Continuum Mechanics and Transport Phenomena
Prof. T. Renganathan
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture - 102
Integral Energy Balance – Part 1

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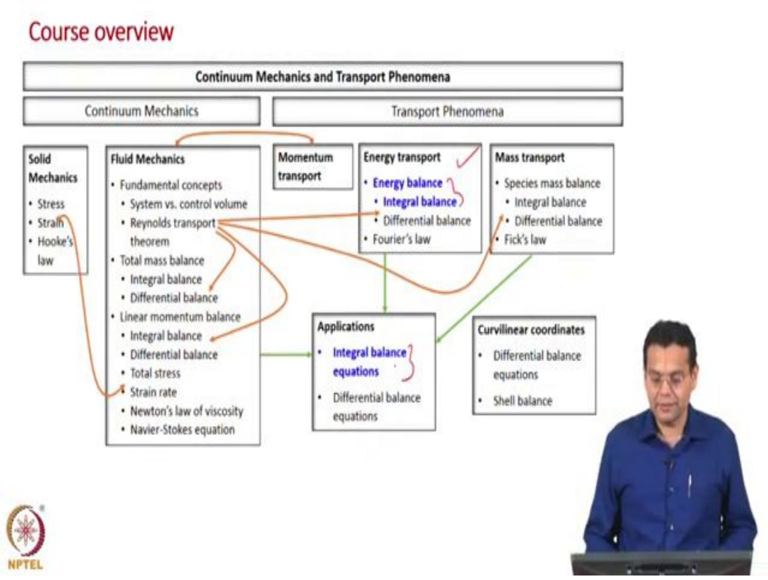

We are in the second part of the course namely, Transport Phenomena and in that we are in the second part namely energy transport. So, this is the block we are going to start now, and that consist of integral energy balance, differential energy balance of course, the Fourier's laws well. And what is the scope of this lecture is the integral energy balance both the derivation and the applications.

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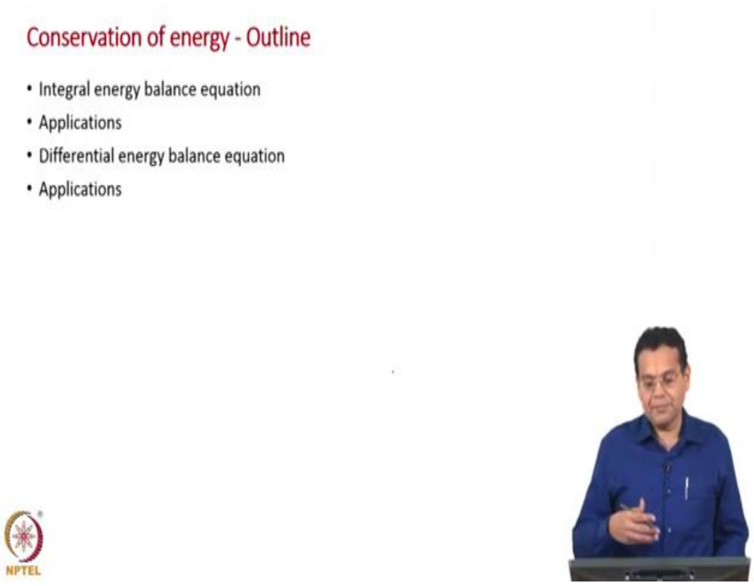

What is the outline for conservation of energy, which we called as energy transport in the previous slide? Of course, integral energy balance equation the derivation and then the applications, then differential energy balance equation the derivation and the applications. So, if you look at these four titles, it is almost analogous to what we discussed in for conservation of mass, conservation of momentum. So, in terms of the topics covered their very much analogous; there also we discussed about integral balance, differential balance applications similarly here as well.

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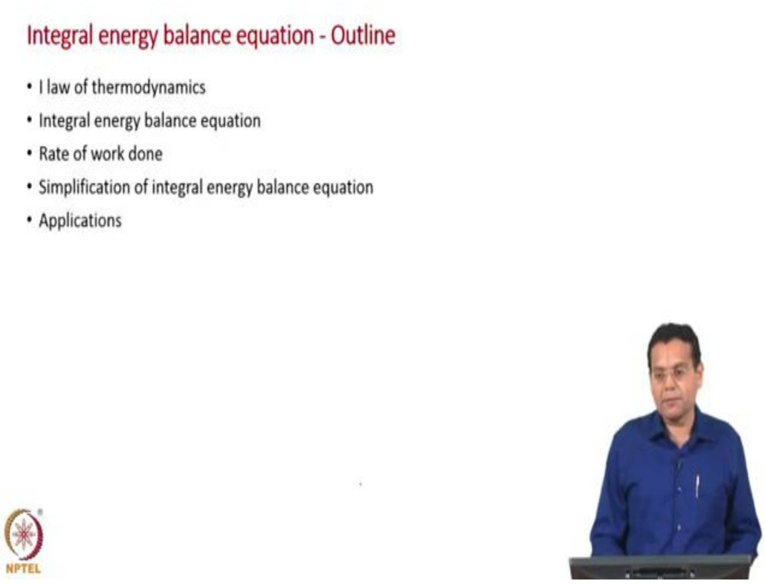

So, now, we are going to discuss about the integral energy balance equation. So, let us look at the outline. As usual we start with the law of physics and that is the first law of thermodynamics. And derive the integral energy balance equation from the first law of thermodynamics using the Reynolds transport theorem. In the integral energy balance equation, we discuss the rate of work done in detail and then we simply the integral energy balance equation just we did it for integral mass balance equation and of course, finally, look at applications.

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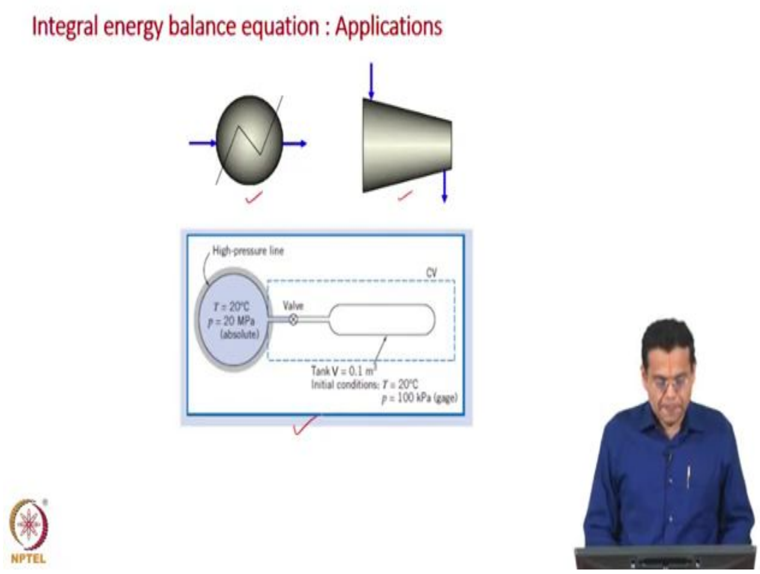

Now, what are the applications which we will be able to solve at the end of integral energy balance equation; few applications are shown here (in the above slide image) which we will discuss.

- First is finding out heat to be supplied, suppose we have a gas to be heated from some temperature some other temperature what is heat to be supplied? Simple heater;
- Second is a compressor suppose you want to compress gas from some inlet condition to outlet condition, some temperature pressure to some other temperature pressure, what is the work to be supplied?

So, these two are steady state examples, which usually come across in process calculation course. The idea is to connect what we are done in the earlier course with this course.

- The third example is little more involved that is example under transient condition, we have a high pressure line and a tank is connected to that and it is getting filled.

And it is a nice example where we see how to know the mass flow rate entering the tank based on the measurement of the increase in temperature. So, it is a transient energy balance, this probably would not have been done as part of a process calculation course.

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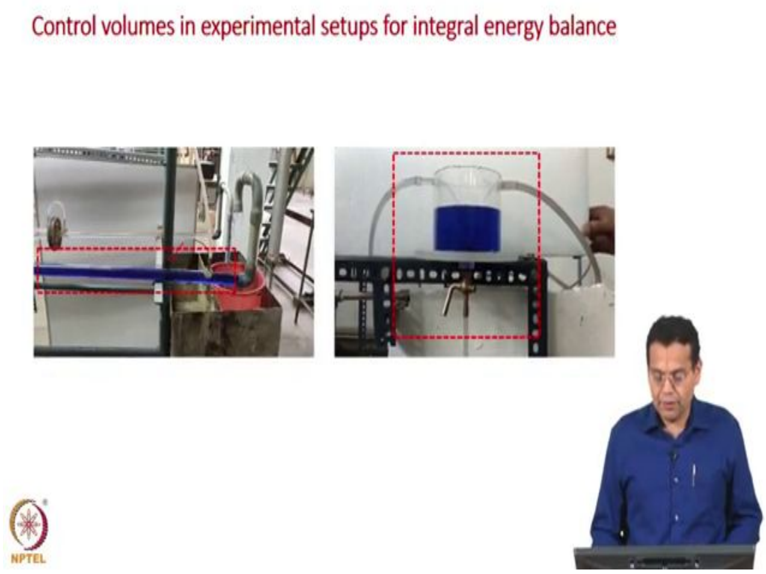

So, these two set ups are well known to us, we have come across few times in this two experimental setups. And what is shown are the control volumes over the two setups; in one case we have a pipe and in the other case we have a tank through which there is inflow and then outflow. And the difference is that the first time when we came across and did integral balance we considered mass entering mass leaving both in the pipe and the tank.

Second time when we came across the setup, we focused on momentum entering momentum leaving, now third time we are coming across the same setup doing balance over the entire pipe or the tank, but now we will focus on the energy being carried by the stream. So, we look at what is the let us say enthalpy carried by the stream, the kinetic energy carried by the stream, potential energy carried by the stream. So, every time when we come back, we look at different property being carried by the stream.

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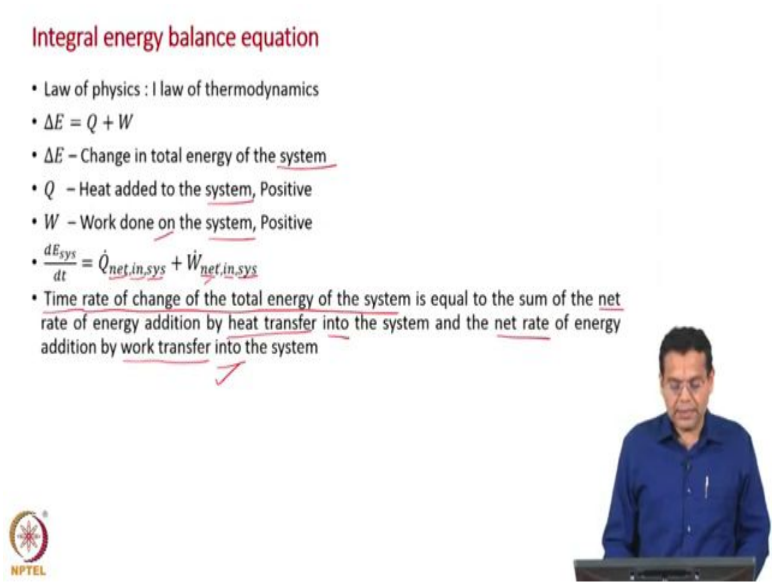

So, let us start deriving the integral energy balance equation. As usual the first step is a law of physics and that is the first law of thermodynamics. Now, let us write the first law of thermodynamics, which is

$$\Delta E = Q + W$$

Let us look at the nomenclature; ΔE represents the change in total energy of the system and Q represents heat added to the system and W represents work done on the system.

Now, what does it tell you whatever heat added to the system and work done on the system goes towards changing the total energy of the system. Few points to be noted, first all terms refer to the system and in terms of sign convention heat added is positive, work done on system is positive. For heat added, the sign convention is universal, but for the case of work done the sign convention is not so universal; work done on the system we are taking as a positive. And what we will do now is, the express this first law of thermodynamics in terms of rate with respect to time.

$$\frac{dE_{sys}}{dt} = \dot{Q}_{net,in,sys} + \dot{W}_{net,in,sys}$$

I will explain the subscript shortly. So, right hand side we have rate at which heat is added and then rate at which work is done on the system. Left hand side rate at which the total energy of the system changes.

So, let us read it out formally, what this equation tells us. So, left hand side time rate of change of the total energy of the system is equal to the sum of the net rate of energy addition by heat transfer into the system, and the net rate of energy addition by work transfer into the system.

So, left hand side it has rate of change of total energy of the system as usual we have been very specific saying it is time rate of change. And right hand side we have two energy addition terms; one is by heat transfer other is by work transfer. Of course, it is rate of energy addition by heat transfer and rate of energy addition by work transfer. It has a term net both for heat transfer and for work transfer. The reason is heat could be added or removed work could be done on the system or by the system. So, net says it is net rate of heat addition and similarly net rate of work done on the system this into the system and on the system are same.

And so, now, let us come to the subscript which I have used there net is of course, representing as we have discussed now. This in represents heat added and work done on the system, in is used to say heat transfer into the system and work transfer into the system instead of out of the system. And of course, subscript system refers that we are writing for the system.

So, very clear description of the terminology there in terms of subscripts. One is net tells about both heat added and removed net heat added; taking into account the heat addition and the heat removal. Similarly, this net represents the net rate of work done on system taking into account work done on and by the system.

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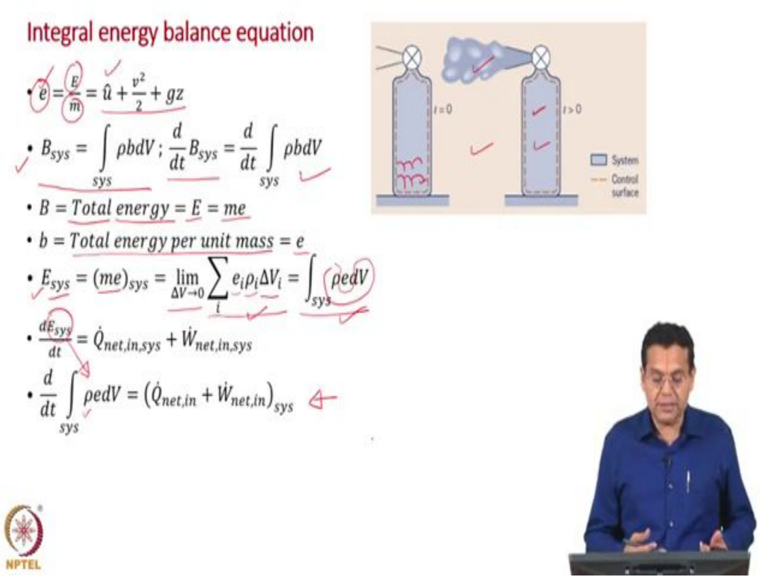

And now we will introduce terminology, we introduce a notation small e which is

$$e = \frac{E}{m}$$

The E is the extensive property of energy and if you divide the extensive property by mass, you get the intensive property of energy. So, when I say energy it represents some of all the energies per unit mass namely internal kinetic and then potential energy. So, E represents the extensive property energy; if you divide by mass you get the intensive property energy.

$$e = \frac{E}{m} = \hat{u} + \frac{v^2}{2} + gz$$

So, of course, we have internal energy per unit mass kinetic energy per unit mass and then potential energy per unit mass when you divide by m. Now, when we derived the Reynolds transport theorem, we introduce the extensive property in terms of integral given by this expression.

$$B_{sys} = \int_{sys} \rho b dV ; \qquad \frac{d}{dt} B_{sys} = \frac{d}{dt} \int_{sys} \rho b dV$$

And we also wrote the rate of change of extensive property in terms of this integral. Just to recall system quickly. So, that we can easily understand this example we have seen several times as an example for system we have a cylinder a time t = 0; the gas inside a cylinder is identified as system are sometime t later if you open the valve part of the gas escapes, but the system still consist of the gas remaining in the cylinder and whatever as escaped that is the system.

Now in the present case B represents total energy which is E, this total represents extensive property this energy represents sum of all the three energies. So, we can even say total-total energy; first total for extensive property, second total for sum of all the three energies. And extensive property here is E and if you write in terms of the intensive property it is

$$E = me$$

And we introduced b as the intensive property in this case it is energy which is total energy per unit mass and in our nomenclature it is e.

$$b = e$$

So, now, what we will do is write down this expression for the present case wherever b is e; e is total energy per unit mass.

$$E_{sys} = (me)_{sys} = \sum_i e_i \rho_i \Delta V_i = \int_{sys} \rho e dV$$

To get the extensive property for the entire system, sum over all the volumes and then we take the volumes smaller and smaller, we consider an infinite number of such smaller volumes. And so, ΔV the volume of each region goes to 0. So, in this limit we can express the summation as this integral what this means is, you are allowing for variation of density and the energy within the system. And ρdV is a mass and e is the energy per unit mass. So, when you integrate you get the total energy of the system.

So, now, let us write down the first law of thermodynamics in terms of rate, which we have written in the previous slide. And then now let us express the E_{sys} in the left hand side in terms of this integral expression. So,

$$\frac{dE_{sys}}{dt} = \dot{Q}_{net,in,sys} + \dot{W}_{net,in,sys}$$

$$\frac{d}{dt} \int_{sys} \rho e dV = \left(\dot{Q}_{net,in} + \dot{W}_{net,in}\right)_{sys}$$

Why do we write in terms of this integral expression? Now you are able to express in terms of the usually measured properties namely ρ, and e is the internal energy per unit mass, kinetic energy per unit mass and then potential energy per unit mass.

So, expressing in terms of this integral to expressed in terms of usually measured or known variables namely density, internal energy, velocity, elevation etcetera. How do you interpret this equation? This is the first law of thermodynamics for a system made of fluid particles that is the way we should visualize.

We usually apply first law of thermodynamics for let us say a gas inside a cylinder with a piston which goes from one state and another state etcetera. The way in which we should visualize this is, it is the first law of thermodynamics for a system and made of fluid particles and the system is flowing, that is why we have expressed this first law of thermodynamics. In terms of rate left hand side also it is rate of change of energy, right hand side rate of heat transfer rate of work transfer.

Reason for expressing this is that this is first law of thermodynamics for a flowing system made of fluid particles that is why we visualize this form of first law of thermodynamics.

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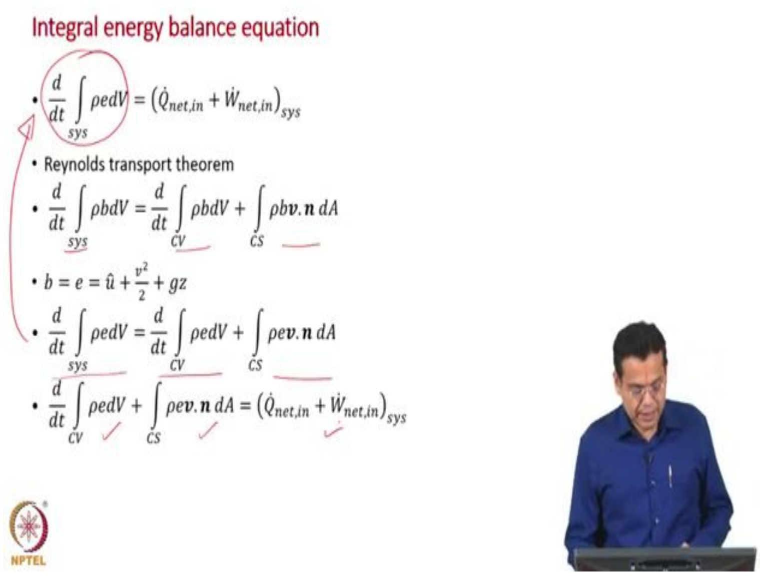

$$\frac{d}{dt}\int_{sys} \rho e dV = \left(\dot{Q}_{net,in} + \dot{W}_{net,in}\right)_{sys}$$

So, let us rewrite that equation. So, now, we will use the Reynolds transport theorem. And this is the general form of Reynolds transport theorem for any intensive property b, which relates rate of change of property by the system and to the rate of change of property for the control volume and net rate at which property leaves through the control surface.

$$\frac{d}{dt}\int_{sys} \rho b dV = \frac{d}{dt}\int_{CV} \rho b dV + \int_{CS} \rho b\, v.n\, dA$$

Now with the present case b is equal to e sum of all the three energies per unit mass.

$$b = e = \hat{u} + \frac{v^2}{2} + gz$$

So, let us rewrite that equation replacing b with e.

$$\frac{d}{dt}\int_{sys} \rho e dV = \frac{d}{dt}\int_{CV} \rho e dV + \int_{CS} \rho e\, v.n\, dA$$

So, now, how do you read this left hand side is rate of change of energy for a system; right hand side rate of change of energy for the control volume and net rate at which energy leaves the control volume through the control surface.

So, now what are we going to do? In the left hand side of the law of physics we are going to use the Reynolds transport theorem. So, that it gets expressed in terms of control volume and control surface. We are doing it the third time, first time for; first time for mass balance and second time for momentum balance, now for energy balance.

$$\frac{d}{dt}\int_{CV} \rho e dV + \int_{CS} \rho e\, v.n\, dA = \left(\dot{Q}_{net,in} + \dot{W}_{net,in}\right)_{sys}$$

So, now, in this form the left hand side tells about rate of change of energy for the control volume ok and the net rate at which energy leaves the control volume through the control surface; right hand side still in terms of system. So, we will have to express this for the control volume.

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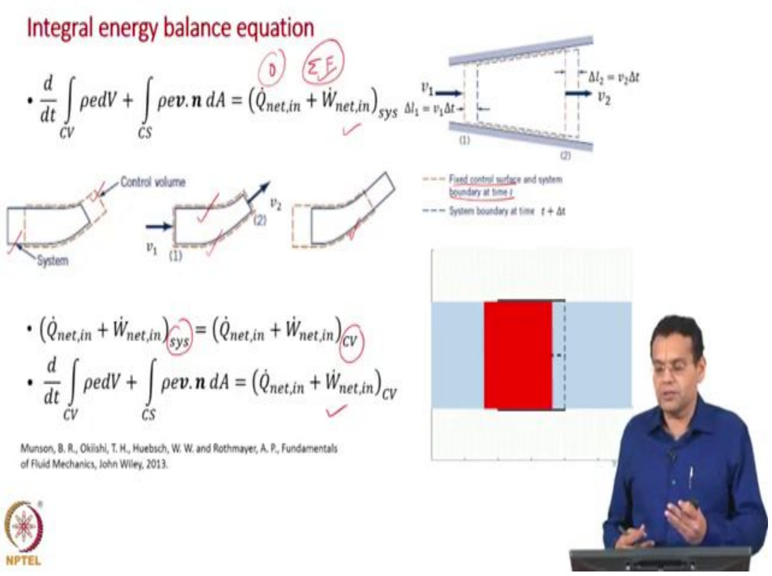

How do we do that? Let us rewrite that equation again,

$$\frac{d}{dt}\int_{CV} \rho e dV + \int_{CS} \rho e\, v.n\, dA = \left(\dot{Q}_{net,in} + \dot{W}_{net,in}\right)_{sys}$$

Now, to express the right hand side in terms of control volume, we have discussed the concept of coincident control volume and system what does it mean? At time t we consider the system and control volume to be coincident that is shown in three different ways here as earlier.

In the derivation of the Reynolds transport theorem for the simple geometry, we took the control surface and system boundary at time t to be coincident with each other. Meaning that, the control volume and system are coincident with each other. Second way of representing the control volume is shown here we could consider system it is just entering this is partly outside partly inside. We could consider a system whose boundary completely coincides with the control volume or a system is partly leaving the control volume which is partly inside partly outside.

What do we choose? We choose system which is coincident with the control volume. So, that it gets well defined, its boundaries are well defined which is same as that of the control volume and in terms of your animation. So, these are different possibilities for the system. We consider the possibility where the system just coincides with control volume and that is the instant we consider. So, different possibilities for system are there we consider the case where at time t the system and control volume coincide with each other, fine.

$$\left(\dot{Q}_{net,in} + \dot{W}_{net,in}\right)_{sys} = \left(\dot{Q}_{net,in} + \dot{W}_{net,in}\right)_{CV}$$

So, now, what happens the right hand side which is for the system now becomes that for the control volume. So, subscript now changes still remember it is net in both for the heat transfer term and the work transfer term, but system now has been replaced with control volume. So, let us now write down the complete internal energy balance equation, where all the terms now correspond to the control volume.

$$\frac{d}{dt}\int_{CV} \rho e dV + \int_{CS} \rho e\, v.n\, dA = \left(\dot{Q}_{net,in} + \dot{W}_{net,in}\right)_{CV}$$

So, left hand side we have the rate of change of energy for the control volume, second term is net rate at which energy leaves the control volume with the control surface. And now right hand side net rate at which heat is added and then net rate at which work is done on the contents of control volume.

So, like to mention we have done this system to control volume the third time. In the case of mass balance, the right hand side was just 0; in the case of linear momentum balance it was sum of all the forces acting on the system. Now, in this case it is the energy added term namely by heat transfer and work transfer.

This system to control volume change is more clear in the case of momentum balance and energy balance rare than the mass balance. Because we will be replacing 0 system with 0 control volume, we may not understand very clearly at that point of time. Now having done two more times for linear momentum balance we replace this sum of forces acting on the system by sum of forces acting on the control volume.

Now, similarly here net rate of energy addition by heat transfer and work transfer term for the system has been replaced for the control volume. So, now it becomes clear how we go from system to control volume; especially from linear momentum balance and the energy balance.

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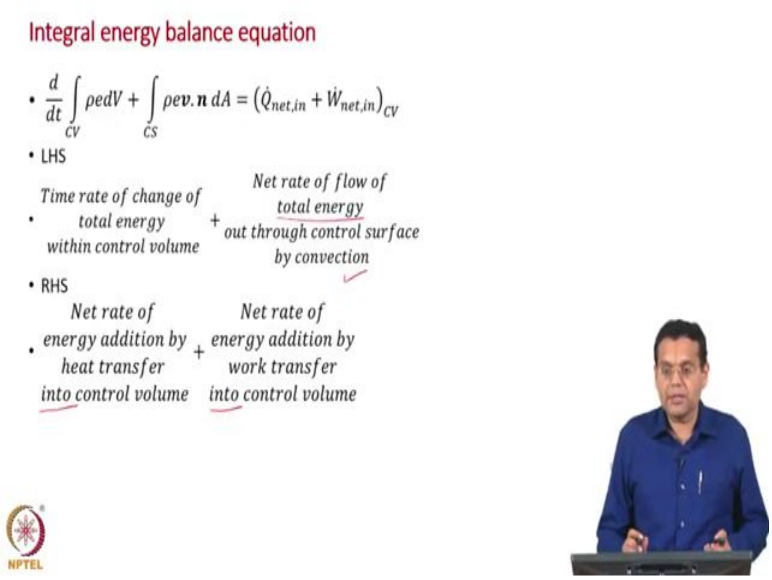

So, let us write down the integral form of energy balance equation, which we have derived in the last slide.

$$\frac{d}{dt}\int_{CV} \rho e dV + \int_{CS} \rho e\, v.n\, dA = \left(\dot{Q}_{net,in} + \dot{W}_{net,in}\right)_{CV}$$

Now, what are the significance of each term, ok? Left hand side, the first term tells about rate of change of total energy within the control volume more specifically time rate of change of total energy within the control volume.

And next term tells about net rate of flow of total energy out through control surface by convention; remember total energy here means sum of all the three energies. And right hand side what does it represent the first term represents net rate of energy addition by heat transfer into the control volume. And of course, next term on the right hand side gives the net rate of energy addition by work transfer into control volume.

So, left hand side we have the transient term, the first term is a transient term and then the second term on the left hand side is the convection term and right hand side we have the heat transfer term and the work transfer term. And, we should note that it is into the control volume based on the convection which I have discussed earlier. And this term by convection has a same meaning as we have discussed earlier for total mass balance and momentum balance. But now, the quantity of interest is the total energy or the property which are focusing in this balance equation is total energy.

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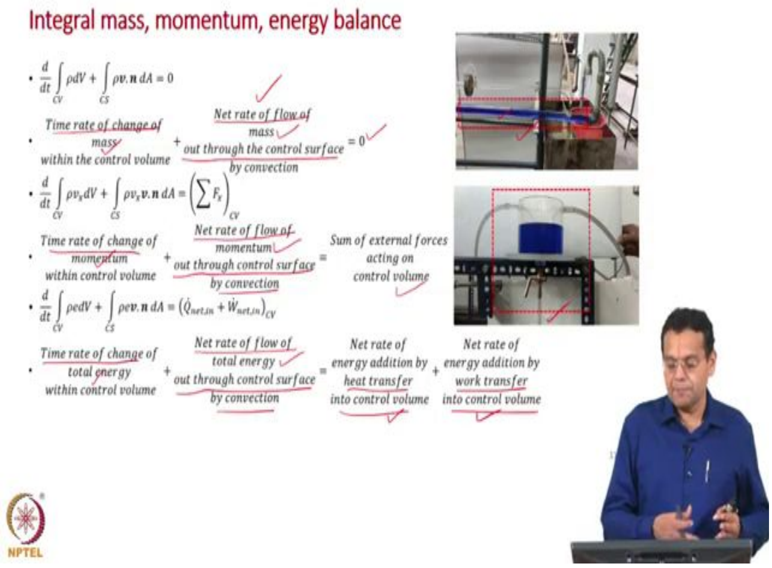

So, let us have an comparison between all the integral balance equations which are discussed so far;

$$\frac{d}{dt}\int_{CV} \rho e dV + \int_{CS} \rho e\, v.n\, dA = 0$$

that is the integral mass balance. Let us write down all the equations and look at the similarities between them and of course, what are the differences as well. So, that is integral total mass balance equation and the significance.

Now, let us write the integral linear momentum balance of course, in the x direction say vectorial equation. So, let us write down the equation in the x direction,

$$\frac{d}{dt}\int_{CV} \rho e dV + \int_{CS} \rho e\, v.n\, dA = \left(\sum F_x\right)_{CV}$$

And of course, right hand side is not 0 its sum of external forces acting on the control volume. So, let us write down the integral energy balance equation which we have derived now.

$$\frac{d}{dt}\int_{CV} \rho e dV + \int_{CS} \rho e \, v.n \, dA = \left(\dot{Q}_{net,in} + \dot{W}_{net,in}\right)_{CV}$$

How do we discuss all these equations? The better understand them we will have to keep in mind the control volumes which we have seen earlier. So, all these equations can be applied for control volumes drawn over entire equipments shown here for experimental setups or for a physical geometry like this; writing total mass balance, momentum balance, energy balance over the entire equipment where there is in flow and out flow.

Now, if you look at the first term on the left hand side; all of them tell about the time rate of change of first equation for mass, second equation for momentum, the third equation for total energy.

So, all of them tell about the rate of change of mass, momentum, and energy within the control volume, well known to us as the rate of accumulation term. The second term on the left hand side tells about the convection term, we have discussed about a convection term is whatever is carried by bulk flow by a stream. Suppose if you have stream entering and then leaving whatever property carried by this stream is the convection term.

And as we discussed now sometime back, the stream can be looked upon as carrying total mass, momentum or energy. In fact, total mass, momentum and energy and that is what is reflected by the second term on the left hand side. And of course, as per our sign convention all of them are net rate of flow out through the control surface. What is the property? It is total mass, linear momentum, and then total energy. The terminology has been maintained same as the property changes from mass to momentum to total energy.

So left hand side is very much analogous a transient term and a convection term; one which accounts for whatever is change happening within the control volume with respect to time and one which accounts for whatever is taken in taken out.

Right hand side the terms are not same because the law of physics is different. For the total mass balance it is zero for the linear momentum balance it is sum of external forces acting on the contents of control volume. And for energy balance it is net rate of energy addition by heat transfer and work transfer into the control volume.

The right hand side terms are of course, they are different, but left hand side terms are very analogous. If we keep this in mind becomes very easy to understand all the integral balance equations and the relationship between them as well. And this view point that if you look at a stream it carries total mass momentum energy, that view point helps us to write the especially the second term on left hand side, because that is by convection.

And if you look at the tank and then say that mass could change, momentum could change or energy could change with respect to time then that explains the first term on the left hand side. Of course, right hand side depends on the property for which we writing the conservation equation and that could be 0 could be forces or could be energy addition.