Fluid and particle Mechanics
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Lecture – 09
Solutions of Navier Stokes - 2

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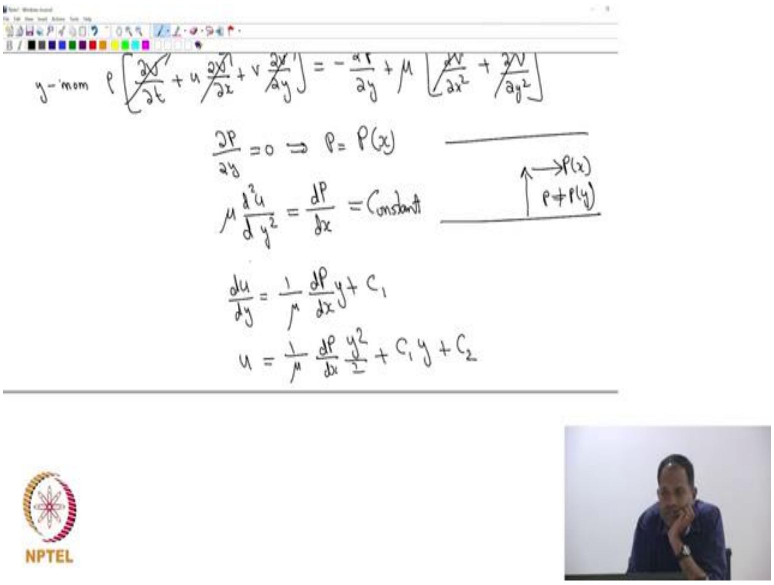

So; that means, now I what I am interested in is now to find out. So, we have already figured out now what pressure is, we understand a lot about pressure. We still do not know enough about velocity and we can figure that out if we integrate this equation because this is a is this now a partial differential equation or a ordinary differential equation. Its an ODE you know how to solve it, you can go ahead and solve it. So, its a second derivative and because this is a constant I can simply integrate it as:

$$\int \mu \frac{d^2 u}{dy^2} = \int \frac{dP}{dx}$$

$$\int \frac{du}{dy} = \int \frac{1}{\mu}\frac{dP}{dx} y + c_1$$

$$u = \frac{1}{\mu}\frac{dP}{dx}\frac{y^2}{2} + c_1 y + c_2$$

So, that is what my velocity field is going to be and C 1 and C 2 will be the constants that are coming out of integration and you have to find that out from the boundary conditions.

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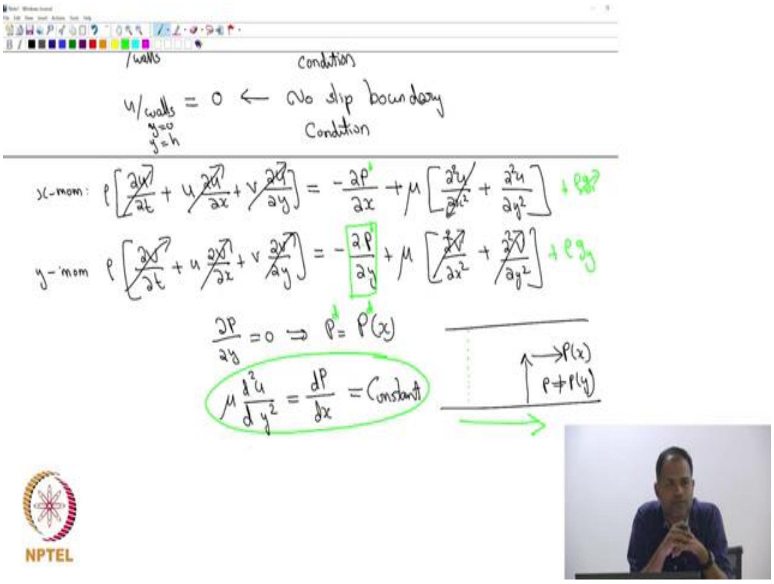

And you know what those boundary conditions are right we already wrote it down, we had written it down as somewhere there u at walls right so; that means, walls is either at y is equal to 0 and let us say some y is equal to h where h will be the width of the channel now. So, let us define all those things. So, that is my let us make another picture.

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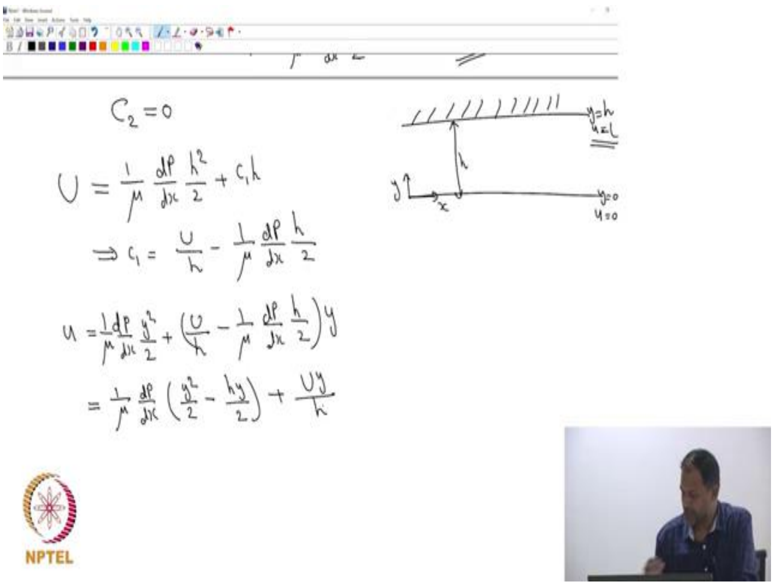

So, that was my original board that is what I called as x this is what I am calling as y let us say that is equal to h. So, this wall is y is equal to 0, this is y is equal to h and when y is equal to 0 you know u is equal to 0 at y is equal to h also that is going to be 0. Now let us do a slightly different problem because that is that will do you know two things in one go. So, what I am going to do now is that the same problem. So, what we have done really is that you have applied a pressure gradient in other words you have applied a large pressure here for example, you know you will put a pipe pump you put a pump, that will exert a large pressure here which will move the fluid in the channel right.

So, along with that let us say that you are not only applying a pressure gradient, you are also going to move this upper plate. So, this upper plate is going to go with a velocity capital U what would that do? That can also drive the fluid right because you take fluid between two plates; you are moving one of the plate the fluid will automatically move.

So, that basic basically what I am trying to say is that, there are two ways of moving fluids typically one is that you put a you know pump, in other words you apply a pressure gradient which can generate a flow or you move one of the boundaries like this and then it will also drive a flow ok. So, we are looking at a situation, now where you have applied a pressure gradient as well as you are moving a boundary ok. And then let us calculate what is the velocity profile and see you know how that how different those velocity profiles are going to be is that clear.

So, I am changing the gear a little bit now. So, so that is the situation. So, we know that so far so, till writing down that equation, we only cared about the. So, we did not really impose the boundary condition, only thing that I am doing is by imposing u is equal to you know capital U at y is equal to h I have just changed the boundary condition. So, this equation whatever we have derived so, far should remain ah. So, right is that clear ok. So, I am going to use that and apply these boundary conditions can you calculate C 1 and C 2 now? So, you have a fluid which is driven by a pressure gradient whose velocity profile is going to be this, but its also that fluid is in between the plates and one of the plate is moving with a certain velocity capital U.

So,

$$At\ y = 0, u = 0 \rightarrow c_2 = 0$$

$$At\ y = h, u = U$$

$$U = \frac{1}{\mu}\frac{dP}{dx}\frac{h^2}{2} + c_1 h$$

$$c_1 = \frac{U}{h} - \frac{1}{\mu}\frac{dP}{dx}\frac{h}{2}$$

Substituting the constants of integration we have,

$$u = \frac{1}{\mu}\frac{dP}{dx}\left(\frac{y^2}{2} - \frac{hy}{2}\right) + \frac{Uy}{h}$$

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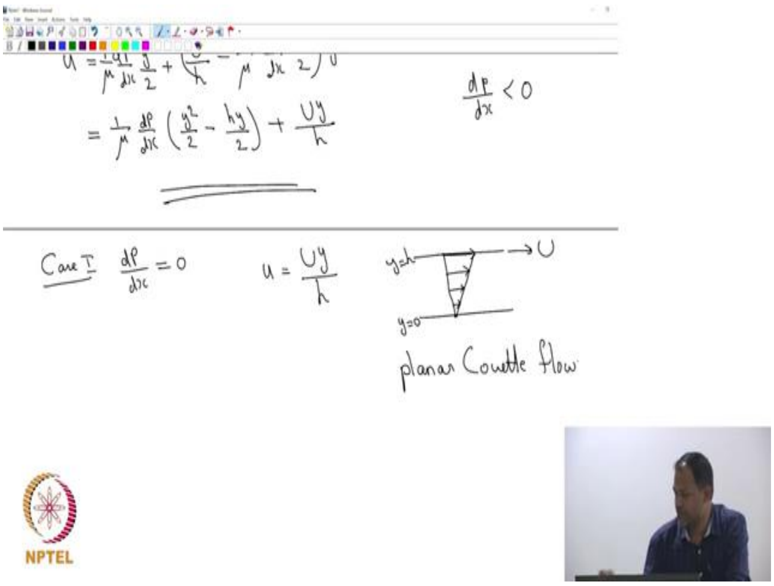

So, dP by dx will it be greater than 0 or less than 0?

Student: Less than 0.

So, you can see if you want to recognize the fact that dP by dx is less than 0 and write down this quantity as a minus dP by dx, this term will reverse that is typically the way you would write it. Let us see different cases:

$$Case\ I: \frac{dP}{dx} = 0 \rightarrow u = \frac{Uy}{h}$$

So, that means, I have a plate this was my y is equal to 0 and this was my y is equal to h at y is equal to 0, u is equal to 0 here this plate is moving with a capital velocity U. So, at cap y is equal to capital H u is indeed equal to capital U. So, it has some finite velocity and the variation is linear so; that means, my velocity profile is going to look like that right.

And that is something that you have already seen probably and this flow is known as one of the simplest flow fields that you can get and this is very important in terms of determining viscosity and so on this particular flow field and you will see that probably next week and week after how do we utilize this further.

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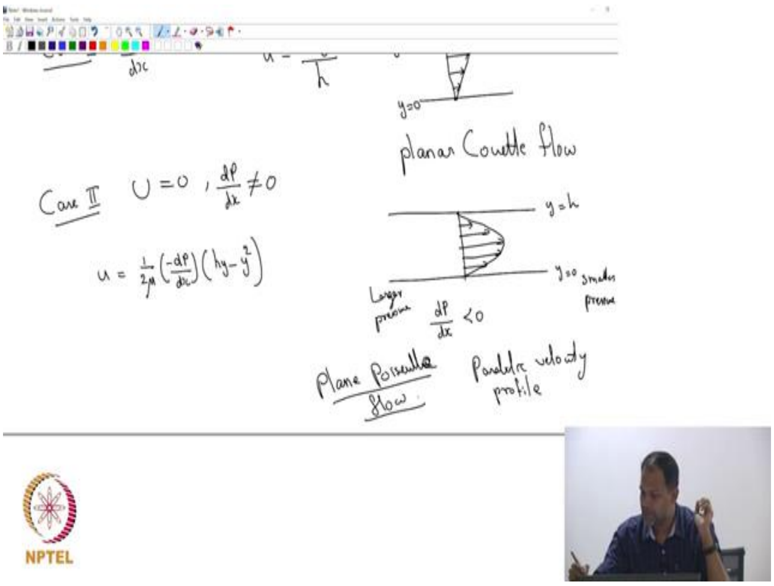

$$Case\ II: U = 0; \frac{dP}{dx} \neq 0$$

$$u = \frac{1}{2\mu}\left(-\frac{dP}{dx}\right)(hy - y^2)$$

So, you have plate y is equal to 0, y is equal to h, you have applied a pressure gradient dP by dx is less than 0 ok. So, because pressure will be larger pressure, here smaller pressure and fluid is flowing from larger pressure to smaller pressure and the flow profile is going to look like this, at y is equal to 0 u is 0 at y is equal to h u is again going to be 0 somewhere in between its going to be a maximum and that maximum can only be at the center then. So, it will look like that parabolic velocity profile. So, this flow is known as plane poiseuille flow doubts.

Student: Sir, suppose we do not make a pressure difference because of a pump or something, due to the fluid flow itself can there be pressure difference generated?

So, its the same thing. So, if you have a. So, in this case what you are asking is that you have generated a flow field, in which case there can already be a pressure difference ok. So, its a, so, because. So, if you look at this equation we ended up saying, so, that one ok. It says that a pressure gradient generates a fluid flow that is what I interpreted it as or you could say that yes there is a flow that will generate a pressure gradient and the way we typically do it is we apply a force and get a motion we do not say that we actually get a motion and because of that that flows existed which is an equivalent description but its probably less physical than interpreting it the other way ok. So, flow will be typically associated with a pressure variation or a pressure variation would generate a flow and.

Student: Sir, why did you neglect the gravity in the y momentum equation?

You could. So, right now I am looking at a pipe, that is actually perpendicular to the gravity ok. So, the only thing that would have happen is that, there can be hydrostatic pressure that is perpendicular to this ok. So, in reality that would mean that when we said this quantity right dP by dy is really rho g if you wanted to include that. So, the pressure will split into two parts, pressure will have a hydrostatic part which will balance the you know gravity force and then there is a pressure variation in the x direction, which is going to balance the viscous forces. Does that make sense? Because hydrostatic pressure will not generate a flow. So, its just that you know just that fact that is why we do not care about gravity at all.

Student: Is there gravity in the x direction?

Gravity is in the y direction.

Student: You should in the y direction you should write the g?

You can and what that will give you is that pressure will just come out to be rho gy which will have nothing to do with what we do next ok. P is only a function of y we did not care because I am saying that. So, read this pressure really can be interpreted as the total pressure minus the hydrostatic pressure.

P is, but we know everything about what is it in y, P is just. So, see the y. So, we have a uniform channel.. So, therefore, any point that you take if pressure is varying that is just according to this what is interesting for us is, what is happening in that direction which is what we did not know, which is what we calculated. So, now, you could if you like you could expand by saying that I am going to add my rho gx term here, I will add my rho gy term here which would give me dP by dy is equal to rho gy, where I am only talking now my gx is 0, I only have gy which gives me P is equal to rho gy.

Student: I am telling that it won't be constant then because this is function of y also there in a gravity like if mu d square u by dy square is a function of y.

So, that is what I am saying that the pressure has two components now, one is due to gravity the other is due to flow what we have calculated is the pressure due to the flow not due to gravity ok. So, in some, so, this pressure. So, let us do it this way, let us say that this is some let us I think this is called a dynamic pressure; that means, we have already removed the hydrostatic part from it. So, we will call it a Pd then its ok? And that is because gravity will not do anything, but if you were looking at a pipe which is perpendicular to gravity, then you should have been careful because these two components will add up in the same direction now ok.

So, we will see how to. So, see that is exactly what you do it in Bernoulli's equation; you have a pressure head, you have a velocity head, you can have a gravity head right if you talk about a pipe in a you know in a horizontal direction, then there is nothing gravity will do ok. Gravity will not generate any velocity, but if it was in the perpendicular direction or in the direction that is parallel to gravity, then gravity could induce a flow and which we will see how we will write down gravity included in terms of pressure field and so on at a later point. So, at the moment let us just say that either we have a situation in which our flow is perpendicular to gravity and therefore, the hydrostatic pressure is not going to do anything. Anything else? I need to do two more cases.

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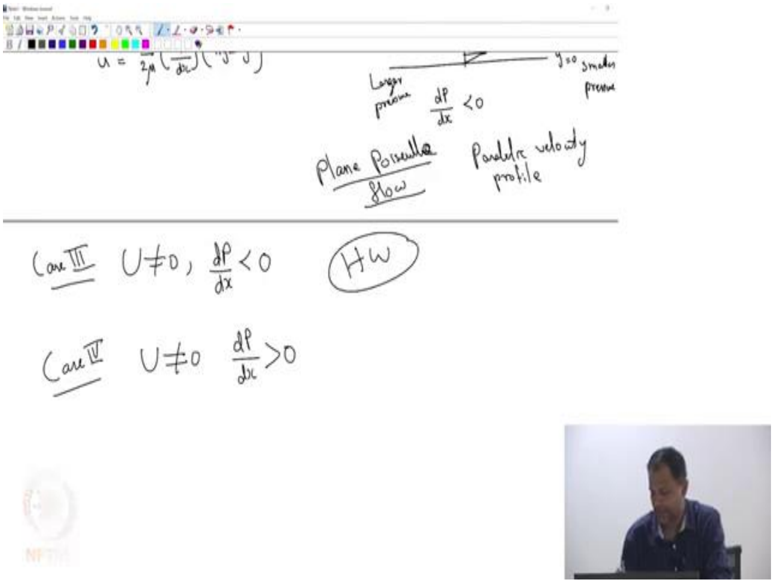

So, the next case you can plot and then come back next class.

$$Case\ III: U \neq 0; \frac{dP}{dx} < 0$$

$$Case\ IV: U \neq 0; \frac{dP}{dx} > 0$$