Continuum Mechanics And Transport Phenomena
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Lecture - 101
Viscous Stress vs. Molecular Momentum Flux Part 3

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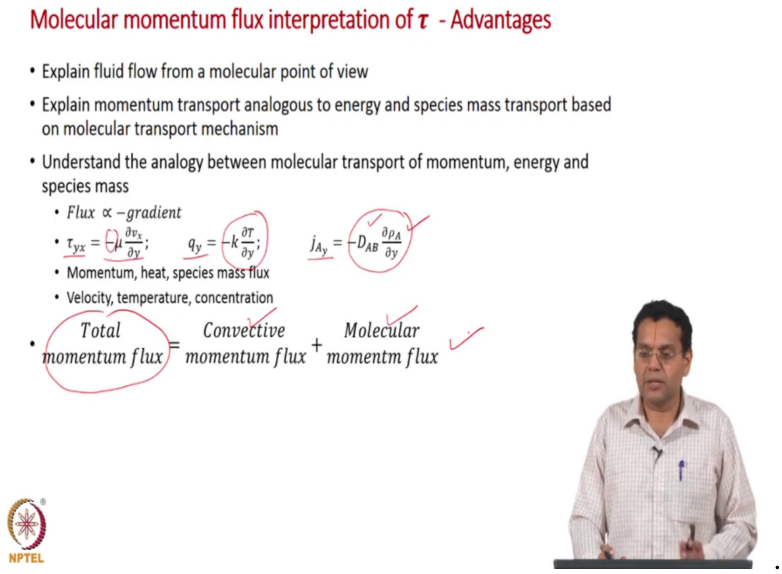

Now, we have interpreted τ in terms of Molecular Momentum Flux; what are the advantages, what are the limitations that is the discussion now.

First major advantage is that you can explain fluid flow from a molecular point of view; the explanation is from molecular point of view that does not mean that we are going to the molecular level of description, remember we said transfer phenomena can be at; transfer phenomena can be discussed at three scales or three levels macroscopic, microscopic and molecular. The first is statement does not mean that we are discussing at the molecular level, only the molecular interpretation can be given, still we are in the continuous hypothesis only. So, explain fluid flow from a molecular point of view.

Now, of course you can appreciate the second bullet after discussing energy transport and then species mass transport; but still we can explain momentum transport analogous to energy and species mass transport based on the molecule transport mechanism. We are talking in terms of molecules moving transporting momentum, the same molecules can transport energy, can transport species mass.

So, we can explain; we can explain the momentum transport analogous to energy and species mass transport based on the molecular transport mechanism. Or the transport of momentum, energy and species mass all can be explained using a molecular mechanism. Because we are able to explain the transport of momentum, energy and species analogously through the molecular mechanism; it also helps us to understand the analogy between the molecular transport of momentum energy and species mass. Because our explanation is common, or explanation using molecular mechanism is same we can understand, it helps us to understand the analogy between the molecular transport of momentum, energy and species mass.

$$Flux \propto -\ velocity\ gradient$$

We have seen our momentum flux is proportional to negative velocity gradient; or in general when we generalise, we can generalise it saying as flux propositional to negative of gradient. What does it mean? For the present case, the momentum flux or molecular momentum flux is propositional to the negative of velocity gradient; of course, in terms of equation becomes

$$\tau_{yx} = -\ \mu \frac{\partial v_x}{\partial y}$$

Later on we will see that, the heat flux or a let us say molecular heat flux is also proportional to a negative of temperature gradient and the equation is

$$q_y = -\ k \frac{\partial T}{\partial y}$$

This we call as the Fourier's law of heat conduction. Still later on we will see that, then molecular species mass fluxes is proportional to the negative of the mass concentration gradient and the equation is

$$j_{A_y} = -\ D_{AB} \frac{\partial \rho_A}{\partial y}$$

Here, k is thermal conductivity and D_{AB} is a diffusivity. So, all the molecular fluxes are proportional to the respective gradients and all of them have the negative sign as well. Suppose if you add a positive sign here, which was based on our old fluid mechanics convention, this analogy becomes a difficult.

Of course, we have also interpreted τ as viscous stress, we would not be able to look at this analogy; because we came to momentum transport and changed our viewpoint of τ as molecular momentum flux, we are able to look at this analogy. If we interpret τ as viscous stress, then this analogy is not possible, because we interpreted τ as molecular momentum flux, this analogy between molecular momentum flux, molecular heat flux and molecular species mass fluxes possible. All of them have similar looking equation; of course, we will call it as a fix law of diffusion.

So, in terms of the transported quantity it is momentum, heat, species mass; of course, the molecular momentum, heat, species mass flux and they are all proportional correspondingly to the velocity, temperature, concentration gradient, concentration thereby used mass concentration.

What is the other advantage? So far, we had τ on the right hand side expressing as viscous stress; because we brought it to the left hand side and then interpreted as a molecular momentum flux, this viewpoint enables us to sum the convective momentum flux and molecular momentum flux and lookup on as a total momentum flux. So, we can now say that, we have a total momentum flux that has a convective contribution and a molecular contribution.

$$Total\,momentum\,flux = Convective\,momentum\,flux + Molecular\,momentum\,flux$$

So, that is why this equation is written as, total momentum flux is equal to convective momentum flux plus the molecular momentum flux and this representation of total momentum in terms of convective momentum and molecular momentum, also analogously carries on to energy transport and species transport as well.

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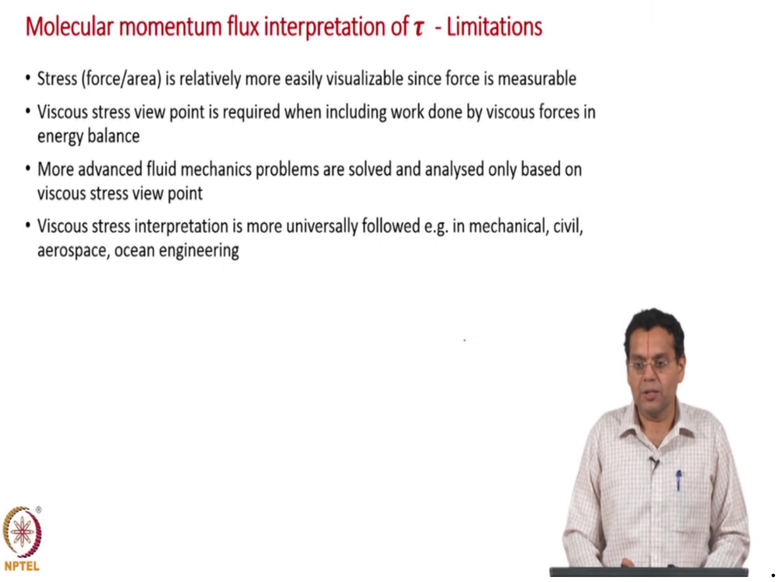

So, this view point once again establishes the analogy. Now so far we have been interpreting τ as a stress, which is force per area, certainly more easily visualizable then when I say molecular momentum flux. When I say τ as stress and force per area, first we can measure directly or indirectly. So, we can relativity, easily you can visualise stress compare to molecular momentum flux. Certainly much more easily, both are in fact difficult to visualise; on relative scale when you say stress and force per area, at least force we can measure in a lab, so relatively easy to visualized compare to a term called molecular momentum flux.

Now once again this, you may not appreciate until we got energy balance and then you may not also include this; but have an when in understand this later on may be advanced course. This interpreting τ as a viscous stress is required when you go to energy balance. Why is that? When you got the energy balance, we will include work done by body forces, work done by surface forces; surface forces are pressure and then viscous stresses.

So, what we do there is or what we include there is, we say work done by or rate of work done by pressure, rate of work done by viscous stresses; we do not say rate of work done by molecular momentum flux, that interpretation is not useful there. So, there when you go to energy balance and include this work done by viscous stresses; this mechanics view point is certainly required there, we cannot avoid that view point.

So, though in terms of a fluid flow, we can either take a view point of stress or momentum transport; when we got energy balance and want to include the work done by the viscous forces, the viscous stress viewpoint will be required, we cannot interpret as work done by molecular momentum flux, that is one of the limitations. Now this again is a limitation which at this interpreted level will not be able to appreciate and realise; but if you take a advance fluid mechanics course or if you are doing research and fluid mechanics, this third bullet plays as a very major role.

If you want to solve and analyse more advanced fluid mechanics problem; when I say more advanced fluid mechanics problem, these are problems I would say at graduate level or research level, at a post graduate level, research level. So, those problems are all solved only based on the viscous stress view point; it is very difficult to solve those problems extending the molecular momentum flux view point.

It helped us to explain from a fluid flow from a molecular view point; but to extend that and solve, more involved problems is difficult and only the viscous stress point will help us to solve such kind of problems ok. Most of the problem we talked about the flow field and then the force, resulting force, the force causing it etcetera. So, we always talk in terms of the viscous force only.

The other limitation is that the viscous stress interpretation is more universally followed; the momentum, molecular momentum flux interpretation I would say is more popular among chemical engineering, field chemical engineering discipline. But if you look at other disciplines, other engineering disciplines be it mechanical, civil, aerospace, ocean engineering the viscous stress interpretation is what is followed. So in that way, the viscous stress interpretation is more universal. And so these are some of the limitations of interpreting tau as molecular momentum flux.

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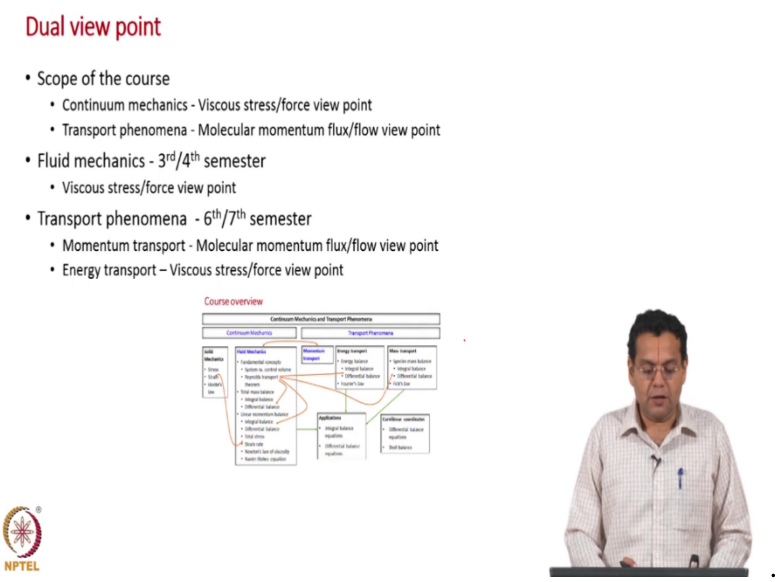

Now the question arises, what is the scope of this lecture for this course? Why did we discussed dual view point, what is the need for this lecture at all.

First, the scope of the course determine this lecture, I would say it is a unique lecture for this course. The scope of the course wanted us to look fluid flow from a mechanics view point, because we had continuum mechanics. The course also had transfer phenomena, so we need to look fluid flow from a momentum transport view point; which means that, we should look from a molecular momentum flux view point. That is why in this particular course we have this lecture, where we try to connect both this viewpoints.

So to put it down as note, continuum mechanics, where we viewed as viscous stressor a force view point; and transfer phenomena, we viewed as a molecular momentum flux or a flow view point, that is why this is required. Suppose if we have a course only on let us say continuum mechanics or fluid mechanics, we would just stop with interpretation as viscous stress only. Suppose we have a transfer phenomena course, we would just stop interpreting as molecular momentum flux; but because we are including both in this course and we need to connect and that is why we are discussing this. Also you would have had a course or you would be taking a fluid mechanics course in let us say 3rd or 4th semester, where the τ will be viewed as viscous stress or force.

Then you would take a transfer phenomena course, let us say in 6th semester, 7th semester, the course name will be transfer phenomena, the book name also may be transfer phenomena;

where you will view τ as molecular momentum flux and what we are done here is to connect. And let us say when you go later on to energy transport, once again you will view tau as viscous stress or a force.

What we have done today is to connect all this whatever you would come across in different courses in a different parts of the course; mechanics view point in fluid mechanics course yeah, momentum flux view point in momentum transport, once again viscous stress view point in energy transport. So we are, this idea of this lecture is specifically keeping these in mind so that you have a better idea of the two different viewpoints.

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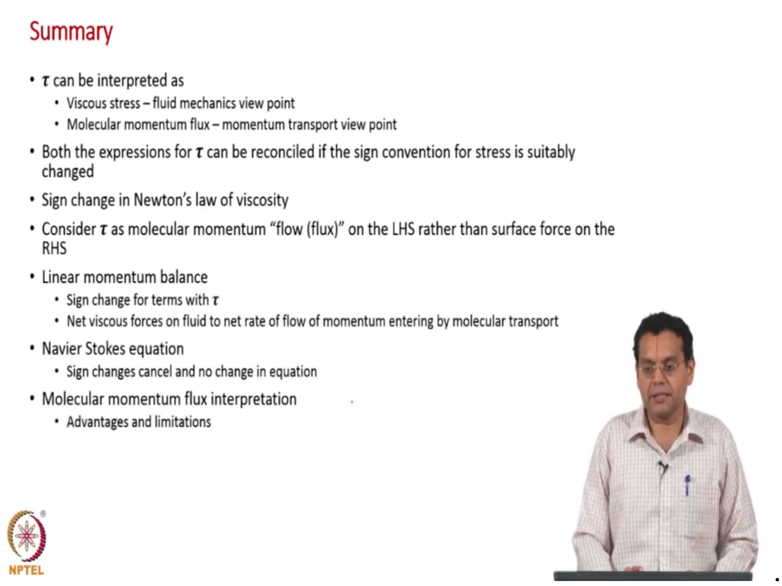

So let us summarize this unique lecture, τ can be interpreted as viscous stress that is a fluid mechanics view point; molecular momentum flux that is a momentum transport view point. I think that is a bottom line for the entire lecture. Both the expression for τ can be reconciled, if the sign convention for stress is suitably changed, we have discussed that. What is a consequence? There is sign change in a Newton's law of viscosity; and what happens in linear momentum balance? We considered τ as a molecular momentum flow or a flux on the left hand side rather than surface forces in the right hand side.

What happens because of that? Sign change for the terms with τ, all other terms of course remains same. So fluid mechanics, it was net viscous forces of the fluid on the right hand side; and if you keep it still on the right hand side, it becomes net rate of flow of momentum entering by molecular transport. If it is in the left hand side is leaving, if it is in the right hand side is negative sign and so it is entering.

So, net viscous forces on fluid on the right hand side that is a fluid mechanics view point to net rate of flow of momentum entering by molecular transport once again on the right hand side that is a momentum transport view point. What happens in a Navier Stokes? Sign changes cancel and no change in equation. And we also discussed the advantages and limitations of the molecular momentum flux interpretation.