Continuum Mechanics and Transport Phenomena
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Lecture – 108
Differential Total Energy Balance
Part 2

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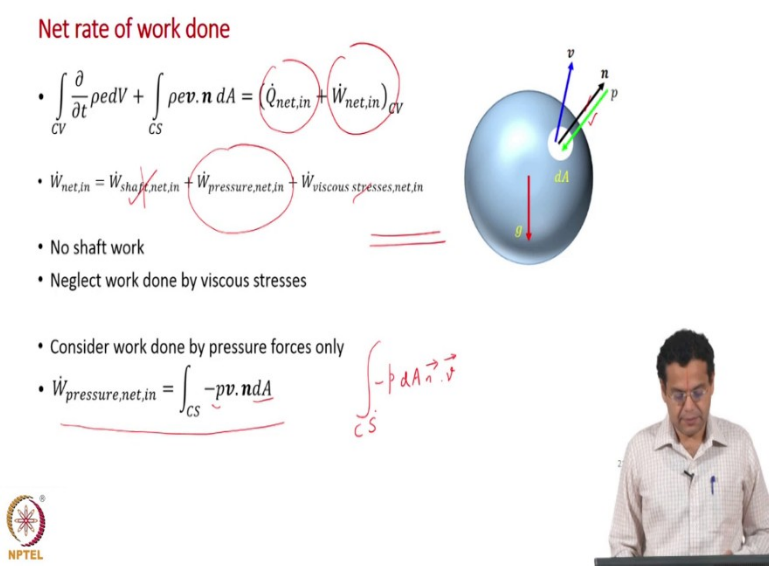

Let us focus on the net rate of work done, that is the integral energy balance equation,

$$\int_{CV} \frac{\partial}{\partial t}\rho e dV + \int_{CS} \rho e v . n\, dA = ¿¿$$

We applied the transient term, the convection term on the left hand side for the small control volume. And, we also discussed how to express the net rate of heat input term for the small control volume.

So, now we will focus on the net rate of work done term on the right hand side of the integral energy balance equation and let us see how do we apply that for a small control volume. So, if we recall our discussion on this net rate of work done when, we discuss integral energy balance equation. We split the work done in terms of shaft work and then work done by pressure, work done by viscous stresses of course all in terms of rate.

$$\dot{W}_{net,\in¿} = \dot{W}_{Shaft.net,\in¿} + \dot{W}_{Pressure,net,\in¿+\dot{W}_{Viscous\,stress,net,\in¿¿}}¿¿$$

Now, let us see what are their terms we are going to take forward, first our controlled volume, I am using this sometimes as a domain, sometimes as a control volume. Let us say this is our domain, our control volume is somewhere inside. So, on there is no shaft to there.

So, within the small control volume we do not have a shaft and so the shaft work is 0, $\dot{W}_{Shaft.net,\in ¿=0¿}$. Now, when we discussed the integral energy balance, we neglected the work done by the viscous stresses. Similarly, here also we are going to neglect the work done by viscous stresses, $\dot{W}_{Viscous\ stress,net,\in ¿=0¿}$.

Now is it justified, when is it significant. It is significant when, let us say you have a very viscous fluid flowing between let us say 2 parallel plates, the distance is very small and let us say the top plate is moving at a very high velocity which means, that the velocity great it will be very large So, in such conditions, the work done by viscous stresses will be significant.

So, the work done by viscous stresses is significant when, you have a very high velocity gradient and your fluid is viscous. And, most of the cases almost may most of the times, this is not significant and so we are going to neglect that. So, there is no shaft work and we are going to neglect the work done by the viscous stresses and we are going to consider the work done by the pressure forces only.

Though we had a split work into three terms, shaft work is not at all there, viscous work done by viscous stresses we are neglecting it, we are neglecting it. And, we are going to consider only the work done by pressure forces. Now, when we derived the integral form of energy balance equation, the rate of work done by pressure force was expressed by this integral expression

$$\dot{W}_{Pressure,net,\in ¿=\int_{CS} -pv.ndA¿}$$

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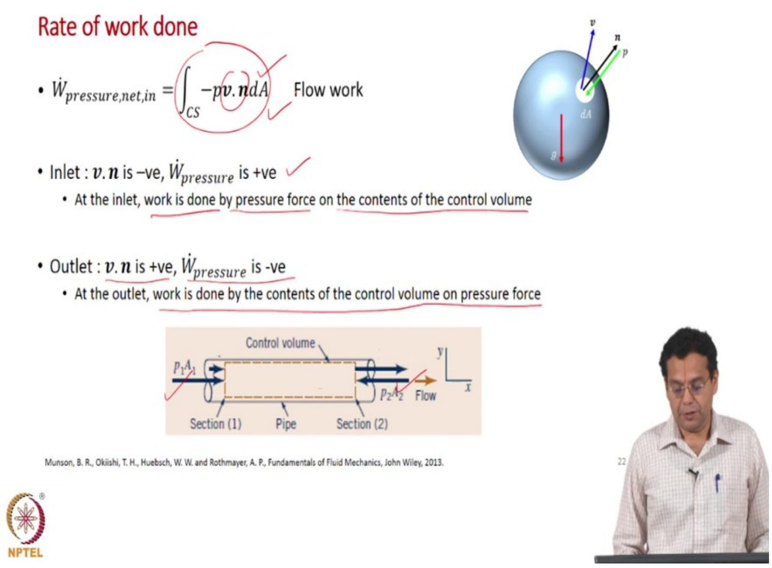

This is a recall slide, if we recall this it will be easier for us to understand the next few slides. What is that we are going to do to in the next slides? We are going to apply this expression for our small controlled volume in fact, for the control surfaces.

$$\dot{W}_{Pressure,net,in} = \int_{CS} -pv.ndA$$

What is that we discussed in this slide is that, this was discussed when we discuss the integral energy balance equation, we applied this equations for the inlet flow through a pipe and the outlet of flow through a pipe. At the inlet what happens? $v.n$ is negative. So, the integral term will be positive and work done is positive means that, work is done on the contents of the control volume by the pressure force.

So, positive or negative is with respect to the fluid. So, when we say work is positive, work is done on the fluid by the pressure, here written as work is done on the contents of the control volume which is a fluid by the pressure force. And, at the outlet $v.n$ is positive. And so, the rate of work done term is negative and what does it mean when, you say negative work is done by the fluid against the pressure force. And, this is what is written here is, work is done by the contents of the control volume on pressure force.

So, to quickly recall we applied this for flow through a pipe, at the inlet $v.n$ is negative, rate of work done is positive which is which means that work is done on the fluid. At the exit $v.n$ is positive, rate of work done is negative, which means the work is done by the fluid.

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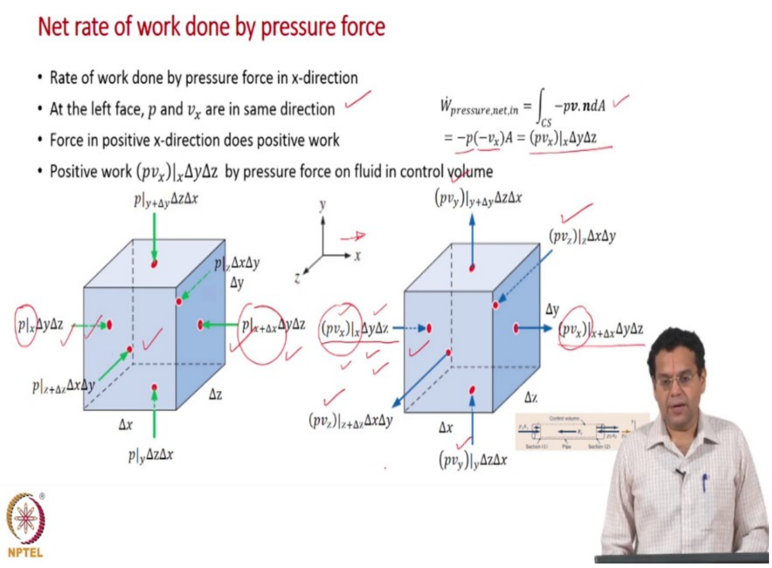

Now, before proceeding further let us understand the two control volumes shown here. The left hand side control volume, the force due to pressure is shown.

Pressure is a compressive force that is why all the arrow marks are facing towards each other. For example, on the left phase, the force is towards the positive x axis, on the right phase the force is towards the negative x axis. We have seen that several times.

So, the left hand side control volume is we are familiar with we have discussed that during the derivation of the differential momentum balance equation. Now, let us move on to the control volume on the right, what has been done is that all the pressure terms has been multiplied with the velocity in the respective directions.

Let us take an example, the left the face we have $(p v_x) \vee ¿_x \Delta y \Delta z ¿$, it has been multiplied by v_x. Similarly, on the right face we have $(p v_x) \vee ¿_{x+\Delta x} \Delta y \Delta z ¿$, we have multiplied by v_x and similarly in the y direction $(p v_y) \vee ¿_y \Delta x \Delta z ¿$, $(p v_y) \vee ¿_{y+\Delta y} \Delta x \Delta z ¿$ and then $(p v_z) \vee ¿_z \Delta y \Delta x ¿$, $(p v_z) \vee ¿_{z+\Delta z} \Delta y \Delta x ¿$. So, the pressure has been multiplied by the velocity in the respective directions.

Now, what does these terms mean. So, let us take one term once again, $(p v_x) \vee ¿_x \Delta y \Delta z ¿$, pressure is force per unit area, you are multiplying with the area over which it acts and hence you get the force multiplied by the velocity, we get the rate of work. So, that all the terms in the control volume in the right hand side, represent rate of work done, it could be either on the fluid or by the fluid. But, all of them represents rate of work done by pressure force.

So, quickly to recall, pressure is force per area multiplying by area you get force, you multiply by velocity, we get rate of work done. And, so all the terms represent rate of work done could be on the fluid or by the fluid and that is what we are going to discuss now. Now, let us take the x direction and write down expression for rate of work done by pressure force in the x direction.

Now, at the left face, the pressure or force due to pressure is along the positive x axis, velocity is also along the positive x axis, always v_x is along the positive x axis. So, the left face pressure is along positive x axis, velocity is also along positive x axis.

Now, when a force is along the positive x axis, it does positive work, what is positive work? Positive or negative work is with respect to the fluid. So, when we say positive work, work is done on the fluid. So, at the x face, positive work, $(p v_x) \vee ¿_x \Delta y \Delta z ¿$. That work is done by pressure force on the fluid in control volume, to be more formally it is rate of work done by pressure force on the fluid in the control volume.

Other quick way of explaining is that, we have force due to pressure along positive x axis, velocity also along positive x axis. And, then we know that rate of work is dot product of these two terms and because they are along the same direction, we get $p v_x$. So, all are equivalent ways of explaining this term. So, to summarize this slide when, you multiply p with the velocity the respective direction, you get the rate of work done. We have taken the left face and seen that it represents the rate of work done by the pressure force on the fluid in the control volume that is why we discussed this pipe. So, this inlet and this face are analogous.

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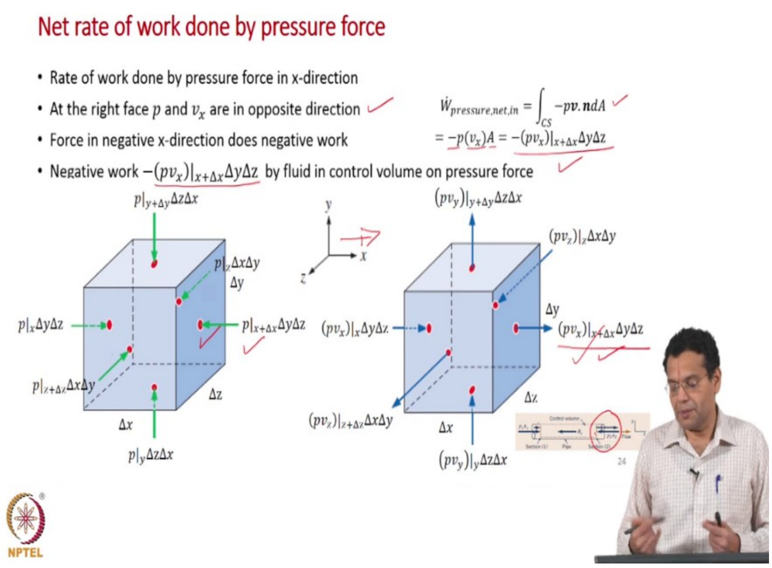

So, now let us go to the right hand side face and find out the rate of work done by pressure force in the x direction ok. Now, right hand side face what happens, the pressure is along negative x axis, velocity as usual is along the positive x axis.

Now, at the right face p and v_x are in opposite direction. So, pressure along negative x axis, velocity along positive x axis. Now, a force and negative x direction does negative work, force due to pressure, in the negative direction does negative work. And, so this term on the right hand side, $-(p v_x)|_{x+\Delta x}\Delta y\,\Delta z$, represents negative work, what is negative work? It is work done by the fluid on the pressure or we say work done by the fluid against the pressure.

So, if you want to take into account the sign than, it is with the minus otherwise $(p v_x)|_{x+\Delta x}\Delta y\,\Delta z$ represents the rate of work done by the fluid against the pressure force.

And, other ways of course, the force due to pressure is along negative x axis, velocity along positive x axis. So, if we take a dot product, you will get $-(p v_x)|_{x+\Delta x}\Delta y\,\Delta z$. So, the summary from this slide is the term here represents rate of work done by the fluid against the pressure. And, that is analogous to the outlet of the pipe that is why we discuss the flow through pipe before discussing these two slides.

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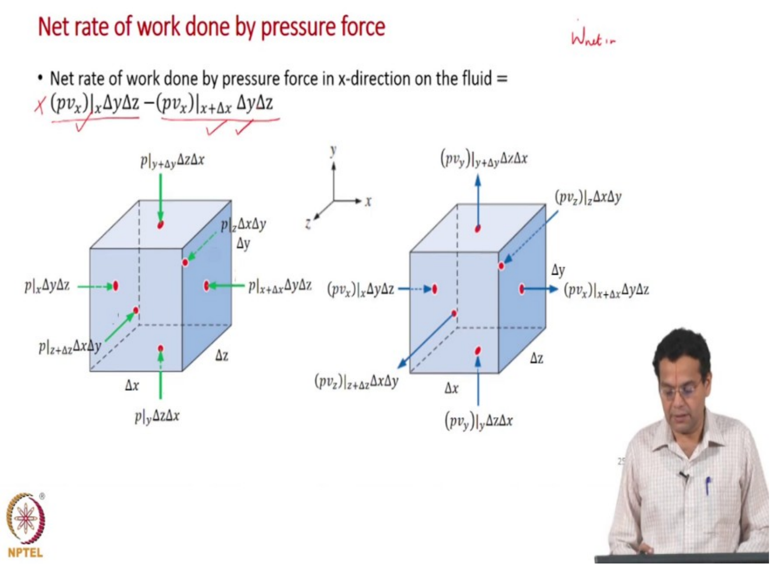

So, now let us find out that net rate of work done rate pressure force.

$$(p v_x) \vee ¿_x \Delta y \Delta z - (p v_x) \vee ¿_{x+\Delta x} \Delta y \Delta z ¿¿$$

Once again we have to be careful here. The right hand side in the integral energy balance equation, we are net rate of work done on the fluid and we said the first term represents work done on the fluid and the second term represents work done by the fluid.

This should also be kept in mind; this convention is in line with the rate of work done term on the right hand side. In the integral energy balance, we said $\dot{W}_{net,\in¿¿}$, what does it represent? Net rate of work done on the fluid that is why, we have on the fluid and then minus by the fluid that should be kept in mind.

This is similar to the net rate of heat input term, where we took in minus out. For the convection term it is out minus in. So, this difference has to be kept in mind, just to summarize for the convection term it is out minus in, for the heat term it is in minus out. For the work term it is work done on the fluid minus work done by the fluid. So, this has to be kept in mind, in line with that is how we started with the first law of thermodynamics.

So, that should be consistently maintained throughout all the derivations. So, the net rate of work done by pressure force in the x direction of course, we are discussing only the x direction.

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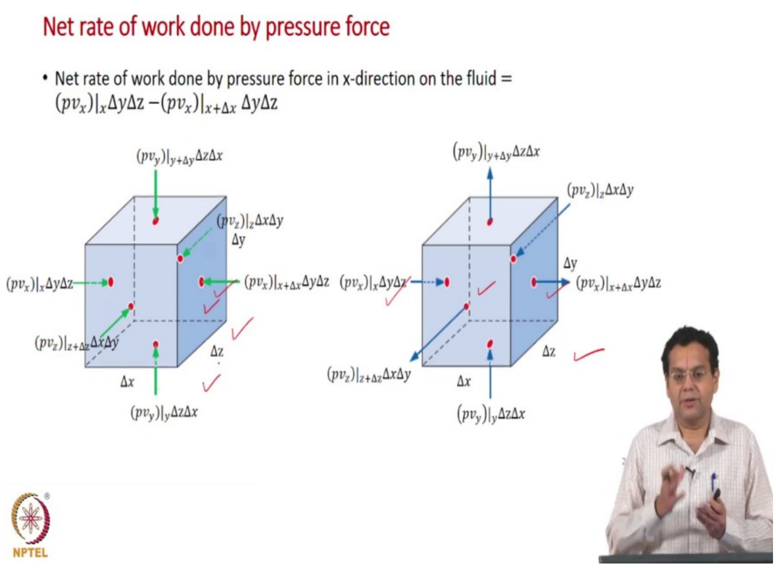

Now, like to discuss this slide, what is shown in this slide are two different representations of the rate of work done terms. Usually a control volume showing rate of work done is not available in most of the books, it is represented in few books. But, the two different representations are followed in the books. So, what you are going to discuss is nothing conceptually new but, we have followed one way of representing, the rate of work done terms, we are discussing two representations.

The right hand side is what we have used which we are discussed so far in the previous slide. How was this shown see, rate of work done on the fluid is shown as an arrow mark entering in. So, that represents work done on the fluid. The right hand side work done by the fluid is shown by arrow mark leaving so, similar in other directions. So, left hand side entering arrow marks shows that work is done on the fluid; right hand side arrow mark shows work done by the fluid. So, it is shown as if something entering, something leaving and similarly other directions so.

And we also interpreted this similar to the convection term, something entering, something leaving, flow entering, and flow leaving. Similarly, we also interpreted the heat input term, heat entering and leaving, this is the convention which we have followed. Now, there is other way of representing the rate of work done by pressure force and that is what is shown in the left hand side. The way in which this is shown is, look at this the terms are the same, p multiplied by velocity in the respective directions.

But, look at the arrow marks here, they are drawn same as the pressure force itself, the two arrow marks are towards each other, in the respective directions. What we have used is, arrow mark entering and leaving, left hand side convention is that, the rate of work done terms are shown as two arrow marks towards each other. How do we interpret this?

Now, this we should interpret this as something similar to force, if you have arrow mark in the positive direction that is positive, positive work which means work is done on the fluid. If you have arrow mark in the negative direction that is negative work, work is done by the fluid. So, when we draw a control volume with rate of work done terms like in the left hand side then, the interpretation is like a force.

Right hand side, if you draw then the interpretation is like the convection terms. So, we are chosen the right hand side But, very few books which give this some books may show the left hand side representation. So, because it is not available in many books, I thought I will discuss this. So, that it becomes clear, how do you represent the rate of work done, in terms of a control volume.

Of course, we can mathematically do it as well, without the control volume also taking dot product of first due to pressure with velocity. That is what essentially we have done.

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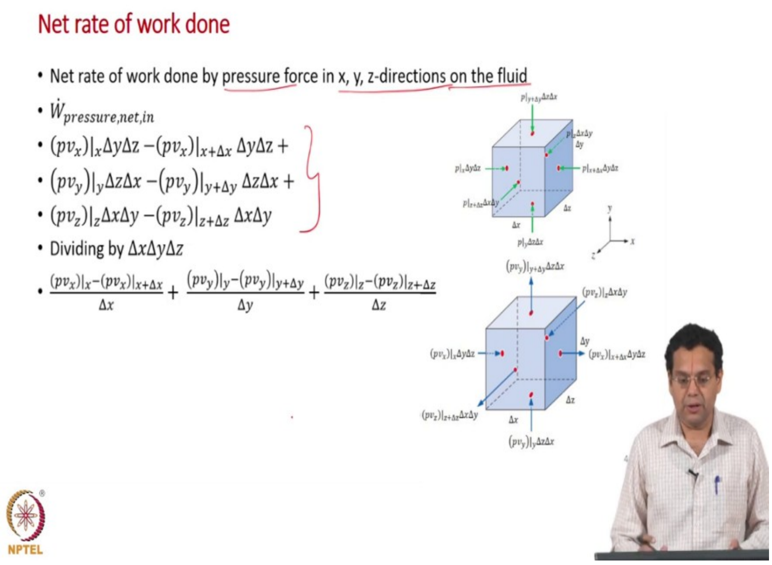

So, let us repeat it for all the 3 directions so, net rate of work done by pressure force on the fluid in x y z directions, just repeat it for all the three directions. And, then divide by $\Delta x \Delta y \Delta z$. This is what we have discussed in the previous slides.

$$(p v_x) \vee ¿_x \Delta y \Delta z - (p v_x) \vee ¿_{x+\Delta x} \Delta y \Delta z ¿¿$$

Similarly y direction,

$$(p v_y) \vee ¿_y \Delta x \Delta z - (p v_y) \vee ¿_{y+\Delta y} \Delta x \Delta z ¿¿$$

Similarly z direction,

$$(p v_z) \vee ¿_z \Delta y \Delta x - (p v_z) \vee ¿_{z+\Delta z} \Delta y \Delta x ¿¿$$

Now, let us divide by $\Delta x \Delta y \Delta z$ and keep it ready for substitution.

$$(p v_x) \vee ¿_x - \frac{(p v_x) \vee ¿_{x+\Delta x}}{\Delta x} + (p v_y) \vee ¿_y - \frac{(p v_y) \vee ¿_{y+\Delta y}}{\Delta y} + (p v_z) \vee ¿_z - \frac{(p v_z) \vee ¿_{z+\Delta z}}{\Delta z} ¿¿¿¿¿¿$$

And, at this stage they represent a rate of work done, net rate of work done on the fluid and moment you divide by $\Delta x \Delta y \Delta z$. They represent net rate of work done on the fluid per unit volume, taking care of all the three directions.

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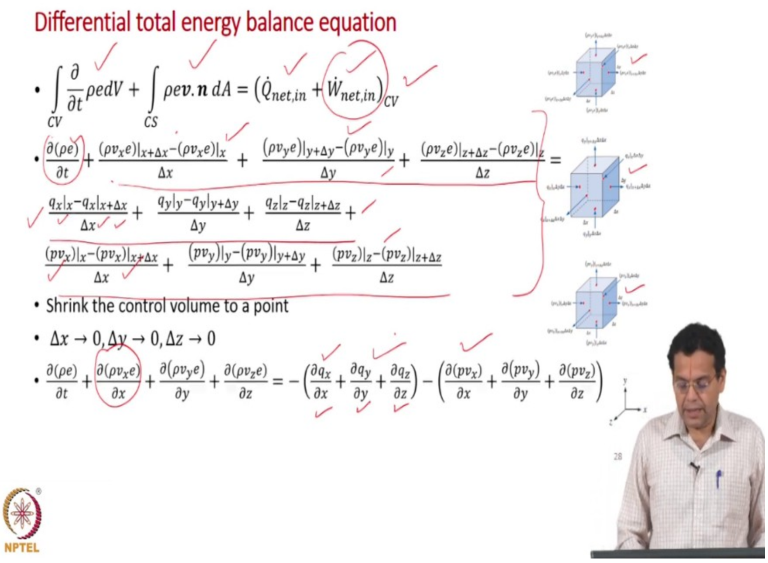

So, now it is time to put them all together, all the control volumes are shown. The first control volume is for convection and of course, for the transient term also. The second control volume is for the heat input term and third control volume is for the work done term. Of course, rate of heat input, rate of work done.

$$\int_{CV} \frac{\partial}{\partial t} \rho e dV + \int_{CS} \rho e v . n\, dA = \text{¿¿}$$

This is the integral form of energy balance equation; we have got expressions for these 4 terms separately. Now, we are going to put them all together. So, remember after dividing by $\Delta x \Delta y \Delta z$. We have already divided by $\Delta x \Delta y \Delta z$. So, the terms which are substituting here are after dividing by $\Delta x \Delta y \Delta z$.

$$\frac{\partial(\rho e)}{\partial t} + (\rho v_x e) \vee \text{¿}_{x+\Delta x} - \frac{(\rho v_x e) \vee \text{¿}_x}{\Delta x} + (\rho v_y e) \vee \text{¿}_{y+\Delta y} - \frac{(\rho v_y e) \vee \text{¿}_y}{\Delta y} + (\rho v_z e) \vee \text{¿}_{z+\Delta z} - \frac{(\rho v_z e) \vee \text{¿}_z}{\Delta z} = q_x \vee \text{¿}_x - \frac{q_x \vee \text{¿}_x}{\Delta x}$$

Now, we shrink the control volume to a point. So, as we have discussed earlier at this stage, all of them represent average values. When, we write this term rho and e are some average value within the control volume when, we write all these 3 terms, they represent average values over the faces. When, we shrink all of them becomes point values so, $\Delta x \rightarrow 0, \Delta y \rightarrow 0, \Delta z \rightarrow 0$. So, now they become instantaneous values and point values as well. So,

$$\frac{\partial(\rho e)}{\partial t} + \frac{\partial(\rho v_x e)}{\partial x} + \frac{\partial(\rho v_y e)}{\partial y} + \frac{\partial(\rho v_z e)}{\partial z} = -\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_x}{\partial y} + \frac{\partial q_x}{\partial z}\right) - \left(\frac{\partial(p v_x)}{\partial x} + \frac{\partial(p v_y)}{\partial y} + \frac{\partial(p v_z)}{\partial z}\right)$$

So, left hand side we have positive derivatives, right hand side we have negative derivatives. This has to be kept in mind and we have got the differential form of energy balance equation, in terms of total energy. As we said the energy balance derivation goes to the series of stages, what differs is the variable for which we write the energy balance for.

So, now we have derived the energy balance equation in terms of total energy, which means left hand side we have total energy because, right hand said we have the heat input terms and then the work done terms.

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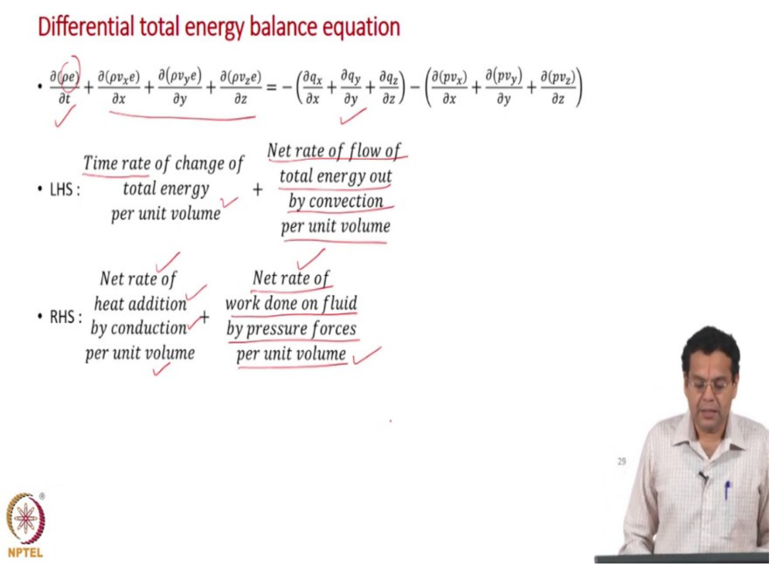

$$\frac{\partial(\rho e)}{\partial t}+\frac{\partial(\rho v_x e)}{\partial x}+\frac{\partial(\rho v_y e)}{\partial y}+\frac{\partial(\rho v_z e)}{\partial z}=-\left(\frac{\partial q_x}{\partial x}+\frac{\partial q_x}{\partial y}+\frac{\partial q_x}{\partial z}\right)-\left(\frac{\partial(p v_x)}{\partial x}+\frac{\partial(p v_y)}{\partial y}+\frac{\partial(p v_z)}{\partial z}\right)$$

Now, let us look at the significance of each term formally, of course, we know significance you have been using it, I would not say I have used the formal terms every time, for the purpose of brevity. But, now it is time to put them very formally. Once you know the significance we can easily understand the terms as well.

Left hand side, first term represents time rate of change of total energy per unit volume. We have been using very formally time rate of change of total energy per unit volume.

Of course, we know that in a differential balance equation all terms are per unit volume basis, what does the next set of three terms put together represent? Net rate of flow of total energy out by convection per unit volume; so, tell us about the flow of total energy and because energy can flow in and out, it represents net rate of flow of total energy out. What is the mechanism, by conviction and it is per unit volume basis.

Right hand side, first term, we have net rate of heat input or heat addition by conduction per unit volume. That is a correct way of representing this term, it is represents heat input or heat addition, what is the mode by conduction, it is net rate because, and it could be heat additional removal. And, so net rate of heat addition by conduction of course, per unit volume

And the last term represent, net rate of work done on the fluid by pressure forces per unit volume. So, it represents work done is it on the fluid by pressure forces and represents rate of work done and it is net rate of course, per unit volume.

So, every word there has a meaning, rate of work done it is net rate, it is on the fluid by pressure forces, per unit volume. So, I would say a very precise way of writing the significance of the different terms in the differential total energy balance equation.

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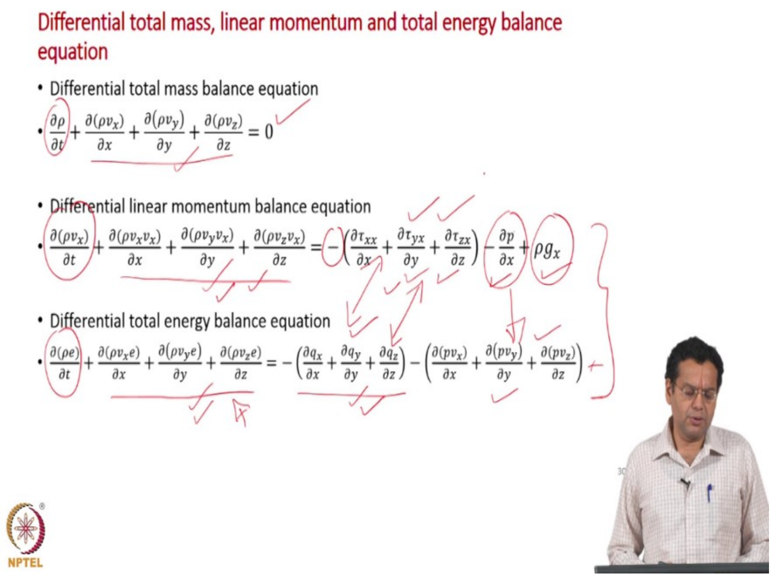

So, now it is time to quickly compare the three differential balance equations which I derived. So, let us list down them first, the differential total mass balance equation,

$$\frac{\partial(\rho)}{\partial t}+\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}+\frac{\partial(\rho v_z)}{\partial z}=0$$

The differential linear momentum balance equation

$$\frac{\partial(\rho v_x)}{\partial t}+\frac{\partial(\rho v_x v_x)}{\partial x}+\frac{\partial(\rho v_y v_x)}{\partial y}+\frac{\partial(\rho v_z v_x)}{\partial z}=\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}\right)-\frac{\partial p}{\partial x}+\rho g_x$$

And the equation which is just know derived namely, the differential total energy balance equation.

$$\frac{\partial(\rho e)}{\partial t}+\frac{\partial(\rho v_x e)}{\partial x}+\frac{\partial(\rho v_y e)}{\partial y}+\frac{\partial(\rho v_z e)}{\partial z}=-\left(\frac{\partial q_x}{\partial x}+\frac{\partial q_x}{\partial y}+\frac{\partial q_x}{\partial z}\right)-\left(\frac{\partial(p v_x)}{\partial x}+\frac{\partial(p v_y)}{\partial y}+\frac{\partial(p v_z)}{\partial z}\right)$$

Let us quickly compare them, the left hand side we have the transient term, which represents time rate of change of be it mass momentum or energy. When you say momentum, it is x momentum, when you say energy it is total energy.

Then, we have the transport of property by convection that is represented by the three terms in the left hand side. What is transported, it could be mass x momentum or energy. So, one is a transient term, other is that conviction in short.

Right hand side of course, for a mass balance it is 0. For momentum balance I have used the transfer phenomena convention or the moment and transport convention that is why we have a negative sign here. If you are following a fluid mechanics convention it would have been positive, which means that τ_{xx}, τ_{yx} etcetera represent components of the molecular momentum flux tensor. And, it represents transport of momentum by molecular transport. Here the left hand side represents transport of momentum by convection, right hand side we are transporting momentum by molecular mechanism. And, we look at the energy balance once again this represents the transport of energy by molecular mechanism, which is nothing but heat.

So, in momentum balance, it represents transport of momentum by molecular mechanism, the corresponding term, the analogous term the energy balance is transport of energy by molecular mechanism. So, left hand side we have convection, right hand side we have molecular transport. Similarly, in the energy balance also left hand side convection, right hand side molecular transport. Now, let us look at the other terms of course, we have the surface force due to pressure and then body force due to gravity in the right hand side of linear momentum balance equation.

And, then in the energy balance equation, we have the rate of work done term of course, that represents on the fluid. Also like to mention when you compare these two equations, these two terms are analogous to each other, they are represent transport by molecular mechanism. Now, the work done by pressure is represented by this term, coming to gravity the work done by gravity is on the left hand side, we will discuss that later. It is including e as potential energy, we will discuss that in detail later.

What is missing on the right hand side is the work done by viscous stresses because, we have neglected it. So, like to repeat it again, the first term in right hand side are analogous, work done by pressure is here, work done by gravity forces include as potential energy on the left hand side, which will discuss on later. We do not have one term corresponding to work done by viscous stresses, if you interpret that as viscous stresses and that is not there because, we have neglected, rate of work done by viscous stresses. So, that is how we should understand this.

A more formal derivation if you look at a typical transport phenomenon book, will also include rate of work done by viscous stresses to our level, we are neglecting it. And, it is not a major limitation because, the differential energy balance equation without the viscous stresses, it is applicable to almost I would say majority of the cases.