Continuum Mechanics and Transport Phenomena
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Lecture - 104
Simplification of Integral Energy Balance

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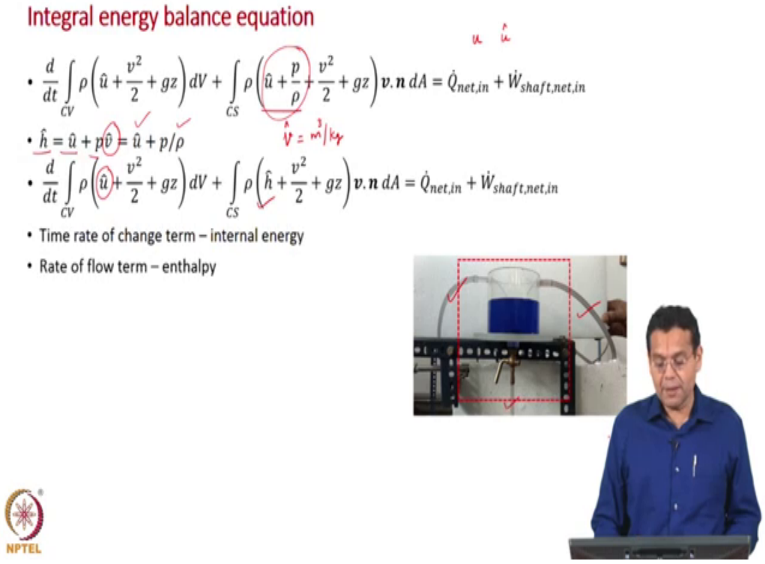

Integral energy balance equation

$u \quad \hat{u}$

- $\frac{d}{dt}\int_{CV} \rho\left(\hat{u}+\frac{v^2}{2}+gz\right)dV + \int_{CS} \rho\left(\hat{u}+\frac{p}{\rho}+\frac{v^2}{2}+gz\right)\boldsymbol{v}.\boldsymbol{n}\, dA = \dot{Q}_{net,in} + \dot{W}_{shaft,net,in}$
- $\hat{h} = \hat{u} + p\hat{v} = \hat{u} + p/\rho$ $\qquad \hat{v} = m^3/kg$
- $\frac{d}{dt}\int_{CV} \rho\left(\hat{u}+\frac{v^2}{2}+gz\right)dV + \int_{CS} \rho\left(\hat{h}+\frac{v^2}{2}+gz\right)\boldsymbol{v}.\boldsymbol{n}\, dA = \dot{Q}_{net,in} + \dot{W}_{shaft,net,in}$
- Time rate of change term – internal energy
- Rate of flow term – enthalpy

Yeah, the Integral Energy Balance in its full form of course, looks scary with so many terms in it, but we will see now how to simplify it.

$$\frac{d}{dt}\int_{CV} \rho\left(\hat{u} + \frac{v^2}{2} + gz\right)dV + \int_{CS} \rho\left(\hat{h} + \frac{v^2}{2} + gz\right) v.n\, dA = \dot{Q}_{net,in} + \dot{W}_{shaft,net,in}$$

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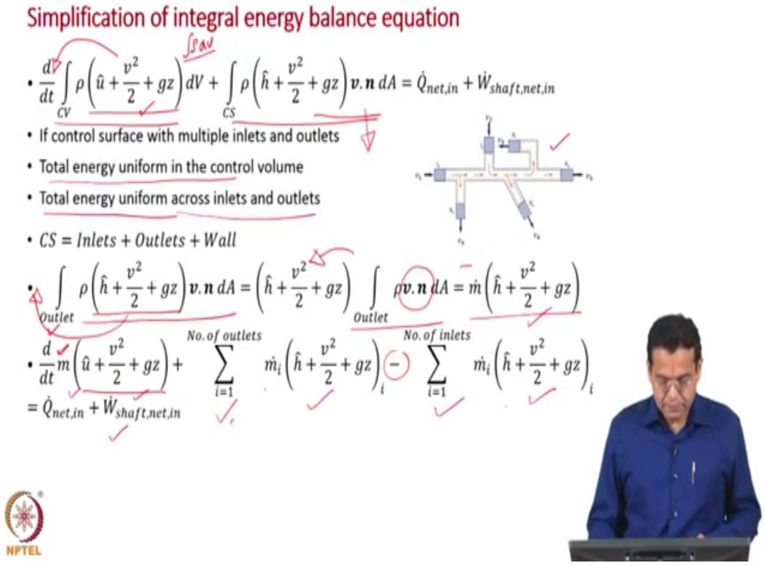

Simplification of integral energy balance equation

$\int \rho dV$

- $\frac{d}{dt}\int_{CV}\rho\left(\hat{u}+\frac{v^2}{2}+gz\right)dV+\int_{CS}\rho\left(\hat{h}+\frac{v^2}{2}+gz\right)\boldsymbol{v}.\boldsymbol{n}\,dA=\dot{Q}_{net,in}+\dot{W}_{shaft,net,in}$
- If control surface with multiple inlets and outlets
- Total energy uniform in the control volume
- Total energy uniform across inlets and outlets
- $CS = Inlets + Outlets + Wall$
- $\int_{Outlet}\rho\left(\hat{h}+\frac{v^2}{2}+gz\right)\boldsymbol{v}.\boldsymbol{n}\,dA=\left(\hat{h}+\frac{v^2}{2}+gz\right)\int_{Outlet}\rho\boldsymbol{v}.\boldsymbol{n}\,dA=\dot{m}\left(\hat{h}+\frac{v^2}{2}+gz\right)$
- $\frac{d}{dt}m\left(\hat{u}+\frac{v^2}{2}+gz\right)+\sum_{i=1}^{No.\,of\,outlets}\dot{m}_i\left(\hat{h}+\frac{v^2}{2}+gz\right)_i-\sum_{i=1}^{No.\,of\,inlets}\dot{m}_i\left(\hat{h}+\frac{v^2}{2}+gz\right)_i$
$=\dot{Q}_{net,in}+\dot{W}_{shaft,net,in}$

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So we are going to discuss about Simplification of the Integral Energy Balance equation, just like we simplified the integral total mass balance equation. We did not simplify much the integral linear momentum balance, we only looked at straightaway in the applications, but we simplified the integral mass balance equation. So, this discussion will be roughly along those lines.

So, let us write down the complete integral energy balance equation;

$$\frac{d}{dt}\int_{CV}\rho\left(\hat{u}+\frac{v^2}{2}+gz\right)dV+\int_{CS}\rho\left(\hat{h}+\frac{v^2}{2}+gz\right)v.n\,dA=\dot{Q}_{net,in}+\dot{W}_{shaft,net,in}$$

Now, if we consider a control surface where there multiple inlets and outlets, this diagram we have come across earlier also now how do we simplify. Now, we will make some assumptions the total energy is uniform in the control volume, but which means within the control volume and total energy is uniform across the inlets and outlets.

What does it mean? Within the control volume, the total energy is uniform and across inlets then outlets the total energy is uniform which means there is no variation across the cross section. Now, how do we usually split the control surface? Usually we split the control surface let us say inlets, outlets and then the wall and wherever we have a wall the convection term does not contribute because velocity is 0.

Now, how do we simplify the term where there are inlets and outlets let us see that.

$$\int_{outlet} \rho\left(\hat{h} + \frac{v^2}{2} + gz\right) v.n\, dA = \left(\hat{h} + \frac{v^2}{2} + gz\right)\int_{outlet} \rho\, v.n\, dA = \dot{m}\left(\hat{h} + \frac{v^2}{2} + gz\right)$$

We assumed that the total energy is uniform across the inlet and outlet which means that $\left(\hat{h} + \frac{v^2}{2} + gz\right)$ term does not vary across a cross sectional area. So, I can take out this term outside the integral sign that is what I have done here and what is left out is the mass flow rate.

So, I can write this as $\dot{m}$ the mass flow rate into sum of all the energy terms and if we do this for inlet, you will have a negative sign why is it? Because $v.n$ is positive for outlet and $v.n$ is negative for inlet. So, now let us simplify use this and simplify the integral energy balance equation.

$$\frac{d}{dt}\dot{m}\left(\hat{u} + \frac{v^2}{2} + gz\right) + \sum_{i=1}^{No.\,of\,outlets} \dot{m}_i\left(\hat{h} + \frac{v^2}{2} + gz\right)_i - \sum_{i=1}^{No.\,of\,inlets} \dot{m}_i\left(\hat{h} + \frac{v^2}{2} + gz\right)_i = \dot{Q}_{net,in} + \dot{W}_{shaft,n}$$

Now, how do we simplify this convection term? The convection term is for inlets and outlets and so we express that as sum over outlet and summed over inlet. And we have seen that we can simplify that term as $\dot{m}$ and the sum of the energies that is what has been used here for every outlet i and same term here of course, the negative sign here, right hand side remains the same.

So, what is the simplification? The total energy is uniform in the control volume and across inlets and outlets. So, the transient term gets simplified and the convection term is over outlets and inlets which is expressed as the mass flow rate into the sum of the energies; we also taken into this sign ourselves.

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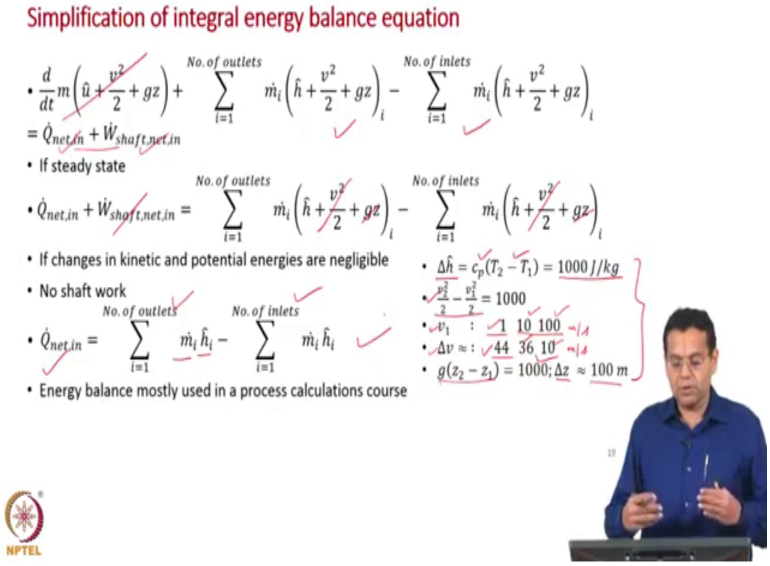

Simplification of integral energy balance equation

- $\frac{d}{dt}m\left(\hat{u}+\cancel{\frac{v^2}{2}+gz}\right)+\sum_{i=1}^{No.\,of\,outlets}\dot{m}_i\left(\hat{h}+\frac{v^2}{2}+gz\right)_i-\sum_{i=1}^{No.\,of\,inlets}\dot{m}_i\left(\hat{h}+\frac{v^2}{2}+gz\right)_i$ $=\dot{Q}_{net,in}+\dot{W}_{shaft,net,in}$
- If steady state
- $\dot{Q}_{net,in}+\cancel{\dot{W}_{shaft,net,in}}=\sum_{i=1}^{No.\,of\,outlets}\dot{m}_i\left(\hat{h}+\cancel{\frac{v^2}{2}}+\cancel{gz}\right)_i-\sum_{i=1}^{No.\,of\,inlets}\dot{m}_i\left(\hat{h}+\cancel{\frac{v^2}{2}}+\cancel{gz}\right)_i$
- If changes in kinetic and potential energies are negligible
- No shaft work
- $\dot{Q}_{net,in}=\sum_{i=1}^{No.\,of\,outlets}\dot{m}_i\hat{h}_i-\sum_{i=1}^{No.\,of\,inlets}\dot{m}_i\hat{h}_i$
- Energy balance mostly used in a process calculations course

- $\Delta\hat{h}=c_p(T_2-T_1)=1000\ J/kg$
- $\frac{v_2^2}{2}-\frac{v_1^2}{2}=1000$
- v_1 : 1 10 100
- $\Delta v\approx$: 44 36 10
- $g(z_2-z_1)=1000;\Delta z\approx 100\ m$

So, now let us proceed further; let us see how do we simplify this further. So, the same equation is written here.

$$\frac{d}{dt}\dot{m}\left(\hat{u}+\frac{v^2}{2}+gz\right)+\sum_{i=1}^{No.\,of\,outlets}\dot{m}_i\left(\hat{h}+\frac{v^2}{2}+gz\right)_i-\sum_{i=1}^{No.\,of\,inlets}\dot{m}_i\left(\hat{h}+\frac{v^2}{2}+gz\right)_i=\dot{Q}_{net,in}+\dot{W}_{shaft,n}$$

So, now this looks certainly simpler we do not have the integral signs ok. The integral sign was take into account either variation within the volume or over the inlet and outlet because we assume it is uniform integral signs are gone and we also expressed in terms of mass flow rate.

Now, if we are operating under steady state condition; the left hand side, the transient term goes to 0

$$\frac{d}{dt}\dot{m}\left(\hat{u}+\frac{v^2}{2}+gz\right)=0$$

And then in a steady state operation usually the unknowns are rate of heat to be supplied or the rate of shaft work. So, let us simplify that equation

$$\dot{Q}_{net,in}+\dot{W}_{shaft,net,in}=\sum_{i=1}^{No.\,of\,outlets}\dot{m}_i\left(\hat{h}+\frac{v^2}{2}+gz\right)_i-\sum_{i=1}^{No.\,of\,inlets}\dot{m}_i\left(\hat{h}+\frac{v^2}{2}+gz\right)_i$$

So, rate of heat addition, and rate of shaft work on the left hand side; why is that? Usually the unknowns are on the left hand side.

Now, we will make few more assumptions and simplify further; if changes in kinetic and potential energies are negligible why is it so? What is that we are going to assume now? Changes in kinetic and potential energies are negligible compared to change in enthalpy; what does it mean? If you have a control volume let us say only one inlet one outlet, what is the change in enthalpy between inlet and outlet? How does that compare with change in kinetic energy change in potential energy.

To understand that comparison and justify this assumption that usually kinetic potential energy changes are negligible; to justify that we have a small example here. Let us say we have air flowing and then its temperature increases by just 1 degree centigrade, just 1 degree centigrade air is flowing.

And so the change in enthalpy will be

$$\Delta\hat{h} = c_p\left(T_2 - T_1\right) = 1000\frac{J}{kg}$$

Here, c_p is 1000 Joule per kg Kelvin; and $\Delta T = 1$ so the enthalpy change in terms of per unit mass is 1000 Joule per kg. Now, let us see what is the velocity change that should happen, so that the kinetic energy change is 1000 Joule per kg. So, equate the kinetic energy change to 1000.

$$\frac{v_2^2}{2} - \frac{v_1^2}{2} = 1000$$

Now of course, the change in velocity require for the kinetic energy changed to be 1000 depends on v_1. So, let us take different cases of let us say inlet velocity v_1 and what is shown here is as a table; the inlet velocity and corresponding change in velocity that should happen so that the change in kinetic energy is 1000 Joule per kg.

v_1	1	10	100
Δv	44	36	10

So, now if the let say inlet velocity is 1 m/s, the change in velocity should be 44 m/s; then the change in kinetic energy is 1000 Joule per kg. If then it is 10 change should be 36, inlet is 100, then change should be 10.

Now, for usual equipments let us say heat exchanger, distillation column or reactors etcetera, let us say the inlet velocity is order of 1 m/s, then the change should be 44 meter per second which hardly happens that is why change in kinetic energies are negligible. But let say suppose we have a steam turbine we have a compressor, then the velocity or somewhere in the range let us say order of 100 meter per second, then the velocity can change by let say order of 10 m/s. So, in those cases you will have to consider the change in kinetic energy term for compressors, steam turbines etcetera.

Coming to the potential energy; what is the elevation required for the potential energy change to be 1000 Joule per kg. So, let us equate potential energy to 1000 and so

$$g\left(z_2 - z_1\right) = 1000; \qquad \Delta z = 100\,m$$

Once again that is a too large a height for usual pros equipments. Remember that is for just 1 degree centigrade change usually temperature change are larger than that. So, once again the potential energy changes also negligible that is why we are going to simplify for a more practical case I would say where changes in kinetic and potential energies are negligible.

So, let us take the case where there is no shaft work; why? For usual equipments where let us say heat exchange, the distillation column there are no shaft work. Shaft work plays a role when you have a pump compressor or you have a stirrer etcetera, then there is shaft work.

So, it gets simplified to

$$\dot{Q}_{net,in} = \sum_{i=1}^{No.\,of\,outlets} \dot{m}_i \hat{h}_i - \sum_{i=1}^{No.\,of\,inlets} \dot{m}_i \hat{h}_i$$

Of course, now becomes very simple; this is the energy balance mostly used in process calculation course. Usually you would have considered steady state process and let us say if it is without shaft work and you would have used the energy balance to find out the heat to be supplied. And this is a form of energy balance which would have used in this simplest case we can probably call this as enthalpy balance; only under this condition and we should note that it is a most one of the most simplest forms of the total energy balance.