Fluid and Particle Mechanics
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Lecture - 70
Tutorial - 09

So, welcome to the Tutorial section of Fluid Mechanics and myself Chaitanya.

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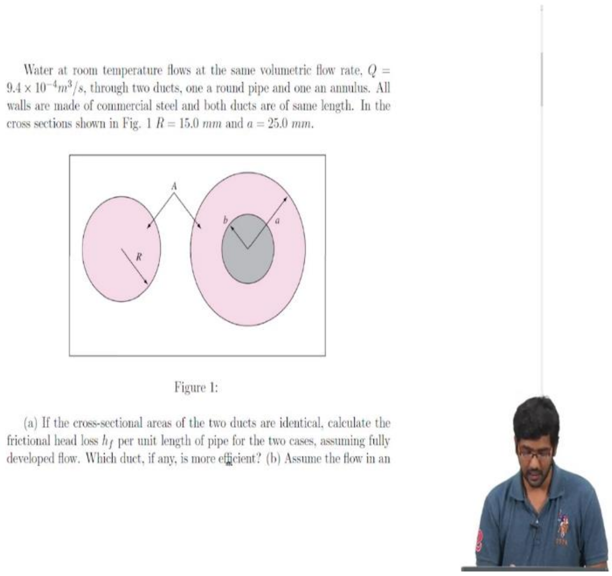

So, the problem that we are going to solve today is this water at room temperature flows at the same volumetric flow rate, cube equals to 9.4 into 10 power minus 4 meter cube per second through two ducts. One is the round pipe, other is the annular pipe which you can see in the figure and all walls are made of commercial speed and both the sort of same length and the dimensions are given. So, our objective is to calculate the head loss in the two pipes and comment on the efficiency of the two pipes. So, let us see how to solve this.

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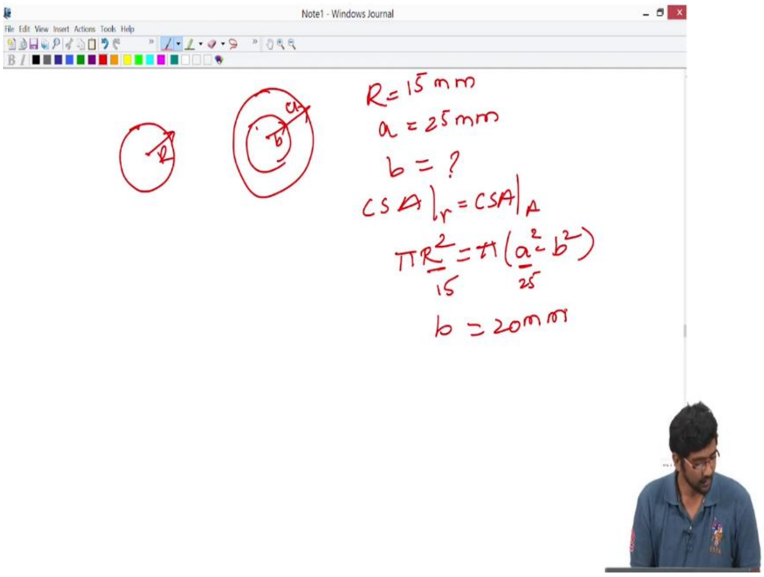

So, the geometry is one is a round one other is the annular pipe. So, if you look at the dimensions R is 15 mm a is 25 mm. So, R is 15 mm, a is 25. So, the first step is to calculate the unknown which is b. So, the information that is given is the cross sectional area of two ducts is equal. So, therefore, the cross sectional area of round pipe is equal to cross sectional area of the annular type.

So, this gives us pi R square is equal to pi into a square minus b square. So, we know R we know a, which is 25 mm and this is 15 mm and therefore, we can calculate b which is going to be 20 mm. So, therefore, we know all the dimensions in the problem, now we need to calculate the head loss in the circular pipe and the annular pipe.

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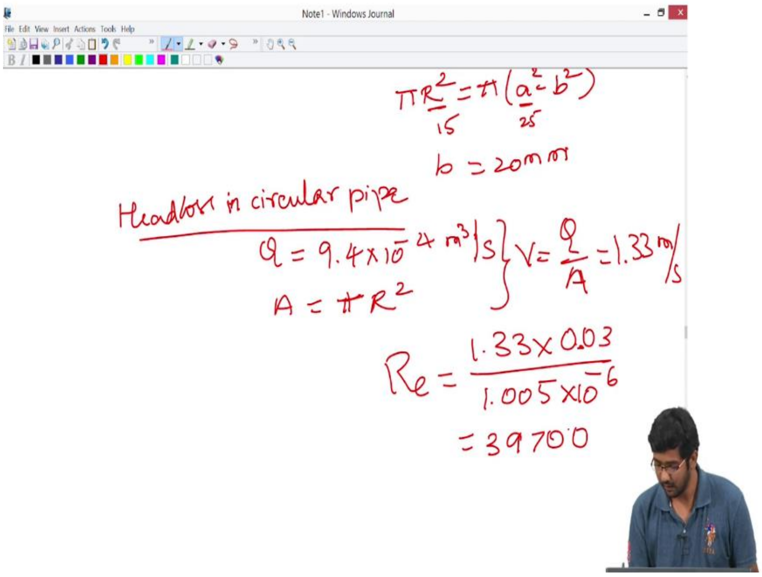

So, let us first calculate the head loss in the circular pipe. So, to do that first we need to calculate the Reynolds number and for that we need the velocity. In the problem the flow rate is given as 9.4 into 10 power minus 4 meter cube per second. And, we know the cross sectional area which is pi R square from this we can calculate V as Q by A which gives us 1.33 meter per second.

So, with this information we can calculate the Reynolds number which is b rho by mu. So, this is going to be 39700 and this is therefore, the flow is turbulent in this problem. Therefore, one needs to use the moody chart to calculate the friction factor and use that to calculate the head loss.

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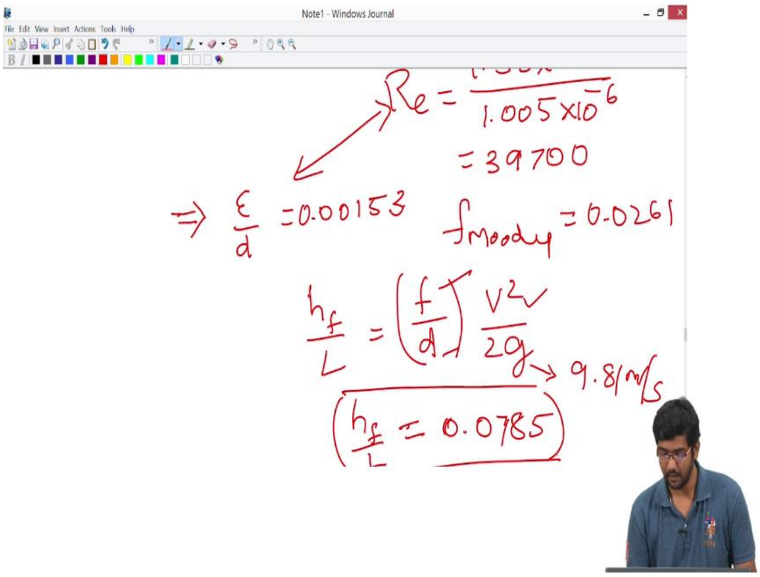

So, for the commercial steel as they have mentioned the pipe is made of commercial steel and the epsilon by the value is 0.00153. So, one can use this epsilon by the value and the Reynolds number, using both of these one can calculate the frictional coefficient from Moody chart which is 0.0261. And, from this one can calculate the head loss using h f by L is nothing, but f by d into V square by 2 g.

So, we have the information of f d and we know velocity and we know gravitational constant 9.81 meter per second. So, substituting all this we can get h f by L as 0.0785. So, therefore, for the circular pipe the head loss is 0.0785.

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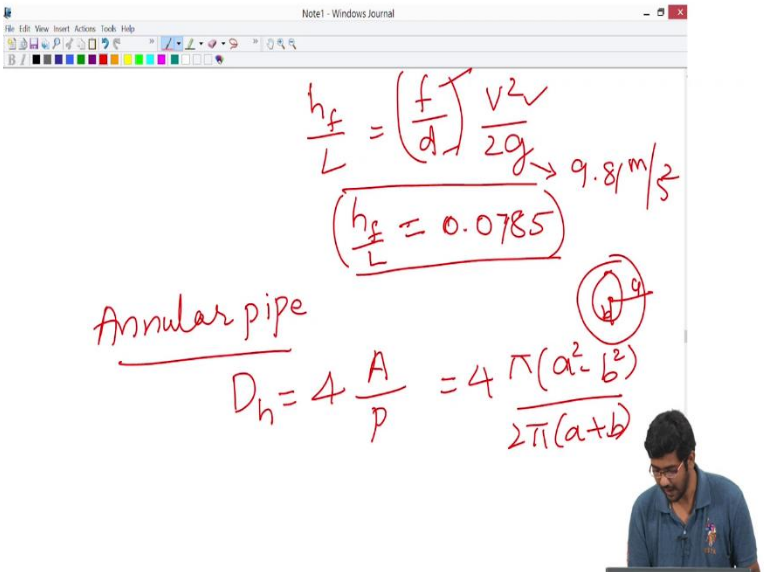

Yeah it is. So, it is meter per second square sorry. So, therefore, we have calculated the head loss in a circular pipe, we will repeat the calculations for the annular pipe, but in the case of an annular pipe we need to use the hydraulic diameter.

So, calculations for annular pipe; so, in the case of an annular pipe we need to use the hydraulic diameter which is defined as 4 times of area by vector parameter. So, for this geometry which is annular pipe this is a and this is what we have to b. The area is 5 times a square minus b square and this is going to be pi times a plus b 2 pi R and for annular is 2 pi R a plus b.

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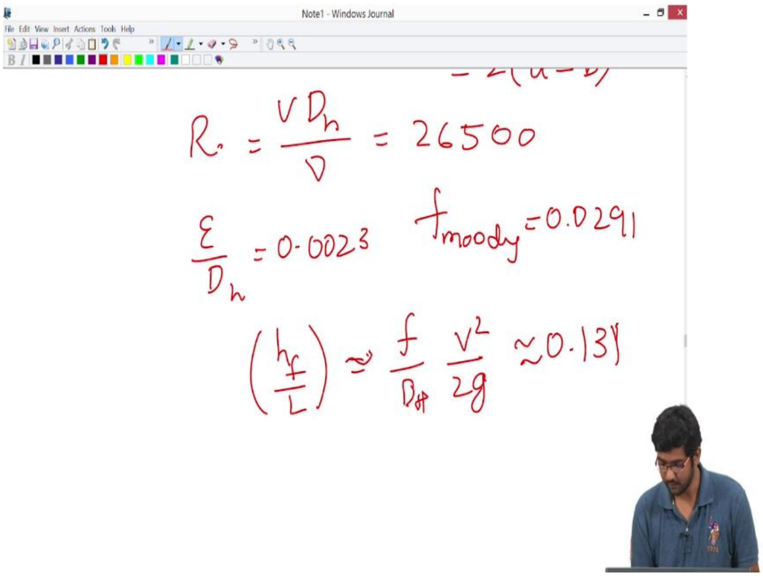

So, this gives us 2 into a minus b. So, this is the hydraulic diameter and so therefore, we calculate the Reynolds number based on this hydraulic diameter which is velocity D h by nu. So, as we have; as it is given in the question, the cross sectional areas of both circular and annular pipe are same; therefore the velocity fluid velocity in both the pipes will be the same because the volumetric product remains same. And, we have we can calculate the hydraulic diameter and we know the kinetic viscosity; substituting all this we will get the Reynolds number as 26500.

Therefore, even in annular pipe the flow is turbulent and here it is going to be 0.023 epsilon by D h value from which we can calculate f moody which is 0.0291. So, now we have the factor from which we will be able to calculate the head loss in the same way as we the for a circular pipe. So, by now it is going to be an approximate value because we are using the hydraulic diameter concept. So, this will be around 0.131.

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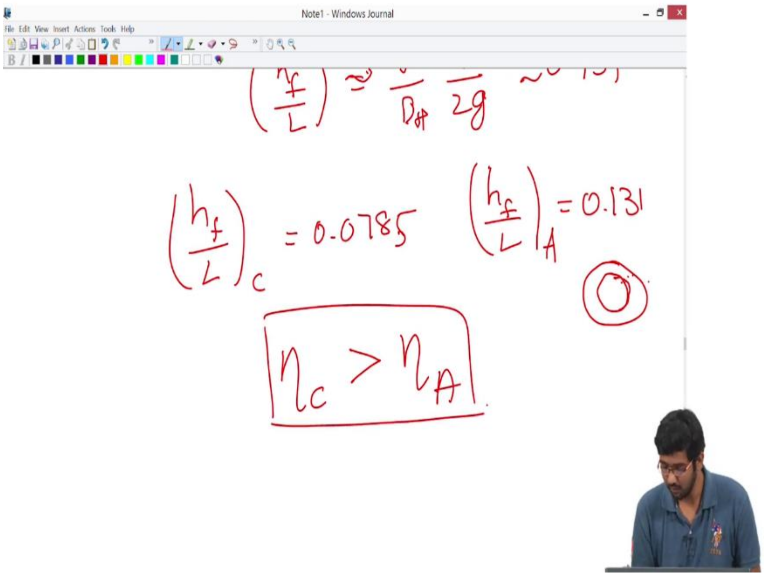

So, if I look at the compare the values of head loss for a circular pipe it is around 0.0785 and whereas, for annulus it is going to be around 0.131. Therefore, it is clear that in the case of an annular pipe the head loss is more and it make sense because in the case of an annular pipe, the fluid is in contact with the more wall area. Because, the fluid is in contact with the inner wall as well as the outer wall therefore, there will be high friction. So, we can say efficiency of circular pipe is greater than efficiency of the annular pipe. So, this is question a.

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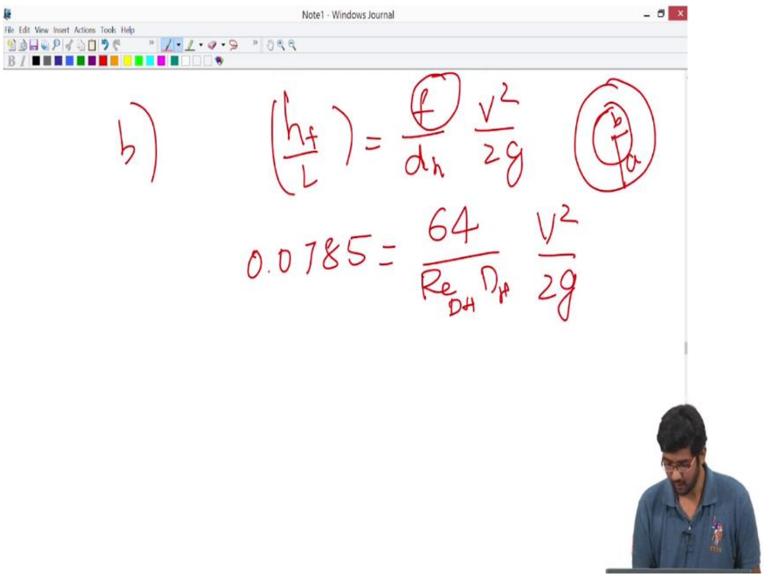

Question b. So, in question b it was asked to assume; in question b it was asked to assume in an annular duct, the flow in an annular duct is laminar and suggest the value of a and b that gives a head loss same as that of a circular pipe. So, here what we do is that we have an expression for head loss 2 g and we were asked that the flow assume that the flow in a annular duct is laminar. Therefore, we can directly calculate f because we have a relation between f and Reynolds number in the case of a laminar flow.

And, we need to suggest the values of a and b which is the radius of inner pipe and the outer pipe that gives the head loss same as that of a circular pipe. So, for the circular pipe we have obtained the head loss as 0.0785. And, in the case of laminar flow f is 64 by Reynolds number which is defined based on the hydraulic diameter and we have one more D H here and we have V square by 2 g.

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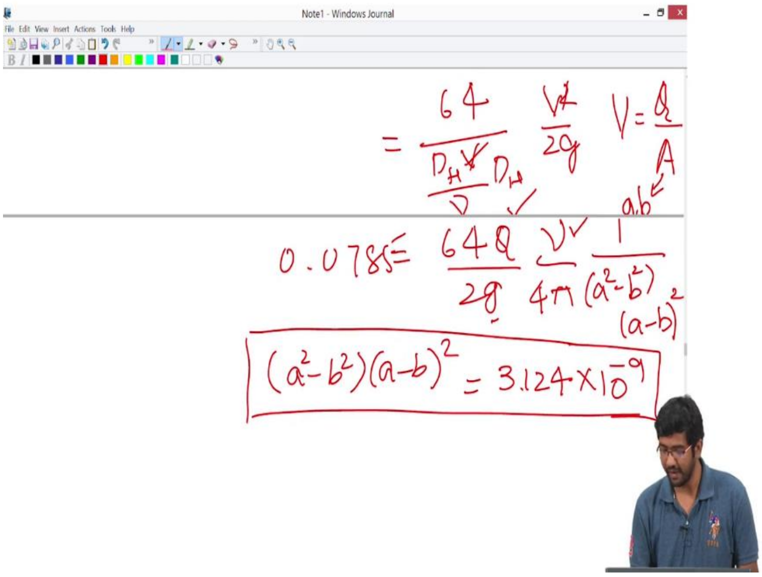

So, if I substitute everything 64 D H velocity by kinematics viscosity into D H to V square by 2 g. So, this V and this V got cancel off and we end up with an equation. So, if I substitute V as Q by A because I know volumetric flow rate, but not the velocity and I know area is unknown because we do not know what is a comma b in this problem. Therefore, I will end up with an equation 64 Q by 2 g to nu by 4 pi into 1 by a square minus b square into a minus b whole square 0.0785.

So, volumetric flow rate is given in the problem, we know the kinematic viscosity and we know the gravitational constant and everything except a and b. So, I will take the unknowns to one side and I will end up with, if I substitute the corresponding values I will get to 10 to the power minus 9. So, this is the equation that governs the values of a and b.

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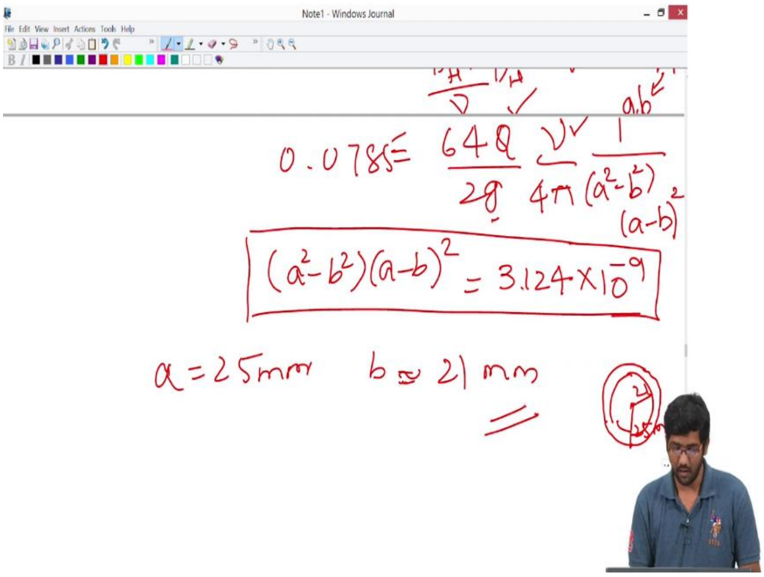

And, if we assume if you take the value of a equals to 25 which is given in the schematic, the value of b that we get is around 21 mm. So, one can get if you want to have the efficiency of an annular pipe same as that of a circular pipe then what one we need to have a very thin annular ring. So, you can just see this is I get 21 mm and this is 25 mm. So, this should be the annular parameter that one should use to get the head loss same as that of a circular pipe. So, therefore, this is about this question.

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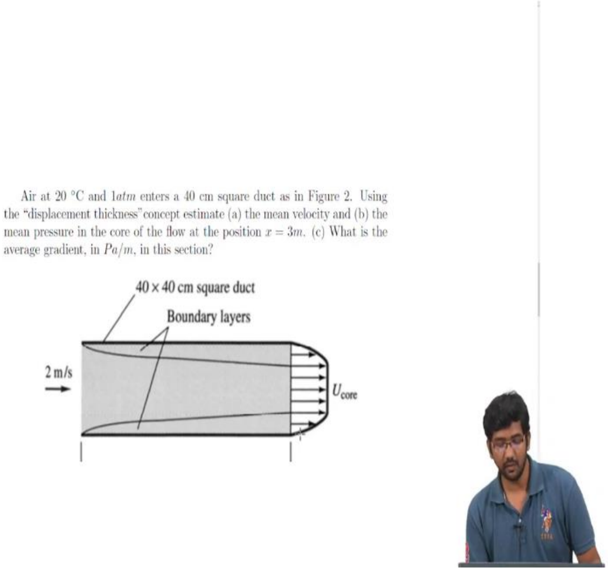

Now, let us move on to then second question. So, in this question we use the concept of displacement thickness to calculate the pressure gradient and the mean velocity. So, the question goes like this air at 20 degree centigrade and 1 atmosphere enters a 40 cm square duct as shown in the figure. And, using the displacement thickness concept we are asked to estimate the mean velocity, the mean pressure in the core of the flow at the position x is equal to 3 meters and what is the average gradient in Pascal per meter in this section.

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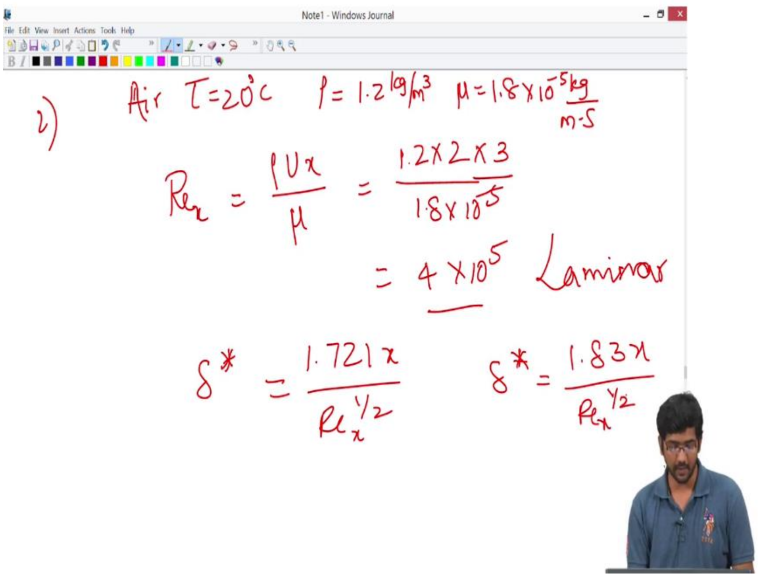

So, let us try to answer this question. So, this is question number 2. So, as they have given air at temperature 20 degree centigrade, the parameters can be obtained from the corresponding tables where density is going to be 1.2 kg per meter cube and the viscosity is 1.8 into 10 power minus 5 kg per meter second. So, we know the air properties, now we were asked to calculate the velocity at the exit. So, if you look at the question so, we are asked to calculate the mean velocity which means the velocity at the exit.

To calculate that we need to impose the continuity equation we will get to that, but first let us calculate the Reynolds number which is defined for this problem as rho U x by mu and we have an inlet velocity and we know density which is. So, density is 1.2, velocity is 2 meter per second which is given and the distance is 3 and viscosity is 1.8 into 10 power minus 5. Substituting all this we will get the Reynolds number as 4 into 10 power 5 which means the flow is laminar, as it is a plate problem.

Now, using the displacement thickness concept as we know the Reynolds number and we know the distance which is x equal to 3 meters, we will be able to calculate the displacement thickness which is delta star. So, there are two formulas to calculate delta star; one is based on the exact calculation which is 1.721 x by R e x power 1 by 2. So, this is based on the exact calculation and if we assume the profile in the boundary layer is parabolic then what we get is something very close to that which is 1.83 x by R e x power 1 by 2. So, the relative error between these two will be 6 per around 6 percent.

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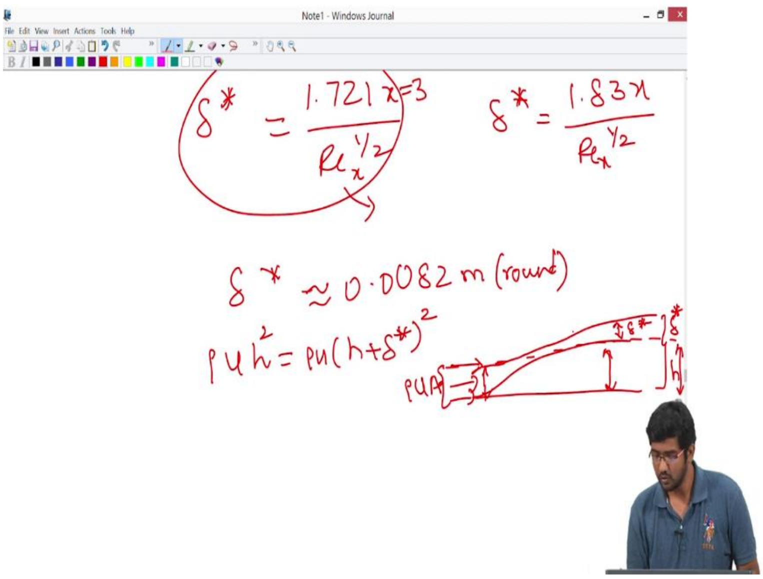

So, now we will use the exact equation maybe you can try using the equation that we have obtained assuming the flow is parabolic. So, we know x which is 3 meters and we know Reynolds number so, that gives us the delta star as 0.0082 meters so, it is a rounded decimal. So, now we need to impose the continuity equation, let us look at the geometry again. We have a fluid entering so, even before that let us look at what is displacement thickness and how we can use it to understand this how to impose the continuity equation.

So, suppose you have a flat plate and the fluid is entering and we know that there will be a boundary layer development and displacement thickness is the measure to quantify the amount by which the outermost streamline gets displaced. So, this is what delta. So, if so, the streamline which is just outside the boundary layer is getting displaced because of the boundary layer affects. So, this displacement is quantified by displacement thickness. So, if you want impose the continuity equation so, if one wants to impose the continuity equation. So, this is fluid which is entering or the mass flux rho u into A.

Now, here the area is changed. So, here the area is based on this height whereas, here area is based on let say this is h so, this is delta star. So, we need to say that rho u h is equal to rho u h plus delta star. So, if we assume it is a square duct it is like h square into h plus delta square. So, this is the case for a flat plate.

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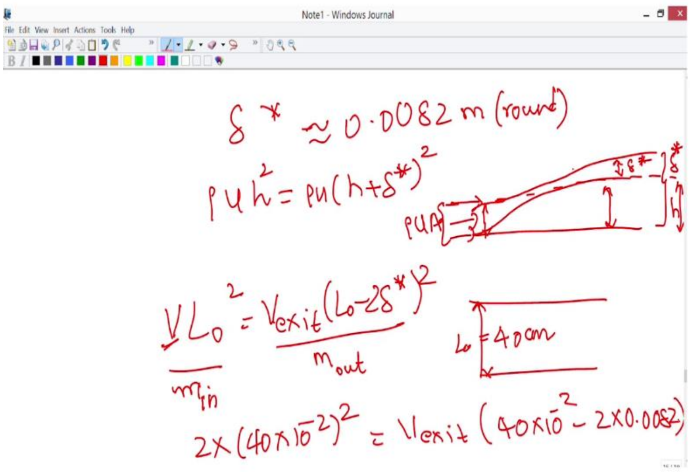

Now, if we use the same concept for the given geometry which is a channel, what we will end up with this and they have given the dimensions of the channel; this is a 2D channel and they have given it is around 40 centimeters. So, here what we get is V into L naught square equals to V exit which is an unknown into L naught minus 2 delta star whole square.

So, you can see so, this is the inlet velocity, we know inlet velocity, we know the dimensions of the channel therefore, we can calculate the inlet mass flux which is rho u into s. And, similarly we know the displacement thickness and using that we can write the out mass; this is the mass flux out and this is the mass flux in. So, we know the entrance velocity which is 2 meter per second, L naught is given in the problem which is 40 centimeters. So, V exit is an unknown, L naught and we have calculated delta star.

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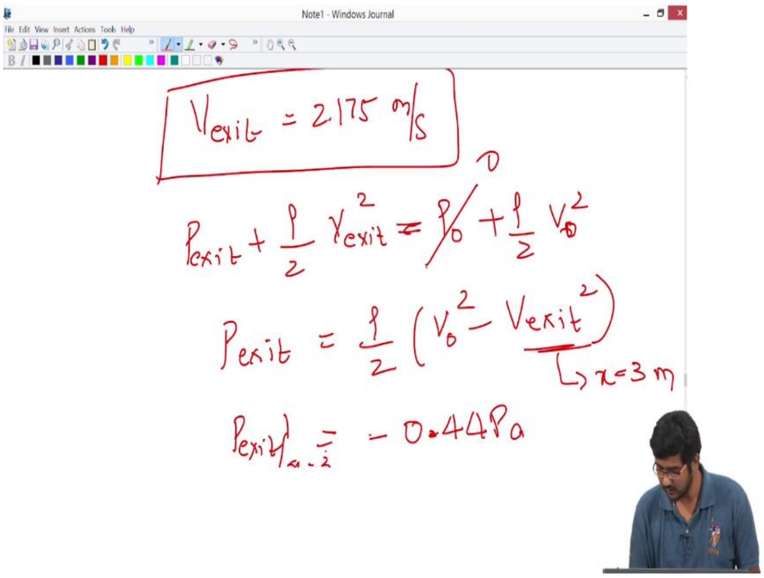

So, from this we can calculate V exit as 2.175 meter per second. So, therefore, we have calculated the a part which is the mean velocity and then we were asked to calculate the mean pressure in the core of the following at the position x equal to 3 meters. So, to calculate the pressure because we have a velocity data and we have an inlet pressure so, we can impose the Bernoulli equation. So, which is P exit plus rho V square by 2 rho V exit square equals to P inlet or I will define as P 0 plus rho by 2 V 0 square. So, as the inlet pressure is atmospheric pressure let us work with the gas pressure which means we are subtracting atmospheric pressure from the given pressures.

Therefore, as inner pressure is already 1 atmosphere, this goes to 0 because we are looking at the gas pressure. So, what we can get is the P exit which is rho by 2 times V inlet square minus V exit square. So, this will be around 0.44 Pascal’s and you should remember that this V exit corresponds to x equals to 3 meters. Therefore, this P exit corresponds to x equal to 3 meters. So, therefore, we have calculated the pressure and the next question is what is the average pressure gradient in Pascal per meter.

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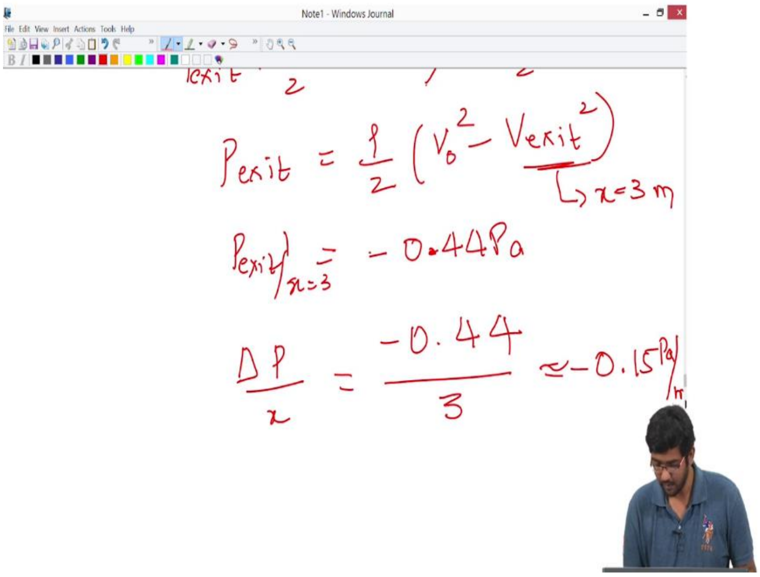

So, the pressure gradient is nothing, but delta P by x, as we are working with the gauge pressures we can write delta P as 0.44 and x in the problem is 3 which gives us around 1.5 Pascal per meter. So therefore, using the displacement thickness concept we have calculated the mean velocity and using that mean velocity and the Bernoulli’s expression, we have calculated the pressure and from that we have calculated the pressure gradient. So, in this way we can use the displacement thickness concept in the case of a channel and calculated the exit velocity and the pressure gradient. So, this ends the tutorial.

Thank you.