it local? Wherever you have a tangent it represents velocity at that particular location. So, what is shown in the figure (referred slide) is a streamline, and I have taken a point x, y, and shown a tangent there. So, according to the definition of streamline; a streamline is a curve that is everywhere tangent to the velocity vector. So, at a point, the tangent represents the direction of velocity. That is why a velocity vector is also shown and that is along the direction of the tangent that completes the first keyword.

Second, as I told you local, why is it local? Because the streamline can be curved as shown in the figure so, the tangent which you draw can keep changing directions. So, one tangent gives the velocity at a particular location, and the other tangent gives the velocity at other location that is why this says a local velocity vector. Thirdly, instantaneous what does it mean? At that particular instant of time, this is the streamline. So, we have a streamline let us say at some instant t, and some other time streamline may be different.

So, that is why you when streamlines are instantaneous at a particular instant of time. I think this should be kept in mind. The way in which time plays a role among these three visualization patterns is slightly different. So, for streamlines, it is instantaneous streamlines and if you have given a set of streamlines just by drawing the tangent you will be able to represent the velocity vectors and their local velocity vectors.

Now, how are you going to quantify this? We have seen in terms of description what are streamlines is. Let us see how do we go about getting an expression for a streamline. What is given to us? A velocity field is given to us. As I told you we have done some simulations, we have done some predictions. We have got the velocity field as a result of that simulation which means that you know the velocity vector and for the present case we will resist ourselves to a 2-dimensional case.

$$dr = dxi + dyj$$

So, we have v_x and $v_{y;}$ two components are there and they could be functions of x, y, and, this result is obtained from a simulation. Now, we want to represent this in terms of streamlines So, that quickly I know how does velocity changes? How does the velocity change from one location to the other etcetera. So, as I told you we start with the given velocity field represented in terms of an expression

$$v = v_x i + v_y j$$

v_x = x component of velocity and v_y = y component of velocity and i and j are the unit vectors along x and y-direction.

Now, we are looking at the streamline in the x, y plane. So, we are interested in the streamline in the x, y plane. Now, what we will do is use the definition of streamline and put it in terms of an equation. What does the definition say? If you draw a tangent to the streamline that is in the same direction as the velocity vector, so, the slope of the streamline at a particular point is given by the tangent.

So, we know that the slope of a curve is given by $\frac{dy}{dx}$, what is specified is it is along a streamline. We are along a streamline, we draw a tangent and the slope of that is represented by $\frac{dy}{dx}$. Now, this should be equal to the velocity vector makes an angle let us say theta with the x-axis and the tangent of that angle should be equal to this slope and we know that the tangent of this angle is given by $\frac{v_y}{v_x}$. So,

$$\left(\frac{dy}{dx}\right)_{along\ a\ streamline} = \frac{v_y}{v_x}$$

So, this equation is the condition for a streamline. The left-hand side is the slope of the streamline and we have defined a streamline such that if you draw a tangent which is the slope will be equal to the tangent of the angle the velocity vector makes with the x-axis.

Another way of looking at this expression is that little more geometrically. I have drawn a small incremental arc in the figure (referred slide) and as a point becomes closer and closer that arc becomes the tangent and, components of this vector are dy in the y-direction dx in the x-direction. Now, we know that the velocity vector is along the same tangent and the components of velocity are v_y and then v_x. So, we can take two similar triangles and equate $\frac{dy}{dx}$ on the left-hand side. Then, equate $\frac{vy}{vx}$ to the $\frac{dy}{dx}$. So, the second one is more geometric first one is just comes from the definition of streamline.

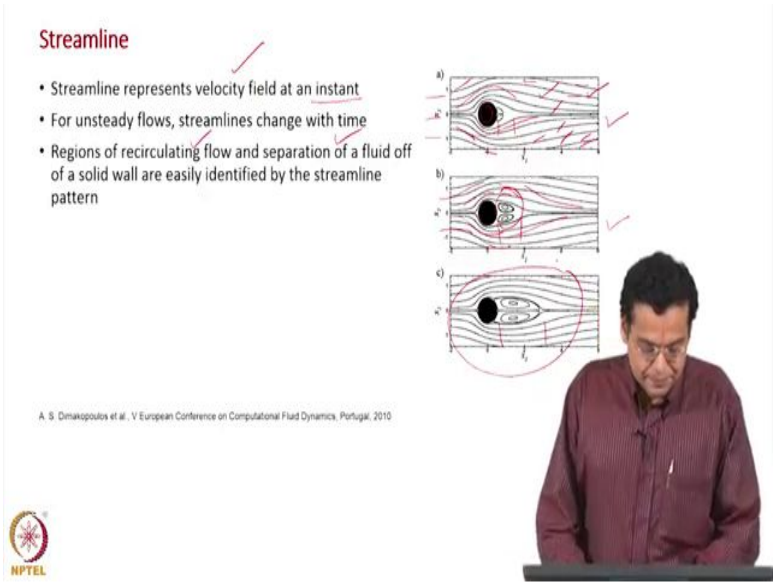
Now, if you extend this to a 3-dimensional case. So,

$$\frac{dx}{v_x} = \frac{dy}{v_y} = \frac{dz}{v_z}$$

So, these conditions can be used to obtain the equation for streamline, how do we do that we will see later. But, right now we have come from a physical definition to a mathematical expression that represents the physical definition.

We will have to integrate this; we will consider only a 2-dimensional case later on as an example. So, integration of this equation will give the equation for streamline. How do we do that we will see take an example later on and then discuss that.

(Refer Slide Time: 10:54)



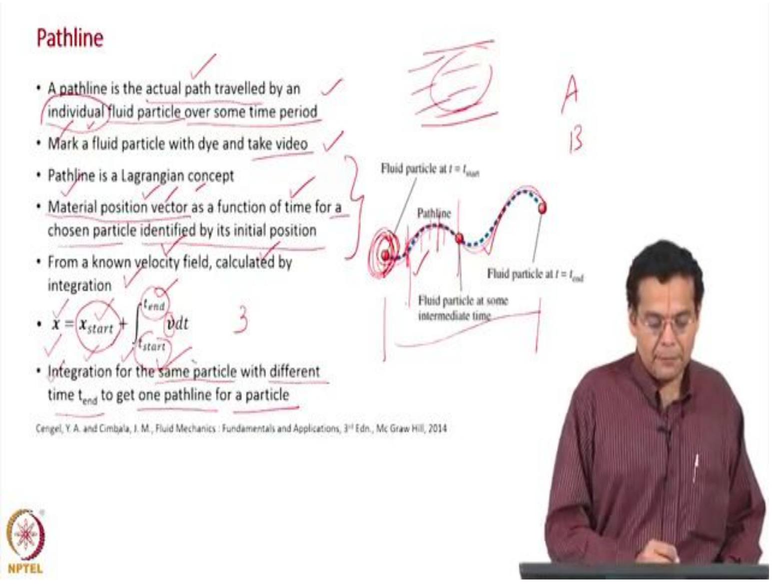
Streamline represents the velocity field at an instant, I want to emphasize that it is at a particular instant, streamlines can change as a function of time and if you are given a set of streamlines then you keep drawing tangent. For example, we have what is shown in the figure (referred slide) are set of streamlines for flow around a circular cylinder. What you see is just front view and you have flow entering here and then flowing over a cylinder.

So, all these are streamlines and if you draw a tangent anywhere you get the entire velocity field that is what this statement tells you. Streamline represents velocity field at an instant. So, wherever you have can draw a tangent then it will represent the velocity vector at all those locations so, you can get the entire velocity field. As usual, as I said earlier why that instant this velocity field can change with time and then you get instantaneous velocity field. So, in that way, it is very a powerful representation of the velocity field.

For unsteady flow streamlines change with time, we have seen that. And, regions of recirculating flow and separation of fluid of a solid or easily identified by the streamline, that is what in fact, is shown here. These streamlines go around the object and then you see some streamlines here which are closed which represents circulation and you can also see that the width of the circulation keeps increasing. So, in that way streamlines are a very powerful representation. They give the velocity field, they also tell you recirculating flow, separation of fluid of a solid wall etcetera, all can be obtained you get a quick picture of how the velocity takes place and that is why we said visualization of flow patterns.

As I told you, in this case, it is flowing over an object, it could be flowing through a pipe whatever geometry and streamlines visually give a quick representation of the entire flow field. So, to explain our title these are the flow patterns and we have seen one method of representing the flow patterns namely streamlines.

(Refer Slide Time: 13:29)



What are this path line, let us look at the definition once again. A pathline is an actual path traveled by an individual fluid particle over some time period ok. Now, we are familiar with the term fluid particle. If you identify a fluid particle as usual by adding a dye and then you track the path followed by the fluid particle and that is called a path line. Path line is the actual path travel by an individual fluid particle; I want to emphasize individual fluid particle will help you to understand later on when you discuss streak line. So, path lines are for individual fluid particles.

Also want to emphasize that over some time period, let us say at $t = t_{start}$ you are at some position, after some time the particle is at some other position. So, over some time period what is a path followed by the fluid particle and what is the path depend on that depends on the local velocity field. So, once again it represents the velocity field. The whole idea is to represent the velocity field in some form, earlier we have discussed streamlines now we are discussing the path line. The path depends on the velocity field; it could which could be a function of time as well.

So, as I told you mark a fluid particle with dye and take a video. Why should you take a video because we are following the particle over a period of time, continuously over a period of time. I want to emphasize this video which you are of course, easily remember so that I can contrast it with streak lines a little later. So, two things to be noted; one, of course, is a path of a fluid particle. I want emphasis on individual fluid particles and taking a video keep this in mind as we go along with streak lines I will contrast with this.

Path line is a Lagrangian concept by the way and I have discussed to be very clear that it is a Lagrangian concept, when you discussed the Lagrangian concept in fact without doing the term path line, what did we say, we identify a particle follow the path, followed by fluid particle. Specifically, I had used the word path line because it was resolved for today's lecture, but without using this terminology we said we take a group of molecules large enough number of molecules dye it and follow the path followed by the fluid particle. In fact, the Lagrangian concept was introduced using a path line without mentioning the name. Now, the time has come to relate these two path line is a Lagrangian concept.

Now, material position vector as a function of time for a chosen particle identified by its initial position. Once again all these terminologies referred to the Lagrangian description. Let us see how is that first of all, for a chosen fluid particle I mention as particle means a fluid particle and then material position vector, why material position vector? We are following a material particle which is a fluid particle and what is its position as a function of the time it could be a, it could move in one direction, it could move in the x-y axis or along x, y, z coordinate. So, it is a position vector of course, as a function of time which varies as a function of time.

Remember Lagrangian description we said the independent variables are a time of course, and then the fluid particle. For example, in the chimney case we said particle A, particle B, and then later on we said, refine saying that we cannot number all the particles, I will give letters for all the particles. We will identify by its initial position that is what this says identified by its initial position.

So, different ways of explaining the path line so that we relate to the Lagrangian concept. So, as I told you streamlines depend on the velocity field, similarly path lines of course, also depend on the velocity field. The whole idea is to represent the velocity field. So, for a known velocity field two ways of looking at it when I say known velocity field, the velocity field could come from a simulation and we want to represent as path lines, but more practically you have a region and there is a velocity field on the path line gets determined by the velocity field existing in that region. We have a pipe and there is some velocity field here, the path line gets determined by the velocity field.

So, when I say known velocity field either an experimental field and you are measuring a pathline or a velocity field which you result which you calculate from a simulation and what you require is a velocity field. In one case you have values or functions in the case of simulation. In the experiment, this region has some velocity field in it and which you do not know and, but you are measuring the path line to indirectly get the velocity field.

Now, how do you get the path line it is a simple integration. Here we should mention that based on my description, this path line is the same as the path line we should have come across in the particle mechanics course. Whatever you have come across in physics course the distance traveled by a particle at different times exactly the same meaning here, but for a fluid particle, that is all the difference.

So, We want the position vector as a function of time

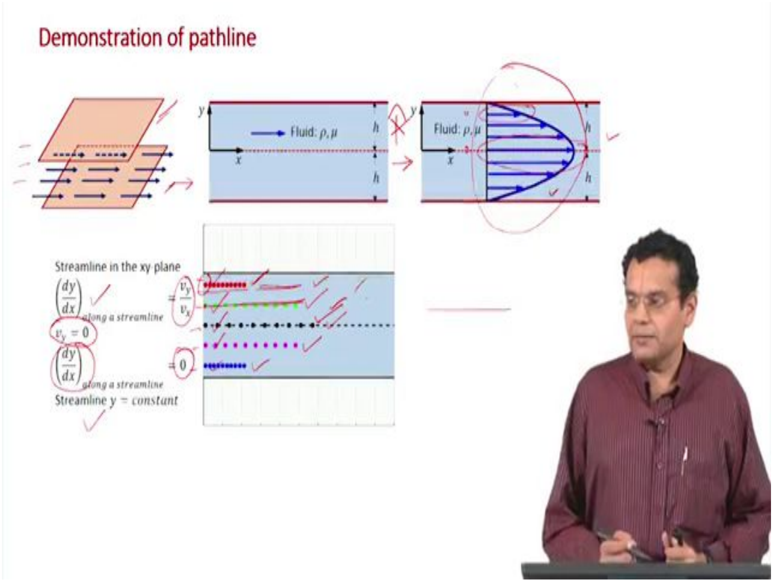
$$x = x_{start} + \int_{t_{start}}^{t_{end}} v\, dt$$

You are integrating from t_{start} to t_{end}. What is that t_{end}? Let us say $t_{start} = 0$, if I take $t_{end} = 1$, I will get the position vector at the first second; if I take $t_{end} = 2$, I will get the position vector at the 2^{nd} second.

So, I want to emphasize here once again; you are given x_{start}, you are given t_{start}. You are identifying the particle by its initial position. So, let us say $t_{start} = 0$ and $x_{start} = 1,1$ or some coordinate system. This t_{end} keeps changing depending on the position you want to find out at the corresponding time. Of course, the vectorial equation we write this into three components; and do the integration. We will see how do we do that in the application part.

Integration for the same particle with different time t_{end} to get one path line for a particle. Every word makes is important here for the same particle. Why is it important? To distinguish this from streak lines that is why it is important. For the same particle with different time t_{end}, so that we get different positions at different times and get one path line for a particle. We are focusing on only one particle. You may focus on different particles. By focusing on one particle you get one path line. So, as shown here, it focuses on one particle and getting the path line for one particle. That is why it says for the same particle with different times t_{end} to get one path line for a particle.

(Refer Slide Time: 21:53)



So, let us see a demonstration, our friendly system is the flow between two parallel plates. We will come across an example repeatedly unless a climax of let us say a fluid mechanics course, fluid mechanics part of continuum mechanics part you will derive this velocity profile ourselves, until then we will accept this velocity profile. So, as I told you we have a region, this is the region between two plates and we have a flow in this direction. And, there is flow only in the x-direction and this region has this velocity field, and our path line will represent this velocity field.

Now, what I have shown here is different fluid particles each will give different path lines. And, they are identified by their initial position that is what we said; we identify a particle at the initial position. Let us say this t = 0, let me start the simulation. Now, these individual lines are the path lines. This is the path line for 1st particle, the path line for 2nd particle, path line at the particle which of the center.

Now, as you see this particle as travels only a smaller distance. This particle the center has traveled a long distance. So, this particle has traveled a larger distance, all are done at the same time. So, as we, as once again going back to object we want to represent the flow field in terms of path line that is what has happened here. These path lines represent the velocity field, a particle at the wall moves slower but a particle at the center moves faster and that is why the centered one has traveled a longer distance compared to a particle which has closer to the wall.

Now, how does it compare with the streamline? Remember the mathematical representation of streamline you said

$$\left(\frac{dy}{dx}\right)_{along\ a\ streamline} = \frac{v_y}{v_x}$$

In this particular case, there is no flow in the y-direction; there is flow only in the x-direction. So, the y component of velocity 0, i.e $v_y = 0$; which means that the slope is 0, when the slope is 0 we know it is just a straight line, and streamline is y equal to constant.

$$v_y = 0;$$

$$\left(\frac{dy}{dx}\right)_{along\ a\ streamline} = 0$$

$$y = constant$$

So, the path line is also a horizontal line, streamline is also a horizontal line. So, path lines and the streamlines coincide with each other. Not any path any streamline; streamline passing through a location and path line through that same location coincide. You cannot say the path line passing through (x,y) coincide with the streamline passing through (x_1,y_1) So, that is not correct. So, now, this is a case of a steady-state velocity field. This velocity field does not change with time hence streamlines coincide with the path line.