Continuum Mechanics and Transport Phenomena
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Lecture - 103
Integral Energy Balance – Part 2

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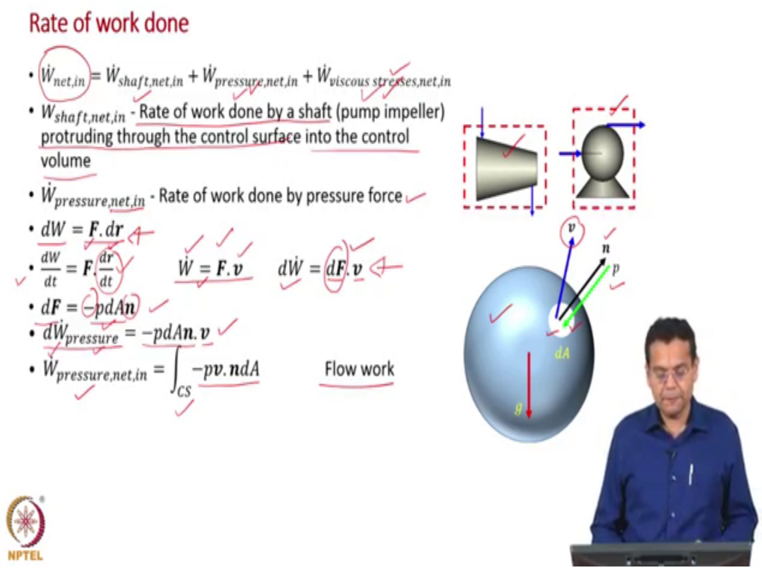

In the integral energy balance equation, right hand side, there are two terms both represents energy addition term, net rate of energy addition, one is by heat transfer, one is by work transfer. So, let us discuss the rate of energy addition by work transfer term that require some discussion. So, let us discuss that; that is why the title says rate of work done.

$$\dot{W}_{net,in} = \dot{W}_{shaft,net,in} + \dot{W}_{pressure,\,net,in} + \dot{W}_{viscous\ stresses,\,net,in}$$

Now, how do we split their rate of work done? $\dot{W}$ represents rate of work done, and of course, net represents net rate of work done and then energy addition into the contents of control volume.

Now, that rate of work done is split into three parts or three contributions. One is the shaft work, other is the work done by pressure, and third contribution is the work done by viscous stresses.

So, rate of work done is equal to rate of shaft work, rate of work done by pressure plus rate of work done by viscous stresses. Let see how do we express all of them.

Now, this shaft work is what we supply or take out. What is the example? The best example is pump. If you take a pump water enters and then let us say water leaves and if you keep adding power, or if you keep adding work to the water let us say in lifting or some pumping over some distance which means that there is a shaft and or a pump impeller which has energy. This work is what we call as the shaft work. It could be a hydro turbine where because of water flow we extract work from water. So, either you supply work take out work or could be a compressor where we once again supply power let us say to increase a pressure.

So, what is shaft work rate of work done by a shaft. In all this cases, we have a shaft protruding through the control surface into the control volume. Though of course, the shaft is not shown explicitly here. We can easily understand that we have a shaft which crosses through the control surface. Why protruding through the control surface, remember whenever we do energy balance even for streams we take only those streams which cross the control surface.

Similarly, here also that work transfer term also should be across the control surface that is why it says rate of work done by a shaft protruding through the control surface into the control volume. So, this is mostly we usually find out this value or we give a known value. So, we do not have an expression for this, usually this is the unknown quantity. For example, to pump water for a over some distance what is the work to be done, what is the rate of work done, what is the power to be supplied.

Now, coming to the next term which is rate of work done by pressure force. Also I like to mention a very fine point when we say pressure we say here viscous stresses because pressure is also force per area viscous is also force per area. When you say pressure force, here you say viscous forces. So, now, $\dot{W}_{pressure,\, net,in}$ represents rate of work done by pressure force.

Let see how do we express this rate of work done by pressure force. Now, to understand that, let us look at this figure. We have come across this figure earlier when we discussed pressure force on the right hand side of integral linear momentum balance. The same figure here as well with one small addition. Let us look at the figure a small control volume is shown, for convenience sphere is shown, but it could be any control volume surrounding control surface also seen.

Now, we take a small area dA and n is the output drawn normal and p is the pressure acting over that small area. So, that part is same as what we have seen earlier and what is shown additionally is the velocity vector. So, there is flow through that small area, so velocity vector is also shown. Of course, as earlier the normal vector is shown, the pressure is also shown.

Now, to write down the expression for the rate of work done by pressure force, let us proceed with this expression which is known to us from physics.

$$dW = F.dr$$

What does this represent, the differential work done by a force in moving an object over a length d r vector is given by the dot product of the F vector within dr vector. Left hand side is the differential work done let us say a force moves a object by a small length in a particular direction. So, if you take the dot product of F vector with dr vector, you get the differential work done.

Now, we have been dealing with rate of course we need rate of work done. So, let us express this equation in terms of rate with respect to time.

$$\frac{dW}{dt} = F.\frac{dr}{dt}$$

So, left hand side we have the rate of work done; right hand side we have the force vector and then $\frac{dr}{dt}$ becomes the velocity vector.

$$\dot{W} = F.v$$

$$d\dot{W} = dF.v$$

So, how do we read this equation, rate of work done is given by the dot product of the force vector and the velocity vector. Now, since we are focusing on a small area, first we will find out what is the differential rate of work done by the pressure, so that is why we write this equation in differential form.

So, let me quickly repeat this. We started with the force moving an object over a small length which is given by $dW = F.dr$, so that tells about that differential work done.

We express that in terms of rate with respect to time. So, left hand side becomes rate of work done and in $\frac{dr}{dt}$ becomes velocity. So, rate of work done is equal to dot product of the force vector and the velocity vector. We are interested in the differential rate of work done, now because we are focusing on a small area. So, we will write it as differential rate of work done is equal to differential force dotted with velocity. This expression is required to proceed further.

Now, this is a very general expression, what we have discussed is a very general expression. What is the differential force relevant to us. Now, the we are discussing the force due to pressure, so that is given by pressure into area over which it acts. So,

$$dF =- pdA\, n$$

Now, coming to the direction n is the outward drawn normal vector, and p is the pressure. And we know that pressure is the compressive force which always acts into the control volume, so which means that direction of p is opposite to that of n. So, this expression including the direction becomes pdA and the direction is $-$n vector. So, the differential force is pdA in terms of magnitude, along with direction it is $-\ pdA\, n$.

So, now let us use this expression for the differential force in this equation and what we get is that differential rate of work done by pressure is

$$d\dot{W}_{pressure} =- pdA\, n. v$$

$$\dot{W}_{pressure,net\,,in} = \int_{CS} -\ p\, v. n\, dA$$

As I told you this is general expression, right now the relevant force is a force due to pressure. So, when we express the differential force in terms of this expression, we get the differential rate of work done by pressure. Now, what these represent, differential rate of work done by pressure over this small area.

Now, for the entire control surface, we integrate over the entire control surface and that is what is shown here. So, now $\dot{W}$ represents net rate of work done by the pressure force over the entire control surface. What is that we have done taken this expression and integrated over the entire control surface, of course with the small rearrangement, so that is comes over well known form. So, this is the expression for the net rate of work done by pressure force. And this is usually called as the flow work; flow work.

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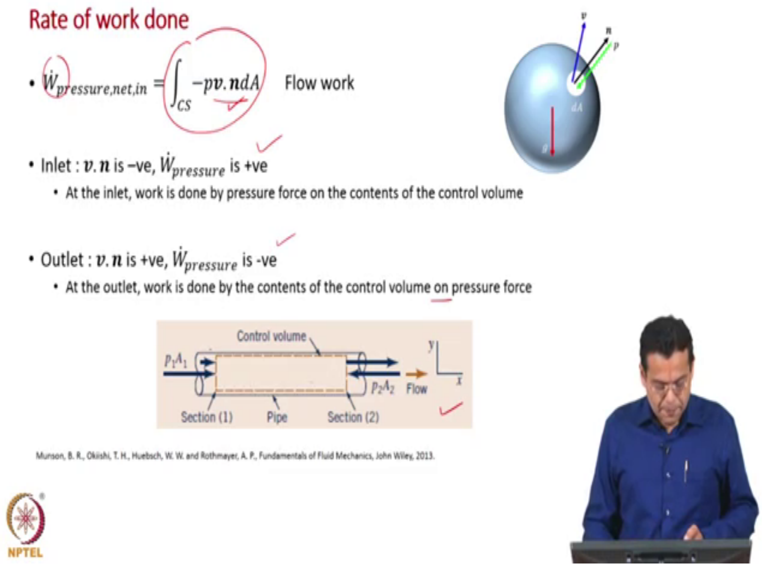

And to understand this flow work what it means, let us write down the expression which we have derived in the previous slide.

$$\dot{W}_{pressure,net\,,in} = \int_{CS} -\,p\,v.n\,dA$$

Now, what is shown here is flow in and flow out through a pipe. Let see what happens to this rate of work done at the inlet surface. At the inlet we know that $v.n$ is negative and so this rate of work done is positive. What does it mean in terms of our sign convention? At the inlet work is done by pressure force on the contents of the control volume, because it is positive so work is done on the contents of the control volume.

As I have told earlier also more precisely we should say contents of the control volume. Now, which means that the pressure does some work in pushing the fluid into the control volume. What happens to the outlet, $v.n$ is positive and the rate of work done by pressure is negative. What does it mean? At the outlet work is done by the contents of control volume meaning the fluid on the pressure force or I used on so that this sentence is opposite to this sentence.

To be more precise, you can use the word against pressure force. So, what happens is the outlet work is done by the fluid against the pressure in leaving the control volume. So, at the inlet the work is done by the pressure in pushing the fluid into the control volume. At the exit the fluid does some work to go out of the control volume against the pressure. This is what we meant by flow work. We are here taken a pipe where there is one inflow one, outflow.

But, if you have a control volume with multiple inlets and outlets, wherever there is inflow work is done by pressure, and wherever there is outflow work is done by the fluid and that is why this takes care of all the regions of the control surface wherever there is inflow and outflow. Of course, wherever there is a wall it would not contribute because velocity is 0.

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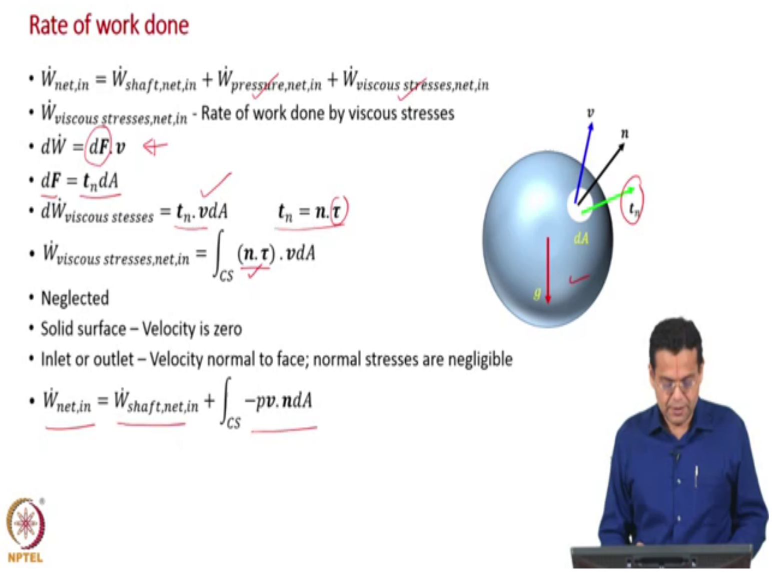

Now, we have seen that the rate of work done is expressed in terms of the rate of shaft work, rate of work done by pressure which I had discussed.

$$\dot{W}_{net,in} = \dot{W}_{shaft,net,in} + \dot{W}_{pressure,\ net,in} + \dot{W}_{viscous\ stresses,\ net,in}$$

The third contribution is rate of work done by viscous stresses, let us see how to evaluate that. Once again the same figure is shown here, as we have seen the previous slide with the only difference is that instead of pressure I have shown a stress vector, stress vector acting on the small surface.

So, let us see how do you express that once you have understood that this becomes easier, so rate of work done by the viscous stresses or viscous forces.

$$d\dot{W} = dF.v$$

As we had discussed earlier and as I told you earlier, this expression is very general just relates the differential rate of work done to the differential force on the velocity. Earlier we applied that for taking the force due to pressure, now we are going to apply that taking the force due to the stress vector or the viscous stresses.

$$dF = t_n dA$$

What is that the differential force is nothing but the stress vector multiply by the area, because we have a small area dA on that we have a stress vector acting. So, juts multiply the stress vector with a differential area, you will get the differential force. Then just substitute we can find out, what is the differential rate of work done by viscous stresses,

$$d\dot{W}_{viscous\ stress} = t_n . v\ dA; \qquad t_n = n.\tau$$

Now, we know that the stress vector can be expressed in terms of stress tensor as a dot product between n vector and the stress tensor. Remember τ is the viscous stress tensor not the total stress tensor why is it, we have taken the work done by pressure separately, so obviously we have take only the viscous stress tensor part. Now, as we have done for rate of work done by pressure, we will integrate over the entire control surface.

$$\dot{W}_{viscous\ stress,\ net,\ in} = \int_{CS} (n.\tau).vdA$$

Now, looks like it's too difficult becoming more difficult, because we are including stress vector expressed in terms of stress tensor and then we have some two dot products, etcetera. Now, however we are going to overcome this very simple, neglect it, very simple. Now, can we neglect it, wherever we have a solid surface velocity is 0, so there is no contribution we are safe.

Now, wherever inlet and outlet usually most of the time the velocity is normal to the area, which means that when you take this $n.\tau$ etcetera, only the normal stresses contribute and the normal stresses usually are negligible, they are in terms of the velocity gradients in the direction of flow and so the normal stress are usually negligible. So, there again we can neglect the contribution by the work done by viscous stresses. So, why are we neglecting on the walls the velocity 0 does not contribute at all, wherever we have inlet outlet mostly the flow is perpendicular to the phase.

So, on the normal stresses play a role here and they are usually negligible and hence we are neglecting. Within the scope of our course and throughout integral energy balance it is neglected, it is negligible. However, expression for the rate of work done becomes rate of shaft work plus rate of work done by pressure.

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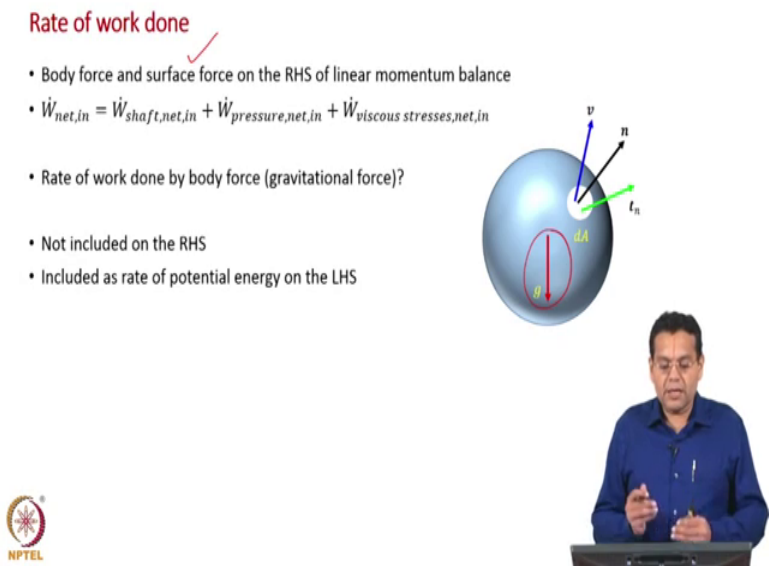

Now, if you observed all the figures shown including the figure which are discussed earlier for linear momentum balance, there is one gravity force acting and purposely I have carried over that gravity force in all the figures as well. But we are not discussed, rate of work done by gravitational force why is it, that is what we are going to discuss now.

If you look at the linear momentum balance, right hand side we have both body force and surface forces, we have both the forces on the right hand side. But now when we wrote the rate of work done, we express that has the shaft work, work done by pressure, work done by viscous stresses,

$$\dot{W}_{net,in} = \dot{W}_{shaft,net,in} + \dot{W}_{pressure,\ net,in} + \dot{W}_{viscous\ stresses,\ net,in}$$

Which means that we have taken into account the work done by surface forces, this has been taken into account. But what is that we are not done, we have not explicitly discussed about rate of work done by body force why is it so.

The reason for not discussing that is that we are already taken into account, the rate of work done by gravitational force in the form of rate of potential energy on the left hand side. So, we are not included in the right hand side the reason is that we have included that has a rate of potential energy on the left hand side. So, effect of gravity can be included in two ways; one is to include that as rate of potential energy on the left hand side along with the convection term, the flow term that is one way that is the method we have followed.

What is the alternate way, do not include on the left hand side, include only internal energy and then kinetic energy without any potential energy term, but include on the right hand side as rate of work done by gravitational force. Two different approaches are there, you have followed the first approach.

Of course, more advanced books let us say post graduate level books, follow the second approach which is more formal, which is in lined with the linear momentum balance also. But usual fluid mechanics books, especially the under graduate ones follow the approach which we have followed.

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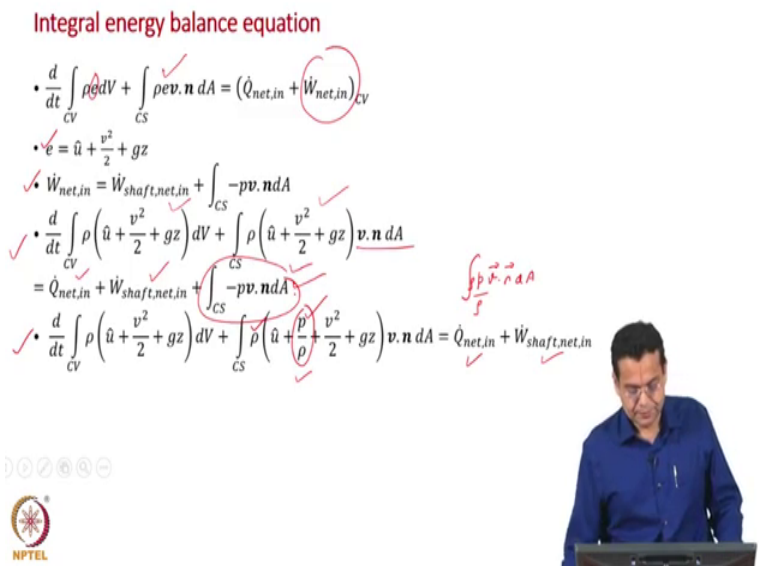

So, let us write the integral energy balance.

$$\frac{d}{dt}\int_{CV} \rho e dV + \int_{CS} \rho e\, v.n\, dA = \left(\dot{Q}_{net,in} + \dot{W}_{net,in}\right)_{CV}$$

What you discuss so far, we have discussed this rate of work done term on the right hand side. Now, we know how to express that so let us substitute for that of course,

$$e = \hat{u} + \frac{v^2}{2} + gz$$

And

$$\dot{W}_{net,in} = \dot{W}_{shaft,net,in} + \int_{CS} - p\, v.n\, dA$$

this expression we have seen in the previous to previous slide, expressing in terms of shaft work and then work done by pressure. So, let us substitute both for e and for the rate of work done term in the integral balance equation,

$$\frac{d}{dt}\int_{CV} \rho\left(\hat{u} + \frac{v^2}{2} + gz\right)dV + \int_{CS} \rho\left(\hat{u} + \frac{v^2}{2} + gz\right) v.n\, dA = \dot{Q}_{net,in} + \dot{W}_{shaft,net,in} + \int_{CS} - p\, v.n\, dA$$

The e is a only a notations, only representation, Only if expressed in terms of internal energy, kinetic energy, potential energy, the equation is complete its in terms of usually measured variables are known variables. So what is that there has been done, e here in the transient term has been expressed in terms of the sum of the energies.

And similarly e in the convection term has also been expressed in terms of the sum of the energy terms, and right hand side we have $\dot{Q}_{net,in}$ that is retained as such. The rate of work done as been expressed in terms of rate of shaft work and rate of work done by the pressure force. Now what do we do, we can do simplification, we have $v.n\, dA$ on the right hand side and left hand side, both terms are to do with convection to do with something with flow. So, let us put them together on the left hand side. So,

$$\frac{d}{dt}\int_{CV} \rho\left(\hat{u} + \frac{v^2}{2} + gz\right)dV + \int_{CS} \rho\left(\hat{u} + \frac{p}{\rho} + \frac{v^2}{2} + gz\right) v.n\, dA = \dot{Q}_{net,in} + \dot{W}_{shaft,net,in}$$

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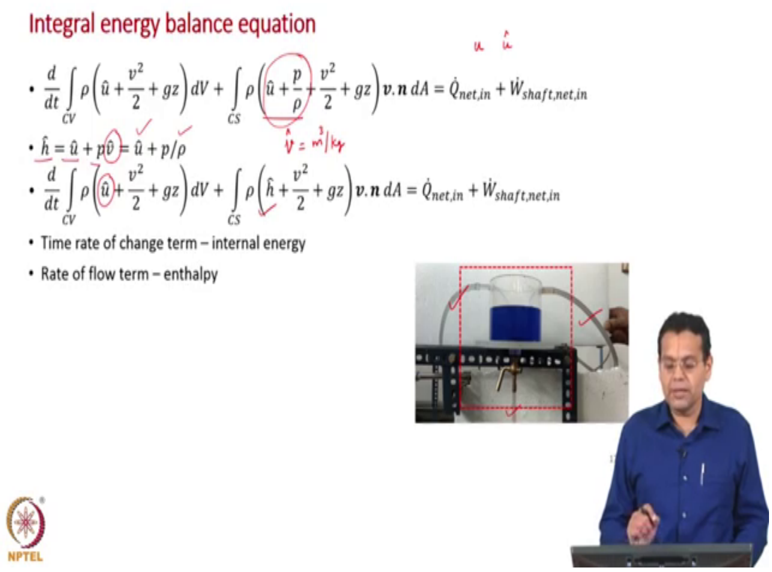

Now, let us proceed further, let us rewrite the equation written in the last slide.

$$\frac{d}{dt}\int_{CV} \rho\left(\hat{u} + \frac{v^2}{2} + gz\right)dV + \int_{CS} \rho\left(\hat{u} + \frac{p}{\rho} + \frac{v^2}{2} + gz\right) v.n\, dA = \dot{Q}_{net,in} + \dot{W}_{shaft,net,in}$$

Now, what we are going to do is express sum of the two terms is enthalpy, let us see how do we do that.

$$\hat{h} = \hat{u} + p\hat{v} = \hat{u} + \frac{p}{\rho}$$

What is enthalpy? Like to mention that this u of course you have been mentioning several time, there is internal energy per unit mass. Two nomenclatures are there usually, when you generally say u it is molar internal energy that is internal energy per unit mole, when you say $\hat{u}$ it is internal energy per unit mass.

So, since all our terms are per unit mass we have used a notation $\hat{u}$. Now, $\hat{h}$ is the enthalpy per unit mass, $\hat{u}$ is internal energy per unit mass of course p is pressure, $\hat{v}$ is the specific volume which is volume per unit mass. So, $\hat{v}$ in terms of unit, it is volume per unit mass which is nothing but $\frac{1}{\rho}$.

So, let us write in terms of enthalpy. So,

$$\frac{d}{dt}\int_{CV} \rho\left(\hat{u} + \frac{v^2}{2} + gz\right)dV + \int_{CS} \rho\left(\hat{h} + \frac{v^2}{2} + gz\right) v.n\, dA = \dot{Q}_{net,in} + \dot{W}_{shaft,net,in}$$

Now, like to emphasize here that the time rate of change term the transient term what you have is the internal energy only. The convection term the rate of flow term or the convection term what to have is enthalpy, this should be kept in mind.

This distinction may not be emphasized in certain books, because most of the time we are interested in doing steady state energy balance, so only enthalpy plays a role. But if you are doing a transient energy balance, that to for a gaseous system this distinction becomes important like to emphasize once again that in convection term we have enthalpy, in the accumulation term or the transient term we have internal energy.

Why is it not important for liquid systems, c_p and c_v are almost same for liquids, so does not matter whether you say change in internal energy or change in enthalpy; both are approximately equal to $c_p \Delta T$. But if you are considering a gaseous system, we know that if you assume a ideal gas law, if you assume ideal gas behavior, then we know $c_p - c_V = R$, so they are different.

So, if you are doing energy balance for gaseous systems, in the accumulation term we have internal energy and in the float term or the convection term it is the enthalpy. Also when you look at the enthalpy, what should come to your mind is internally energy plus flow work, moment you look at enthalpy. So, let us say if you are looking at a stream flowing in and out, what we say is first time when we came across we said the stream contains energy; the three parts are internal energy, kinetic energy and then potential energy.

Now, slightly refine it and say that the stream carries enthalpy, internal energy and then potential energy. When you say enthalpy, you are including the effect of internal energy and the flow work. So, if this physical significance is clear, then we knowing clearly that the in the convection term, we should include enthalpy. So, there is no such flow work term in the rate of accumulation term that is way that is internal energy.

And of course, you have an example very nice application where we distinguishes internal energy term, where we distinguish the internal energy term in the transient term and enthalpy appearing in the convection term.