**Continuum Mechanics and Transport Phenomena**
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**Lecture – 114**
**Heat Conduction in Slab**

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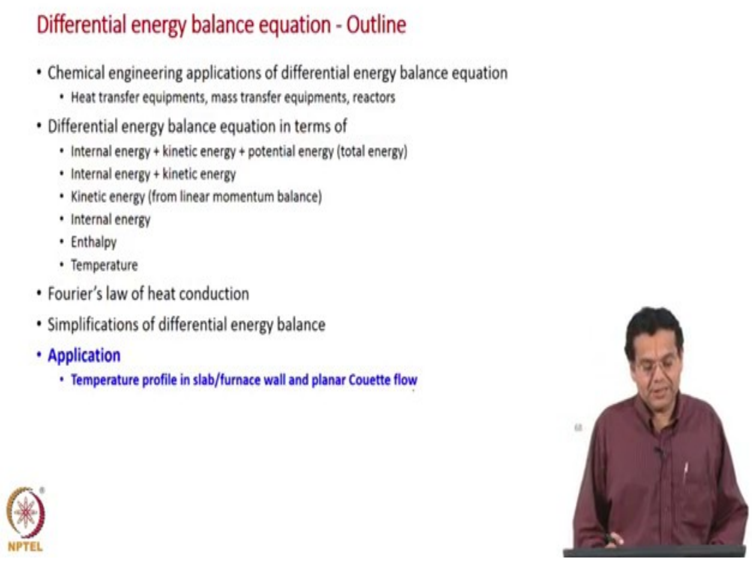


We have derived the differential form of energy balance equation starting from total energy and expressed in terms of temperature. Also closed it using the Fourier's law of heat conduction simplified it, and now it is time to look at the applications. Let us look at simple applications, of course, the main application is to find out the temperature profile we are going to find out the temperature profile in a slab and then extent that to a furnace wall and then in a planar Couette flow.

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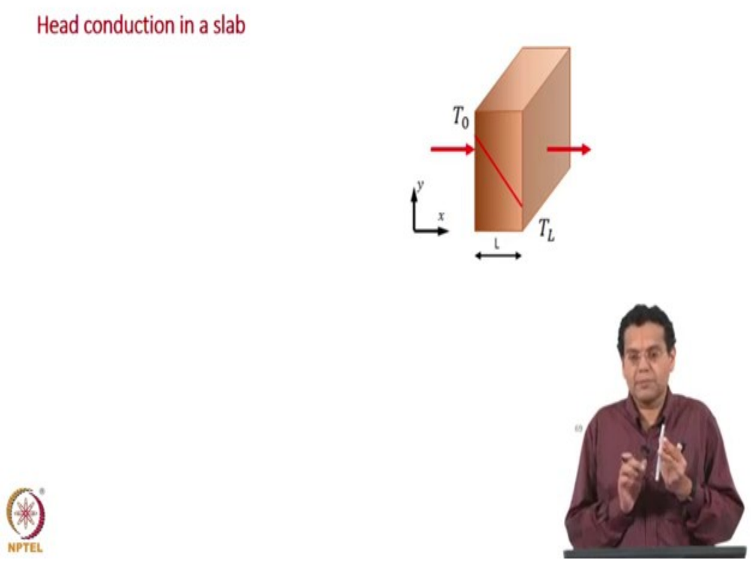


So, let us start with the first application which is heat conduction in a slab, what we have is a slab like and, then the two surfaces are maintained at two different temperatures, let say $T_0$ and then $T_L$. The thickness of this slab is L and what is that we are interested in, what is the temperature profile across the thickness of this slab, how temperature varies with the thickness of the slab.

We will consider the case where the left hand side surface is at higher temperature, and the right hand side surface at a lower temperature. And, so the heat flows from higher temperature to lower temperature along the positive x axis and takes place by conduction.

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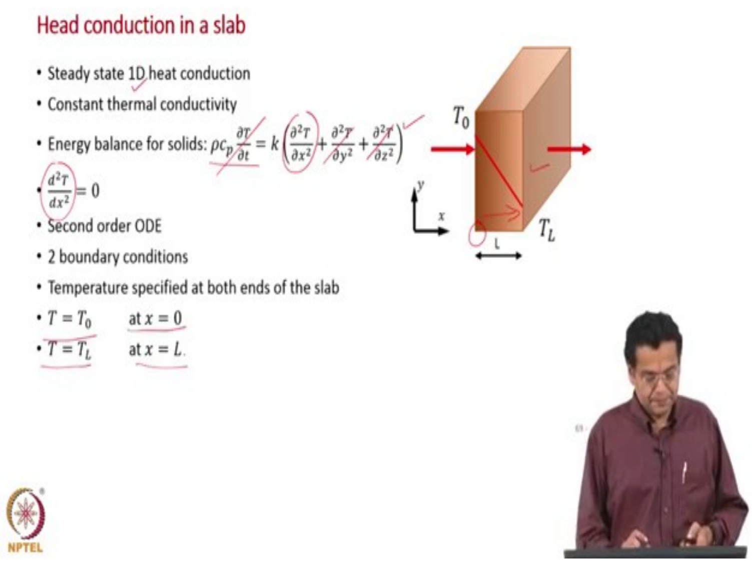


We will consider steady state condition and we are going to consider only one dimensional heat transfer and that is why it says, steady state one dimensional heat conduction. So, we will start with an assumption that the thermal conductivity is a constant meaning that it does not vary with temperature.

Now, we have seen different simplifications of the energy balance equation, so now we should take the form which is applicable for solids, because we considering a solid slab material.

$$\rho c_p \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

So the energy balance for solids was shown to be this form. Now, this gets further simplified because it is steady state, so the left hand side becomes 0, because it is one dimensional these two terms become 0. And, we are left only with the temperature variation along the x direction.

$$\frac{d^2 T}{d x^2} = 0$$

This is second order in differential equation. We require two boundary conditions to solve this second order ODE and the temperatures are specified both ends of this slab and those are the boundary conditions at $x=0$, the left hand the left surface of this slab.

So, the temperature is

$$T = T_0 \, at \, x = 0,$$

$$T = T_L \, at \, x = L$$

So those serve as the boundary conditions.

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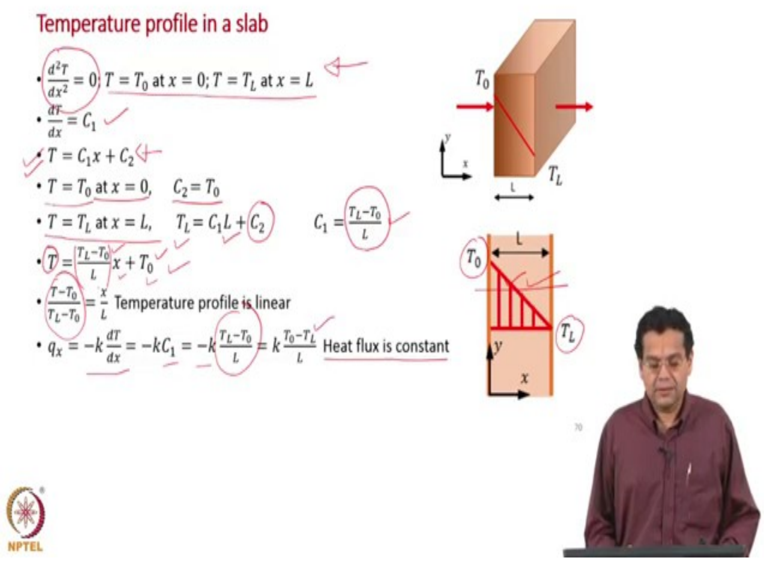


So, let us summarize them here

$$\frac{d^2T}{dx^2} = 0\,;\, T = T_0 \, at \, x = 0\,;\, T = T_L \, at \, x = L$$

The energy balance has finally, simplified to just a simple second order ordinary differential equation with the right hand side 0, does not even look like energy balance now. But, we know the physics behind it we know how we arrived at that particular equation, and these are the boundary conditions which we have discussed.

So, we integrate once it becomes

$$\frac{dT}{dx} = C_1$$

Integrate once again we get

$$T = C_1 x + C_2$$

Now, we need to evaluate the two constants, we will use both the boundary conditions at $T = T_0 \, at \, x = 0$. So, if we substitute in this equation we get

$$C_2 = T_0$$

Now, let us use the second boundary condition $T = T_L \, at \, x = L$.

$$T_L = C_1 L + C_2$$

So, if you rearrange we get the constant $C_1$ as

$$C_1 = \frac{T_L - T_0}{L}$$

So let us substitute both the constants in the equation. And, we get

$$T = \left( \frac{T_L - T_0}{L} \right) x + T_0$$

This gives the temperature profile across the thickness of this slab at x = 0 we have $T_0$. And, x = L we get $T_L$ and that is a temperature profile shown in the figure.

We can also express this in dimension less form, so

$$\frac{T - T_0}{T_L - T_0} = \frac{x}{L}$$

So, we can express the temperature profile in this dimensionless form as well, unless evaluate the heat flux heat flux is given by

$$q_x = -k \frac{dT}{dx} = -k C_1 = -k \frac{T_L - T_0}{L} = k \frac{T_0 - T_L}{L}$$

What do we observe? The heat flux is a constant. Because L is a constant, the end temperatures are constant; we assume the thermal conductivity to be constant. So, all variables here are constant the heat flux is just a constant; it does not vary throughout the thickness of this slab you have only one heat flux value.

We should also know that in the previous slide we have discussed all the physics and what you have discussed in this slide is just pure maths. At this stage the physics ends and then the maths starts, so that should also be kept in mind. So, that we clearly understand what is the physics of the problem, what is the maths of the problem.

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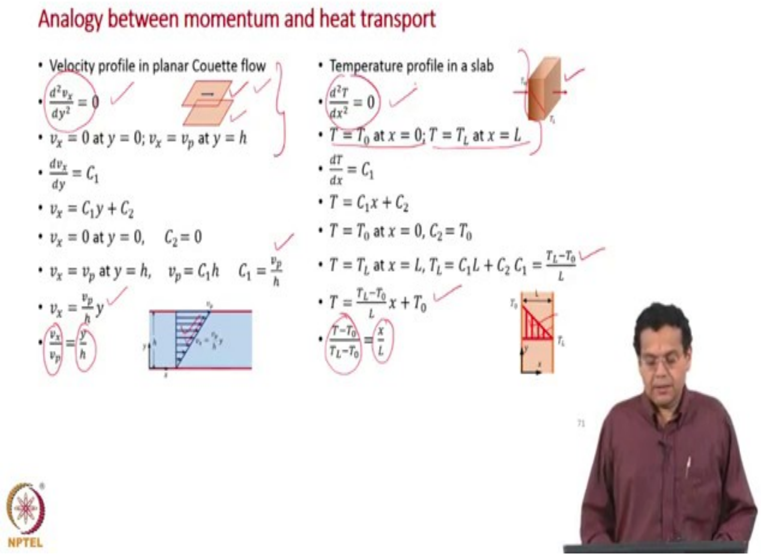


If we carefully recall our discussion on the previous two slides many steps many words also I would say would be same as what we discussed earlier for the case of planar Couette flow. So, let us now discuss the analogy between momentum and heat transport by using the two examples, one is this planar Couette flow. And then the other one is the present example of heat conduction in a slab.

Let us see how they are very much analogous, so left hand side we have velocity profile in a planar Couette flow, the flow between two parallel plates the bottom plate fixed, the top plate set in motion. Right hand side we have temperature profile in a slab we have a slab the left and right surfaces are maintained in two different temperatures

So, the plates we have two different velocities 0 and then velocity of the plate, and in the slab we have two different temperatures. What is the governing equation? The governing equation just becomes

$$\frac{d^2 v_x}{d y^2} = 0$$

And, the governing question in heat transfer is

$$\frac{d^2 T}{d x^2}=0$$

The boundary conditions for the case of planar Couette flow are

$$v_x=0\, at\ y=0 ; v_x=v_p\, at\ y=h$$

For slab

$$T=T_0\, at\, x=0 ; T=T_L\, at\, x=L$$

Of course, the direction is different first it is along the y axis here is along the x axis.

Now, how do we solve, so the physics of the problem ends here, the governing equation and the boundary conditions. Now, we start solving them

For Couette flow we integrate once we get

$$\frac{d v_x}{dy}=C_1$$

Then for slab we integrate once we get

$$\frac{dT}{dx}=C_1$$

Integrate again we get

$$v_x=C_1 y+C_2$$

$$T=C_1 x+C_2$$

We evaluate the two constants we use the boundary condition; the first boundary condition at the bottom plate and find

$$C_2=0$$

And for slab once again we use a boundary condition at the left surface and find out the constant

$$C_2 = T_0$$

Next we use the second boundary condition which is velocity of the top plate and find the constant as

$$C_1 = \frac{v_p}{h}$$

Similarly for slab, we use the boundary condition in terms of temperature at the right surface and find out the constant

$$C_1 = \frac{T_L - T_0}{L}$$

Substitute the constants and then get the expression for the velocity profile,

$$v_x = \frac{v_p}{h} y$$

And, substitute the constants get the expression for the temperature profile.

$$T = \left( \frac{T_L - T_0}{L} \right) x + T_0$$

At this stage they may look different, but moment we express in terms of dimensionless form they become very much analogous;

$$\frac{v_x}{v_p} = \frac{y}{h}$$

The dimensionless velocity for the Couette flow,

$$\frac{T - T_0}{T_L - T_0} = \frac{x}{L}$$

For slab again dimensionless temperature, both the profiles are linear.

We use the arrow mark to show velocity, but for temperature as a scalar, so we do not use an arrow mark. And, in both the cases we should know that the term which remained in the conservation equation was due to molecular transport. In the case of a fluid flow it was the molecular transport of momentum, and in the case of a slab it is the molecular transport of heat.

So, only those terms remained and the solution, procedure, the profile all are very much analogous, what is the reason we discussed the previous lecture that momentum and heat transport are mathematical analogous when it is one dimensional case. Because we are discussing one dimensional case both momentum and heat transport are mathematically analogous.