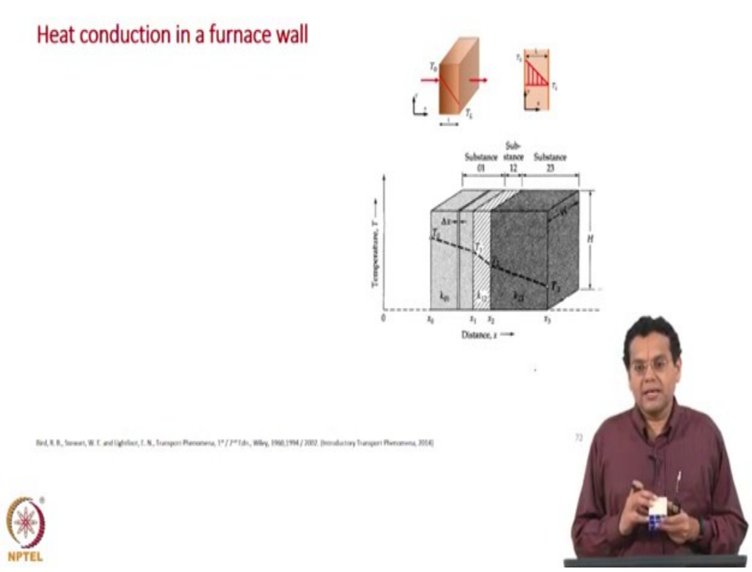
Continuum Mechanics and Transport Phenomena
Prof. T. Renganathan
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture – 115
Heat Conduction in Furnace Wall

(Refer Slide Time: 00:15)

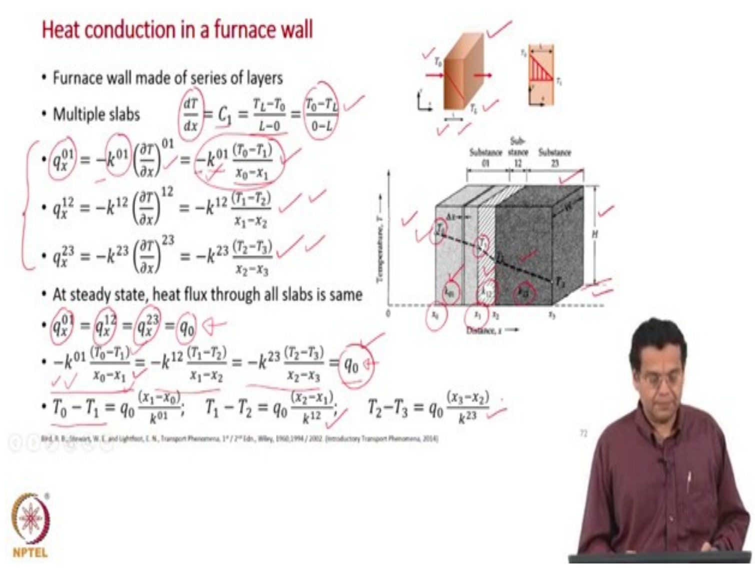


So, now let us extend the discussion what they had for a singles slab to a series of labs. What is an application? This can be taken as an example for a series of slabs. The application would be in a for a furnace wall, furnace is an equipment where we have very high temperature inside and we use it for carrying out high temperature reactions.

And, because the temperature is high inside and outside is ambient there can be very high heat loss through the walls of the furnace. So, usually furnace walls are made of several layers and they are made of materials which have low thermal conductivity. So, that they can provide good insulation and that is what is shown in the image in the above slide snap, here it is shown as a series of slabs.

But, what we can imagine is this to be a furnace wall let us say high temperature here and let us say a low temperature here. So, that is the application what we are going to do is extend the discussion for single slab to multiple slabs. And, what is the object to find out that heat flux through a series of slabs like this, so let us do that.

(Refer Slide Time: 01:55)



So, as I told you furnace wall made of series of layers or series of slabs, so we discuss in terms of multiple slabs. We will use a this result which you have seen in the case of single slab that is the temperature gradient or the slope of the temperature profile

$$\frac{dT}{dx} = C_1 = \frac{T_L - T_0}{L-0} = \frac{T_0 - T_L}{0-L}$$

So, now let us write down the expression for the heat flux in each of the layers, this figure is from Bird, Stewart, Lightfoot, three layers are shown and each layer is made of a different material called as substance 01, 12 and 23. We will just call them as 1, 2, 3 for easy discussion and each material has it is own thermal conductivity k^{01}, k^{12}, and then k^{23}.

Left surface is at a temperature T_0 naught and the right surface is at a temperature T_3, so now let us write the expression for heat flux in the first layer. So, heat flux in the first layer is equal to minus thermal conductivity in the first layer multiply by that temperature gradient in the first layer. So, we have

$$q_x^{01} = -k^{01}\left(\frac{\partial T}{\partial x}\right)^{01} = -k^{01}\frac{(T_0 - T_1)}{x_0 - x_1}$$

So, this equation gives the heat flux through the first layer in terms of the temperature difference and $x_0 - x_1$, is the thickness of that layer and also in terms of the thermal conductivity of the first layer.

So, let us similarly write down the expression for the second layer and the third layer.

$$q_x^{12} = -k^{12}\left(\frac{\partial T}{\partial x}\right)^{12} = -k^{12}\frac{(T_1 - T_2)}{x_1 - x_2}$$

$$q_x^{23} = -k^{23}\left(\frac{\partial T}{\partial x}\right)^{23} = -k^{23}\frac{(T_2 - T_3)}{x_2 - x_3}$$

So, these are the equations for the heat flux through the second layer and through the third layer.

The temporary difference for the second layer is $T_1 - T_2$ and the temperature difference for the third layer is $T_2 - T_3$ in the corresponding thicknesses. Now, like in the last case we are considering steady state condition, so the heat flux through all this labs should be same; slab 1, slab 2, slab 3 all the heat flux should be same.

So, let us put down that in terms of a equation

$$q_x^{01} = q_x^{12} = q_x^{23} = q_0$$

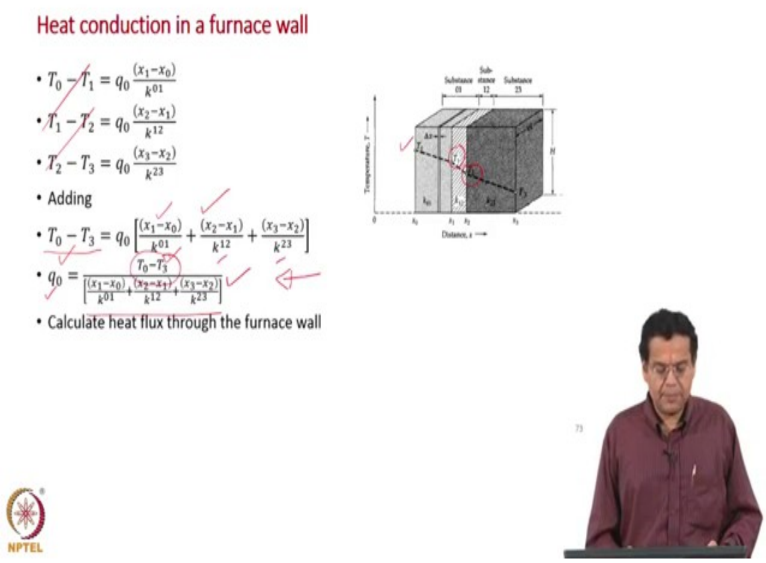
So, now, let us express this equation in terms of the thermal conductivity and the temperature difference and the thickness using these 3 expressions.

$$-k^{01}\frac{(T_0 - T_1)}{x_0 - x_1} = -k^{12}\frac{(T_1 - T_2)}{x_1 - x_2} = -k^{23}\frac{(T_2 - T_3)}{x_2 - x_3} = q_0$$

Now, what we will do is from this equation we will write expressions for the difference in temperature across the 3 slabs. So, from this expression we write

$$T_0 - T_1 = q_0\frac{(x_1 - x_0)}{k^{01}};\, T_1 - T_2 = q_0\frac{(x_2 - x_1)}{k^{12}};\, T_2 - T_3 = q_0\frac{(x_3 - x_2)}{k^{23}}$$

(Refer Slide Time: 07:29)



So, let us write down all the three expressions for the temperature difference across the 3 slabs and you understand why we are doing this,

$$T_0 - T_1 = q_0 \frac{(x_1 - x_0)}{k^{01}};$$

$$T_1 - T_2 = q_0 \frac{(x_2 - x_1)}{k^{12}};$$

$$T_2 - T_3 = q_0 \frac{(x_3 - x_2)}{k^{23}}$$

Now, what we do is just add all these three questions,

$$T_0 - T_3 = q_0 \left[\frac{(x_1 - x_0)}{k^{01}} + \frac{(x_2 - x_1)}{k^{12}} + \frac{(x_3 - x_2)}{k^{23}}\right]$$

The reason is that we know only the n temperatures, we know only T_0, we know only T_3 the n temperatures are known. The interface temperatures; interface temperatures meaning, the temperature at the interface of the two materials T_1, T_2 are not usually known that is why what we have done is eliminated T_1, T_2 from the above equations.

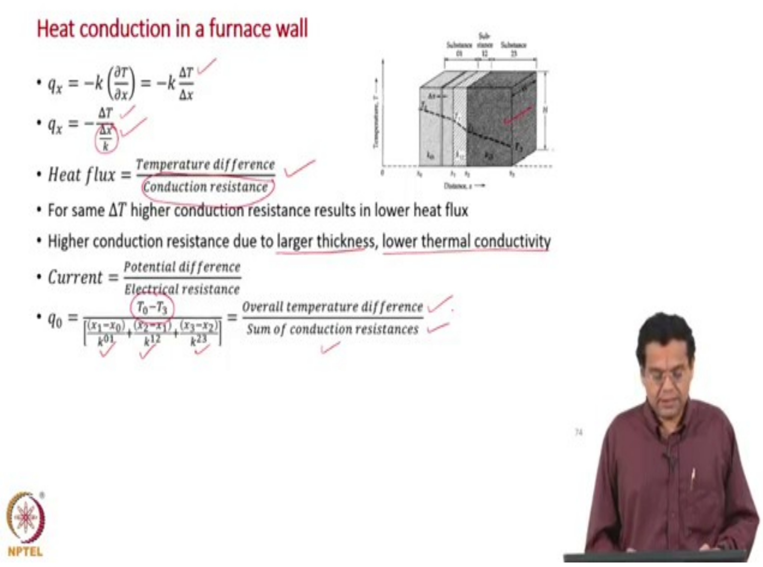
So, when you add left hand side we have you have only $T_0 - T_3$, right hand side of course, q_0 is common and then multiply by thickness by thermal conductivity for each slab, slab 1, slab 2 and slab 3. As I said our objective is to find out the heat flux through this series of slabs, so let us rearrange this expression for q_0. So,

$$q_0 = \frac{T_0 - T_3}{\left[\frac{(x_1 - x_0)}{k^{01}} + \frac{(x_2 - x_1)}{k^{12}} + \frac{(x_3 - x_2)}{k^{23}}\right]}$$

So, what is the use we can calculate the heat flux through the furnace wall [inaudible], we know T_0, we know T_3, we know the thickness of each slab or each layer, we know the thermal conductivity of the material of this slab. So, simple substitution we can find out what is the heat flux through the furnace wall.

Other way of using this equation would be suppose let us say you want to limit the heat flux to a particular value then q naught is known. Of course, a temperature difference is known to us then you can find out the thickness of one of the layers. Now, this equation can be interpreted in a different way let us see how do we interpret that.

(Refer Slide Time: 10:09)



For that let is write the Fourier's law of heat conduction in terms of difference. So,

$$q_x = -k\frac{\partial T}{\partial x} = -k\frac{\Delta T}{\Delta x}$$

Now, let us do a small rearrangement on the right hand side

$$q_x = \frac{-\Delta T}{\frac{\Delta x}{k}}$$

How do we interpret this equation left hand side we have heat flux right hand side we have temperature difference in the numerator. The denominator we can call this as conduction resistance, because that determines the resistance to conduction.

$$Hear\ flux = \frac{Temperature\ difference}{Conduction\ resistance}$$

Suppose the temperature difference is same the higher the conduction resistance lowers is the heat flux. So, what can result in higher conduction resistance either the thickness can be large or it can be a material which has a very low thermal conductivity.

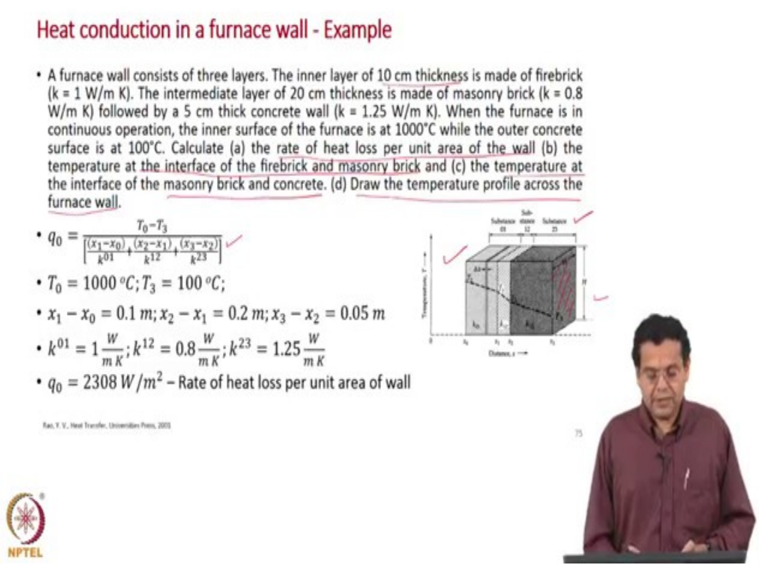
So, either the thickness is a large or the thermal conductivity is low then the conduction resistance will be high and for the same temperature difference heat flex will be reduced.

So, it acts as a resistance to conduction heat flux and this expression is similar to our well known expression for a current expressed as potential difference by electrical resistance. So, now, if you go back to our expression for heat flux through the three slabs we can interpret that as

$$q_0 = \frac{T_0 - T_3}{\left[\frac{(x_1 - x_0)}{k^{01}} + \frac{(x_2 - x_1)}{k^{12}} + \frac{(x_3 - x_2)}{k^{23}}\right]} = \frac{Overall\ temperature\ difference}{Sun\ of\ conduction\ resistance}$$

So, the heat flux through this series of three layers can be expressed as the overall temperature difference divided by some of the conduction resistances. And this also very quiet popular representation of heat flux or any flux expressed as ratio of the driving force by the resistance.

Example: (Refer Slide Time: 12:43)



Let us take a numerical example to illustrate the theory which are discussed now for heat conduction of furnace wall. Let us read the example the furnace wall consists of three layers, the inner layer of 10 cm thickness is made of firebrick is thermal conductivities 1 W/m. K.

The intermediate layer of 20 cm thickness is made of masonry brick, the thermal conductivity is 0.8 W/m. K, followed by a 5 cm thick concrete wall whose thermal conductivity is 1.25 W/m. K. As I told you there are multiple layers, three layers here each is made of different a material they have their own thermal conductivity and of course, different thickness as well.

And, the figure shown in the above slide image is only a qualitative representation of this furnace wall. When the furnace is in continuous operation the inner surface of the furnace is at 1000 ^{0}C, while the outer concrete surface is at 100 ^{0}C. Calculate the rate of heat loss per unit area of the wall, the temperature of the interference of the fire brick and masonry brick that is the first two layers.

And the temperature at the interface of the masonry brick and concrete that is an interface between the second and third layers. And we are also asked to draw the temperature profile across the furnace wall,

Solution:

Now, let us find out the numerical values we have discussed the theory.

$$q_0=\frac{T_0-T_3}{\left[\frac{(x_1-x_0)}{k^{01}}+\frac{(x_2-x_1)}{k^{12}}+\frac{(x_3-x_2)}{k^{23}}\right]}$$

So, we have seen this expression for the heat flux expressed in terms of the temperature at the inner surface and outer surface and in terms of thickness of each material and thermal conductivity of each material.

So, it is just a matter of simple substitution

$$T_0=10000\,^0C\,;T_3=100\,^0C$$

$$x_1-x_0=0.1\,m\,;x_2-x_1=0.2\,m\,;x_3-x_2=0.05m$$

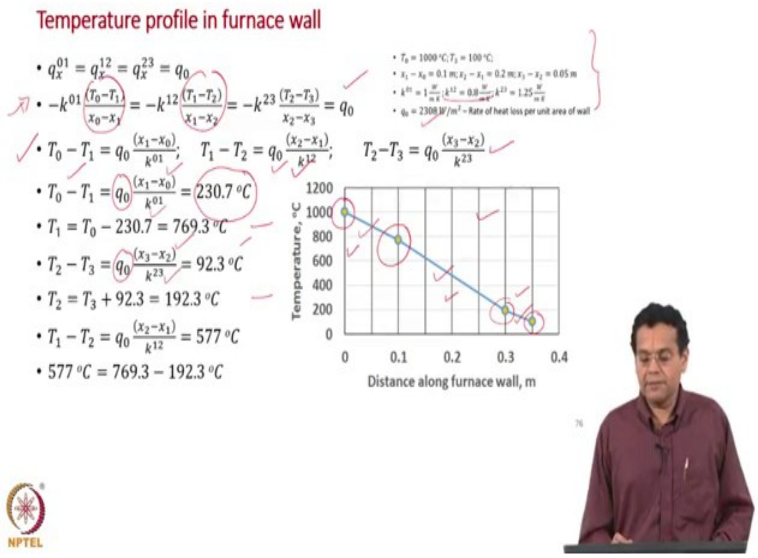
$$k^{01}=1\frac{W}{m\,K}\,;k^{12}=0.8\frac{W}{m\,K}\,;k^{23}=1.25\frac{W}{m\,K}$$

So, simple substitution will give us

$$q_0=2308\frac{W}{m^2}$$

That is a rate of heat loss per unit area of wall. So, when we say area we should take this area that is the area of heat transfer that is the area that is about 2300 W/m^2. Now, so that answers part a we will have to find out the interface temperatures let us do that.

(Refer Slide Time: 15:33)



The values from the previous slide are shown here for convenience; now let us write down the expressions which we discussed in the theory behind this example.

$$q_x^{01} = q_x^{12} = q_x^{23} = q_0$$

$$-k^{01}\frac{(T_0 - T_1)}{x_0 - x_1} = -k^{12}\frac{(T_1 - T_2)}{x_1 - x_2} = -k^{23}\frac{(T_2 - T_3)}{x_2 - x_3} = q_0$$

That is an expression for the equality of heat flux through all the 3 layers and now this equation expresses that in terms of the temperature gradients.

$$T_0 - T_1 = q_0\frac{(x_1 - x_0)}{k^{01}}; T_1 - T_2 = q_0\frac{(x_2 - x_1)}{k^{12}}; T_2 - T_3 = q_0\frac{(x_3 - x_2)}{k^{23}}$$

And, this equation are written for the temperature difference and we need these equations, because we need to find out the interfaced temperatures and that is why these equations are shown here again. So, let us take the first equation, so

$$T_0 - T_1 = q_0\frac{(x_1 - x_0)}{k^{01}}$$

And, we know the value of q_0, we have just now determined and we know the thickness of this first layer. We know it is thermal conductivity if you substitute you get

$$T_0 - T_1 = 230.7^0 C$$

So, that is a temperature difference across the first slab and we know T_0, so we will evaluate T_1

$$T_1 = T_0 - 230.7 = 769.3^0 C$$

Now, let us use the third equation the equation for the third layer,

$$T_2 - T_3 = q_0 \frac{(x_3 - x_2)}{k^{23}}$$

And we know thickness, we know the thermal conductivity, we have found out q_0 in the previous slide if you substitute you get

$$T_2 - T_3 = 92.3^0 C$$

And we know T_3, so

$$T_2 = T_3 + 92.3 = 192.3^0 C$$

And the second equation for the second layer is not required, but we can just use it for crosschecking let us do that let us use this equation.

$$T_1 - T_2 = q_0 \frac{(x_2 - x_1)}{k^{12}}$$

If you substitute the values you will get

$$T_1 - T_2 = 577^0 C$$

And if you find out the difference between T_1 and T_2 it is in fact, 577 degrees centigrade it is only a crosscheck. And, the temperature profile is shown in the above slide image within each slab the profile is linear and we also found out the interface temperatures, that is why we are able to plot the temperature profile.

But, within each law it is just linear that is because we have assumed thermal conductivity to be constant. If thermal conductivity depends on temperature this profile would not be linear and what about the slope if you look at these equations, those slopes are inversely proportional to the thermal conductivity q_0 is a constant.

So, the slopes are inversely proportional to the thermal conductivity, so the slope of this temperature profile in the second slab is higher than that in the first slab. And, the slope of this temperature profile in the third slab is lower than the slope of temperature profile in the first slab. Because, second slab the thermal conductivity is lower than that of the first slab, so slope is higher. And for the third slab thermal conductivity is higher, so slope is lesser in comparison to the first slab.