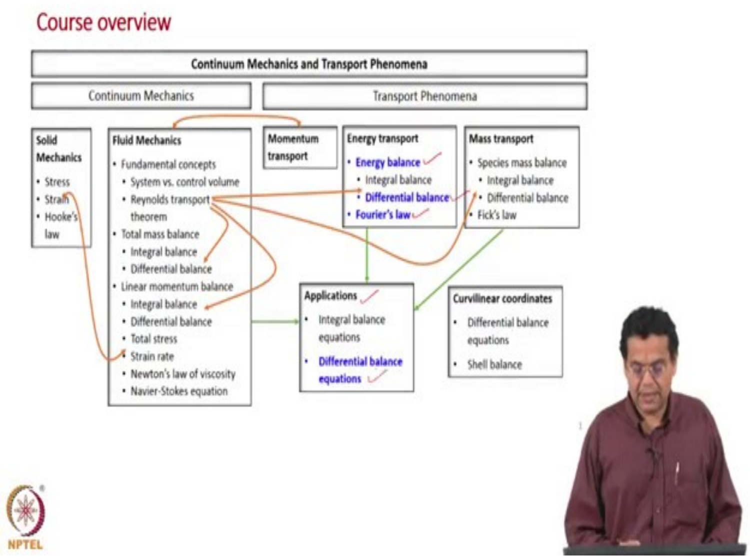
Continuum Mechanics and Transport Phenomena
Prof. T. Renganathan
Department of Chemical Engineering
Indian Institute of Technology, Madras

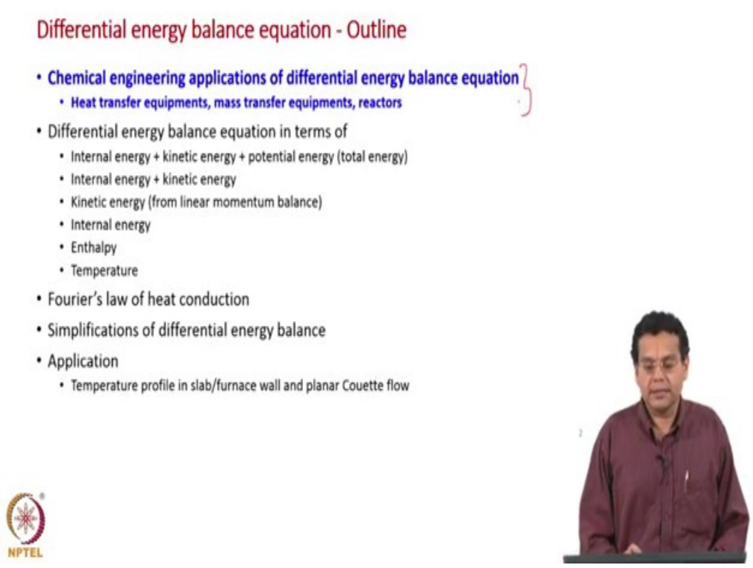
Lecture - 106
Differential Energy Balance:
Introduction

(Refer Slide Time: 00:15)



We are in the energy transport part of transfer phenomena, we have derived the integral energy balance equation, starting from the first law of thermodynamics, and looked at applications. Now, we will derive the differential form of the energy balance equation and also discuss Fourier's law of heat conduction and look at applications of the differential energy balance equation. And, those are the terms that are highlighted here, energy balance within that differential balance and Fourier's law and then, in applications differential balance equations.

(Refer Slide Time: 00:59)

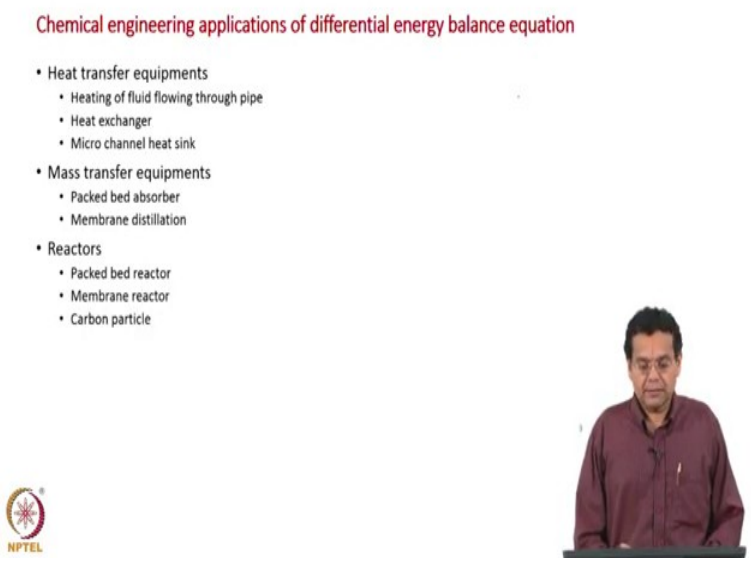


So, what is the outline for the differential energy balance equation, we will start with the chemical engineering applications of the differential energy balance equation, as applied to heat transfer equipments, mass transfer equipments, and reactors. Then, you start deriving the differential energy balance equation; it goes through a series of stages. First, we derive in terms of the total energy that is internal, kinetic, and potential energy put together.

From that we get an equation in terms of internal and kinetic energy then, we derive a separate differential energy balance equation for kinetic energy starting from the linear momentum balance equation. And then, we can get an equation for the internal energy, and from the differential energy balance equation for internal energy, using thermodynamic relationship, we express in terms of enthalpy and then temperature.

We discussed Fourier's law of heat conduction; we also simplify the differential energy balance equation to forms which are usually used. And finally, look at the application of the differential energy balance equation. The main application is to find out the temperature profile. So, we find the temperature profile in a slab, extend that to a furnace wall and then we find temperature profile in the planar Couette flow. Now, we are going to start with the applications of the differential energy balance equation and that is why this bullet is highlighted.

(Refer Slide Time: 03:05)

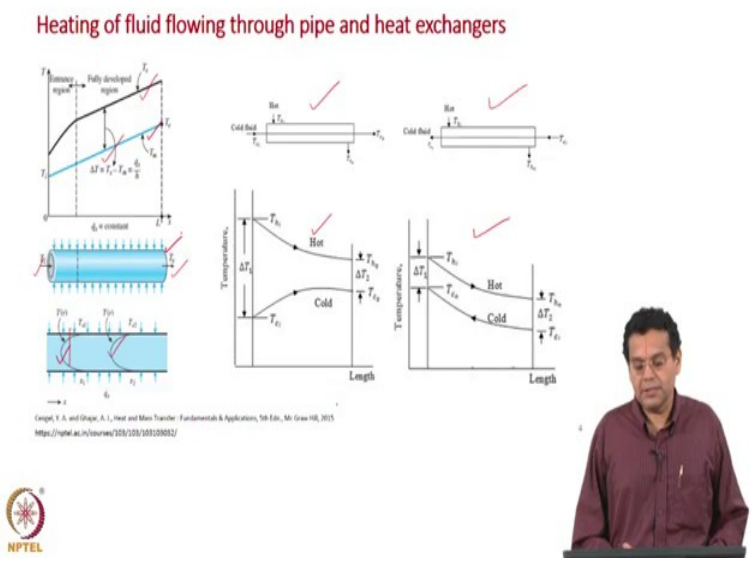


Before discussing differential linear momentum balance equation, we discussed the chemical engineering applications of the differential linear momentum balance equation along with the continuity equation. So, similarly here we will discuss the chemical engineering applications of differential energy balance equation before starting to derive that. So, that we know where is it apply and why do we need a differential energy balance equation.

We are going to see the application with respect to heat transfer equipments, mass transfer equipments, and reactors. Under heat transfer equipments, we are going to look at the heating of fluid flowing through a pipe, heat exchanger and then microchannel heat sink. Under mass transfer equipments, we will discuss application of and the energy balance to a packed bed absorber and then membrane distillation.

We should also know that the main application of that energy balance equation, in the chemical engineering literature, is for reactors, if you search let us say differential energy balance equation, many of the papers which you come across will be the application for reactors. So, we will take up a packed bed reactor, a membrane reactor, and then a carbon particle.

(Refer Slide Time: 04:51)



So, let us start with the heat transfer equipments, the first application is shown here (left hand side image). We have a pipe through which water flows, we know the inlet temperature and we need to find out the exit temperature. And, this pipe wall is subject to a constant heat flux for example, you can wind with the heating tape, which will supply constant heat flux.

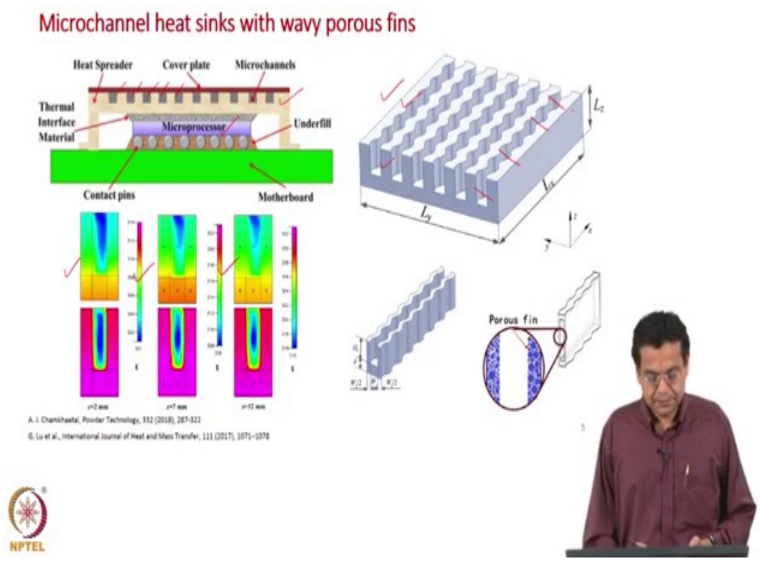
So, we can solve the energy balance equation to find out the exit temperature, what is shown, is the temperature profile along the radius of the pipe. So, how do we get? We solve the energy balance equation and get the radial temperature profile. And, then we can average this temperature and then also get the variation of the average temperature along the length of the pipe and that is what is shown here.

And, we can also get the actual variation of the temperature of the surface of the pipe and under these conditions the temperature difference between them will remain constant. And, so we can use energy balance equation to find out the exit temperature given the inlet temperature,

The next application is for heat exchanges, different configurations of heat exchanger are possible, where exchange of heat takes place between hot fluid and a cold fluid. Two simple configurations are shown in the above slide image (top left side figure) cocurrent heat exchanger and a counter current heat exchanger. In the cocurrent heat exchanger both the cold fluid and the hot fluid, enter at the left hand side and leave at the right hand side. In the counter current arrangement, the flow in opposite direction. Cold is flowing from right to left and hot is flowing from left to right.

Now, we can solve the energy balance equation to find out the temperature profile in these heat exchanges. What is the practical application, you can use the energy balance equation for the design of these heat exchangers, and also, you can predict their performance or use it for the design of these heat exchangers.

(Refer Slide Time: 07:35)



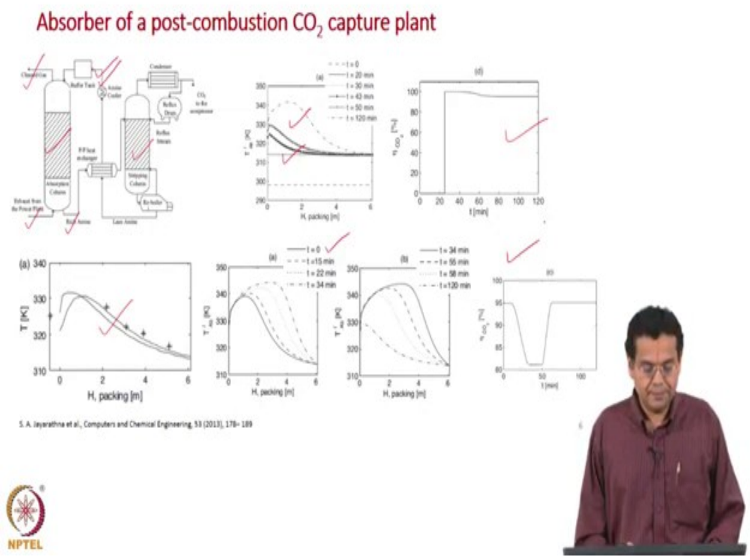
The next example is a micro channel heat sink, which is used to remove the heat generated in microelectronic equipments and that is what is shown in the above slide image, a micro-processor is shown, in which heat is generated. And, micro channel heat sink is shown and the channels through which a fluid is sent, it could be a liquid, it could be air or could be a nano fluid.

Which helps to remove the heat that is being generated, we already seen this example when we discussed applications for the linear momentum balance equation. The focus then was on the pressure drop and the velocity profile but, now our objective is on the heat transfer part, the study shown they have considered a micro channel heat sink of such a geometry, where the walls are wavy and the fin is porous.

The idea of choosing such a geometry is to enhance the rate of heat removal. Now, suppose we want to design a micro channel heat sink, which is very efficient, which means that it should remove heat very quickly. Then, that performance of the heat sink depends on the temperature profile in the heat sink and which can be obtained by solving the energy balance equation of course, you do have flow.

So, you solve along with the continuity and the Navier-Stokes equation and such temperature profiles are also shown in the above slide image at different lengths along the channel. So, you can use the energy balance equation to get the temperature profile and arrive at very efficient design of this micro channel heat sink.

(Refer Slide Time: 09:43)



Now, let us move on to mass transfer equipments, the first example is an absorber. What is the application? In power plants, we burn coal and generate a power but, the flue gas coming from the power plant has lot of CO_2 in it. So, it cannot be let out to the atmosphere, we will have to capture the CO_2 and then let out to the atmosphere. How do you capture is what is shown in the above slide image. So, the exhaust from the power plant is sent upwards through an absorption column and it is brought in contact with the amine solution for example, mono ethanolamine solution.

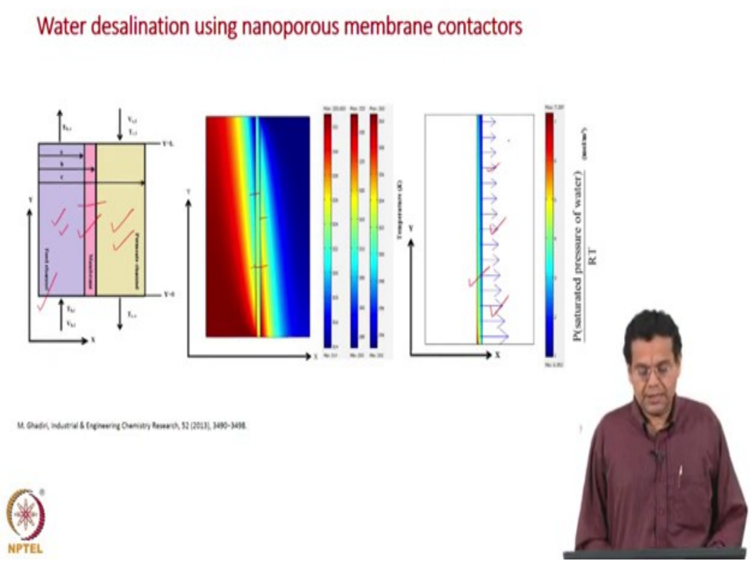
And, transfer of CO_2 takes place from the gaseous phase to the liquid phase and the relatively clean gas is let out to the atmosphere. What happens to the amine solution? It becomes rich in CO_2, we sent to a stripping column which strips off CO_2 from the amine solution and it is recycled back to the absorber.

Our focus here in this slide is on the absorption column, in the present study let us say we want to predict the performance of the absorption column. What do we mean by that, let us say we want to predict the percentage removal of CO_2 or we want to designs absorption column for the best removal of CO_2. Then, we need to solve the conservation equations, what are the conservations equations? The energy balance equation and the species balance equation.

So, the equations are been solved and the temperature profile along the length of the packed column is predicted and compared with experimental data. Not alone that, in this work they also study how the temperature varies along the length of the column under transient conditions. What happens when you start the plant, and when you give a change in gas flow rate or liquid flow rate. And, the authors predict how the liquid phase temperature will vary along the length of the column at different time instance.

More importantly our main variable of interest is the percentage removal of CO2. So, the authors also predict how the percentage removal of CO_2 varies with time, under startup condition and then when there is step change in gas and liquid flow rate. So, you can use the conservation equations to predict the steady state performance and the dynamic performance of the absorption column.

(Refer Slide Time: 12:41)



The next example is on distillation not the conventional distillation but, membrane distillation. Which is used to separate water from a aqueous solution, let us say brine solution and the application would be to get water from seawater and such application is shown in the above slide image, which is called as membrane distillation because, we use a membrane to achieve this operation.

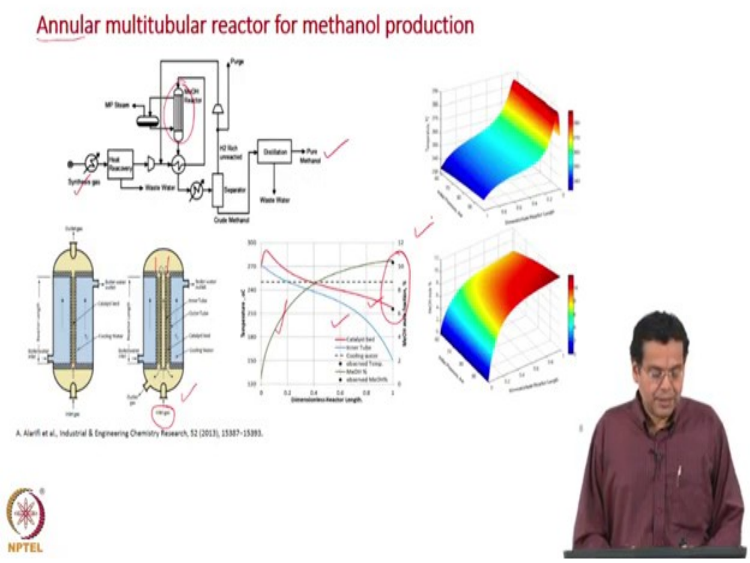
So, the membrane distillation unit is schematically shown, through the feed site, we send the brine solution and the permeate site we send water and a hydrophobic membrane separates these two sites. And, the driving force for the operation is the temperature difference between these two streams; the feed stream is at a higher temperature, and the water stream is at a lower temperature.

So, in the feed site water vaporizes, it diffuses through the hydrophobic membrane and when it reaches on the other side it condenses. So, in that way we achieve separation of water from the salt solution. And, we use a hydrophobic membrane. So, that liquid water does not pass through it and suppose we want to design this membrane distillation unit, study the performance of the membrane distillation unit, what do we mean by that? The main variable of interest is how much of water has got transferred from the feed side to the permeate side.

So, to predict it is performance, we need to solve the conservation equations, what are the conservation equations? We solve the continuity and Navier-Stokes equation because, you do have flow here. And, we solve the energy balance equation to get the temperature profile, temperature plays a major role as we have seen, that is a driving force. And, because water vapor diffuses through the membrane, we solve the species balance also.

So, the temperature profiles in the feed site, the permeate site and the membrane as well is shown in colored images. And, then the concentration of water vapor in the membrane, how does it spatially vary and more importantly, what is the flux of water, how much of water passes through the membrane, along the length of the membrane. So, all the conservation equations to predict it is performance or arrive at an optimal design.

(Refer Slide Time: 15:43)

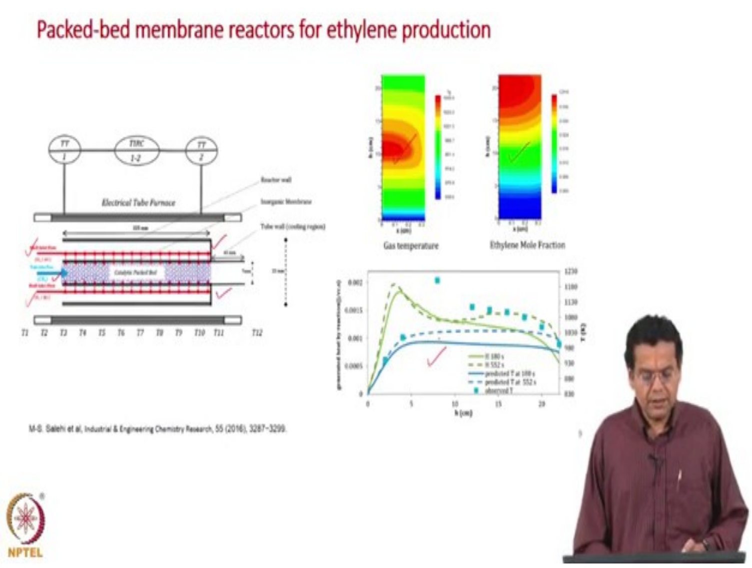


Let us move on to reactors, first example is a packed bed reactor, the application is for the production of methanol from synthesis gas, for which the flow sheet is shown in the above slide image, our focus is on the methanol reactor. And, one of the designs is shown here; the feed gas enters at the bottom and rise up through the inner tube and then flows down through the annular tube. That is why the title says annular multi-tubular reactor, we are focusing on one such tube there are several such tubes in the reactor. So, feed gas enters the inner tube and then flows down through the annular region, where we have packed the catalyst, it is a catalytic reaction and because it is exothermic we use a coolant.

Now, to efficiently design this reactor predicts it is performance, once again we will have to solve the conservation equations, we have to solve the energy balance and species balance equations. And, in this study the equations have been solved and the temperature profile in the tube side, in the annular region vary of the catalyst bed, they have been predicted. And we can also predict what is the concentration of methanol along the length of the reactor, they have also compared with experimental data.

So, once the model is validated, we can use the model to study the effect of different parameters for example, the author study what is the effect of pressure, on the temperature profile, the methanol concentration profile. So, that you can arrive at the optimal operating conditions and what is the objective to maximize the production of methanol.

(Refer Slide Time: 17:57)

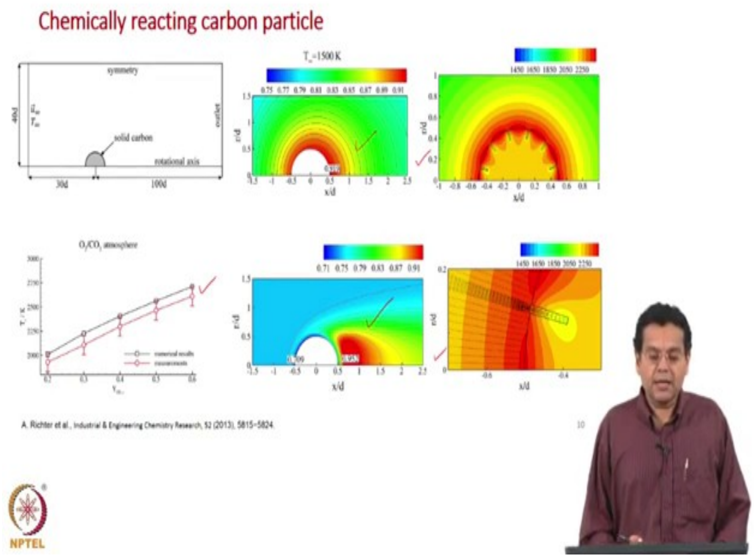


So, that is where the conservation equations play a role, in this case energy balance along with the species balance equation. Next example is also a packed bed reactor but, it is a packed bed membrane reactor, the application is for the production of ethylene from natural gas or methane. Methane is sent through the tube side, which is packed with the catalyst and then through the shell side we send oxygen, oxygen and methane react to give ethylene. Now, the cell side is closed. So, what happens, oxygen enters into the tube, how does it enter? The surface of the tube or the walls of the tube is made of an inorganic membrane.

So, oxygen diffuses through the membrane and reaches the tube site. So, in that way oxygen is fed to the reactor along the entire length of the reactor. That is why it is called a packed bed membrane reactor, you have a packed bed inside the tube and then the walls of the tube is made of a membrane, it is also plays a role for allowing the oxygen to enter inside of the tube. Now, to design such a membrane packed bed reactor or to predict it is performance, you will have to solve all the 4 conservation equations, continuity, momentum, energy and species equation also.

And, in this work our focus is inside the tube only. So, the temperature profile in the packed bed is predicted and what is shown here is the ethylene mole fraction that is our final variable of interest. So, how does ethylene mole fraction vary in the reactor that is also shown the authors predict the temperature profile along the length of the reactor as a function of time as well. So, use the conservation equations including the energy balance equation to design such reactors and predict their performance.

(Refer Slide Time: 20:13)

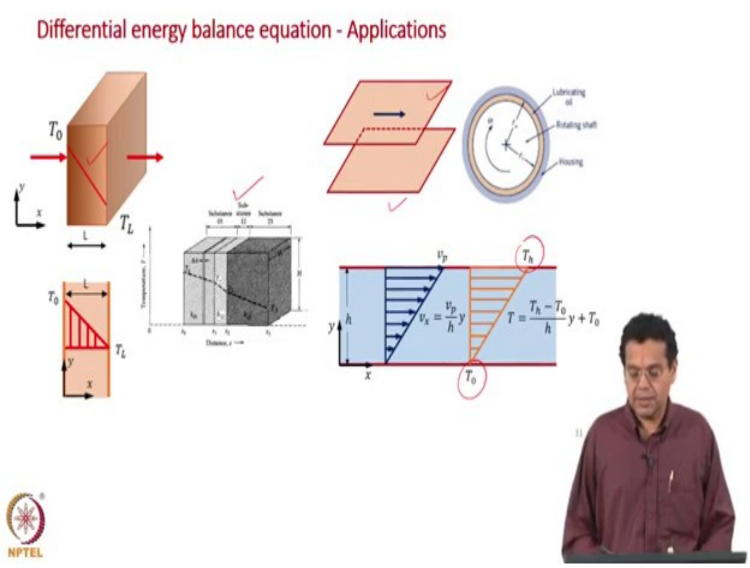


And, the last example is not a reactor but, just a single particle which could be inside a reactor. Let us say we have a carbon particle let us say a few 100 microns and that is in atmosphere of oxygen and CO_2. So, carbon particles surrounded by oxygen and CO_2 of different composition, we want to know what is the rate at with the carbon particle burns and once again the all the conservation equation are solved, all the 4 conservation equations are solved, including the differential energy balance equation.

And, the authors predict the temperature at the surface of the carbon particle and compare with experimental results. They also predict what is the concentration of CO_2 surrounding the particle, what happens at low velocity, what happens at high velocity and then they consider a porous particle. And, see the temperature distribution inside the particle and the temperature distribution outside the particle.

So, by solving the energy balance equation of course, along with other conservation equations, we can do a detailed study even at the particle level that is why this example is being discussed. Other example were at this scale of reactor, let us say few meters but, this is order of few microns and this where is this carbon particle of course, that could be in inside the reactor.

(Refer Slide Time: 21:57)



And, eventually we can predict the performance of the reactor. So, those are the applications, which I would say are high end applications as far as this course is concerned. I would say all of them are researchable applications most of them, what is it we are going to discuss in the present course. As usual our applications are going to be very simple, let us look at them.

We are going to apply that differential energy balance equation to find out the temperature profile in a slab, whose true surfaces are at two different temperatures. And, then we will extend that to a series of slabs, which would represent a furnace wall made of different layers and here our objective would be to find out the heat flux through such a wall made of different layers.

And, then we move on to our well known example of the planner couette flow, bottom plate fixed top plate moving but, now with the difference, we will consider the case where the bottom plate and the top plate are at two different temperatures. Our objective would be to find out the temperature profile just like we focused on the velocity profile now; we will see how to get the temperature profile between the 2 plates. So, those are the simple applications which, we will discuss as part of this course.