Fluid and Particle Mechanics
Prof. Sumesh P. Thampi
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture - 67
Boundary Layer - Momentum Integral Analysis – 2

So we are looking at the boundary; sorry boundary layer flow and we started out by looking at an integral approach ok. We basically looked at what does the mass conservation tell us, what does the momentum conservation tell us. And that is we defined drag force and we defined the drag force the force exerted by the plate on the fluid as an integral, so the first integral that you see is what they wrote down is that the all right.

And then we said we do not know what the profile is going to be, but it is a reasonable approximation that we you know use a parabolic profile for the velocity field. And if so we can calculate what is the drag force on the party, on the plate or the force exerted by the plate on the fluid. So, we said if you take that second expression U as a quadratic expression that is something that satisfies the boundary condition, that is something that varies quadratically with the with Re. So, we can use that expression and try to calculate what is the drag force ok, so that is where we ended up, so we could continue.

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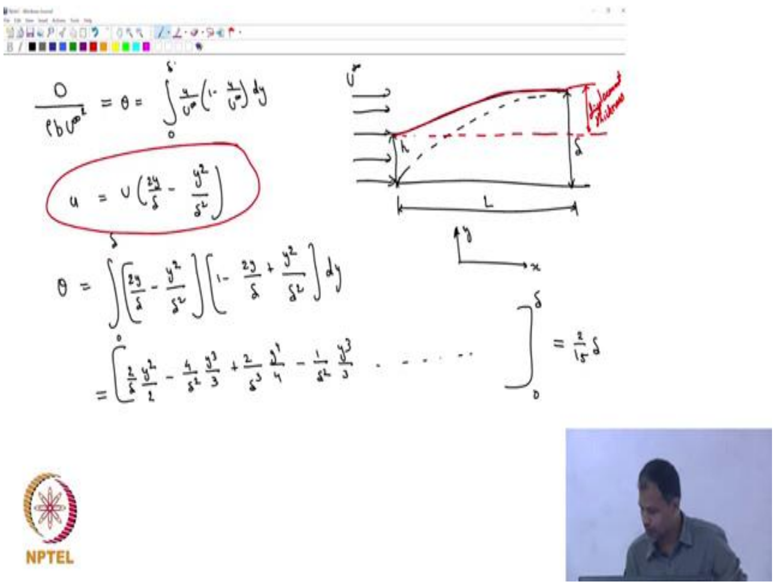

We will first calculate this so called momentum thickness theta equal to integral 0 to delta U, so we need U by U infinity. So, that is 2 y by delta minus y square by delta square times

1 minus U minus U by U del U infinity that is 1 minus 2 y by delta plus y square by delta square d y.

Equal to you could go ahead multiply an integrate q y q by delta into y is y square by 2 minus 2 into 2 4 by delta square y square that is y cube by 3. Then plus 2 by delta cube y cube that is y raised to 4 by 4 minus 1 by delta square y square that is y cube by 3 and so on keep going all of them and the apply a limit and see what we get 2 by 15 delta.

$$\theta = \int_0^{\delta} \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right]\left[1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right] dy = \left[\frac{2}{\delta}\frac{y^3}{3} - \frac{4}{\delta^2}\frac{y^3}{3} + \frac{2}{\delta^3}\frac{y^4}{4} + \cdots\right]_0^{\delta} = \frac{2}{15}\delta$$

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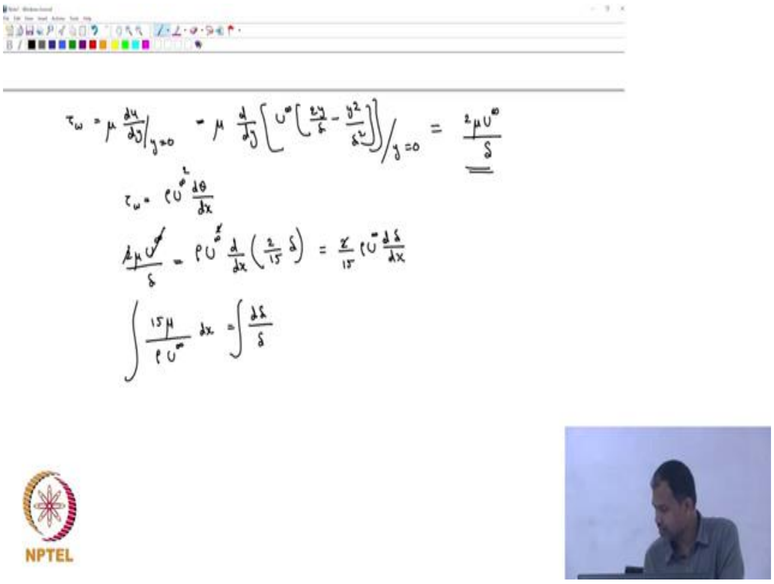

So, that is theta, now we can calculate the wall shear stress tau w as mu into du by d y at y is equal to 0. So, that is mu into d by d y of U that is U infinity into 2 y by delta minus y square by delta square at y is equal to 0. So, what does is that give you ok, while we are writing it down. Because we know tau w in terms of theta what is the expression what is the connection between tau w and theta?

$$\tau_w = \mu \frac{du}{dy}|_{y=0} = \mu \frac{d}{dy}\left[U_\infty \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right]\right]|_{y=0} = \frac{2\mu U_\infty}{\delta}$$

We derived tau w is equal to rho U square d theta by d x ok and we have got an tau w now as this. That means, 2 mu infinity divide by delta is equal to rho mu infinity d by d x of 2 by 15 delta. So, one of them will go away this is equal to 2 by 15 rho U d delta by d x, remember delta is a boundary layer thickness and it changes with x. Therefore, so we will go we have 15 mu U infinity by rho U infinity d x is equal to d delta divided by delta 15 mu by rho infinity d x is equal to d delta by delta.

$$\tau_w = \rho U_\infty \frac{d\theta}{dx}$$

$$\frac{2\mu U_\infty}{\delta} = \rho U_\infty \frac{d}{dx}\left(\frac{2}{15}\delta\right) = \frac{2}{15}\rho U_\infty \frac{d\delta}{dx}$$

$$\int \frac{15\mu}{\rho U_\infty} dx = \int \delta \, d\delta$$

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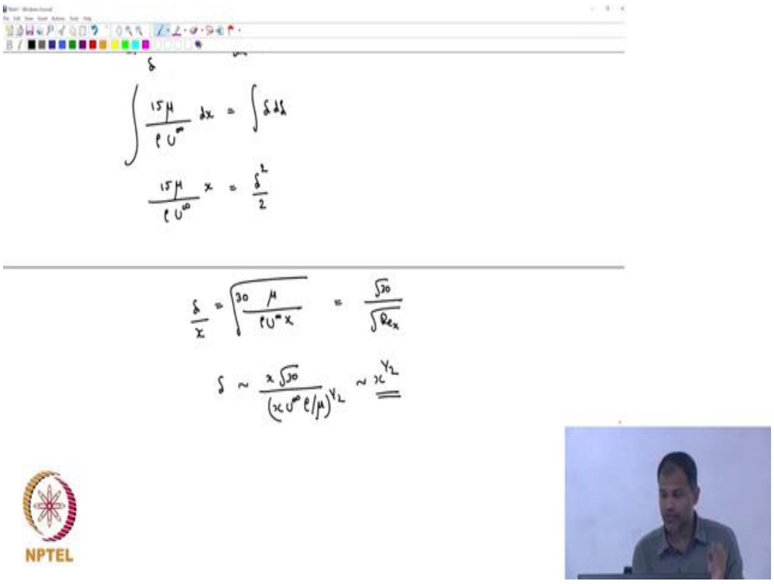

And integrating do not to get 15 mu by rho u infinity into x is equal to lon delta.

What is that, did I make a mistake?

It is a that way, so we can not be a log right, so it is delta d delta integral that is right. So, that is delta square by 2 how could it that is because we have supposed to get a rule and that is where I get delta equal to 30 rho U infinity x divided by mu square root. Now, there is something else missing what is that I will missing.

I am ending within with a dimensional quantity equal to a non dimensional quantity.

We no fine ok, this x is down stress right, so I should be basically doing that when I move, so on the f write basically you have a root of 30 times R e x ok. So, this is what we had said when we started talking about boundary layer this thing that the thickness of the boundary layer grows with x and for a laminar fluid laminar boundary layer it goes approximately a square root of Reynolds x ok. So, if you assume that the profile is actually parabolic you basically end up doing that or ended seeing that.

$$\frac{15\mu}{\rho U_\infty}\, x = \frac{\delta^2}{2}$$

$$\frac{\delta}{x} = \sqrt{\frac{30\,\mu}{\rho U_\infty x}} = \sqrt{\frac{30}{Re_x}}$$

So, where should I change is 30 by because I have in done mu delta by x is equal to x square 13 by rho U infinity x this is right. So, therefore, I should really have root 30 divided by root of R e x is this right. So, that we have delta going as x root 30 divided by x U infinity rho by mu to the power of half and that means as x to the power of half.

So, the other thing is that you know actually the Reynolds as the Reynolds number increases the boundary layer thickness should go down ok. So, it has to be a one by relation that also it tells ok, the Reynolds number increases the thickness is there. So, that mean, so see what we assume was this, this is what we assume for the laminar velocity profile. So, if you could you know write down an expression for the turbulent velocity profile. Then you would be able to see how does it actually change in a turbulent boundary layer as well, what is this scaling in turbulent boundary layer? X to the power of 6 by 7 or something, so you could a get now, the other thing is this stream line that we do the other day. So, that stream line is getting displaced because the fluid below is getting retarded. so that is a argument that we had ok. So, there is a, so how much it gets you know displaced this distance ok. So, if comes at h and it reaches a delta, so that h minus del delta minus h is called displacement thickness, so this thing is called displacement thickness.

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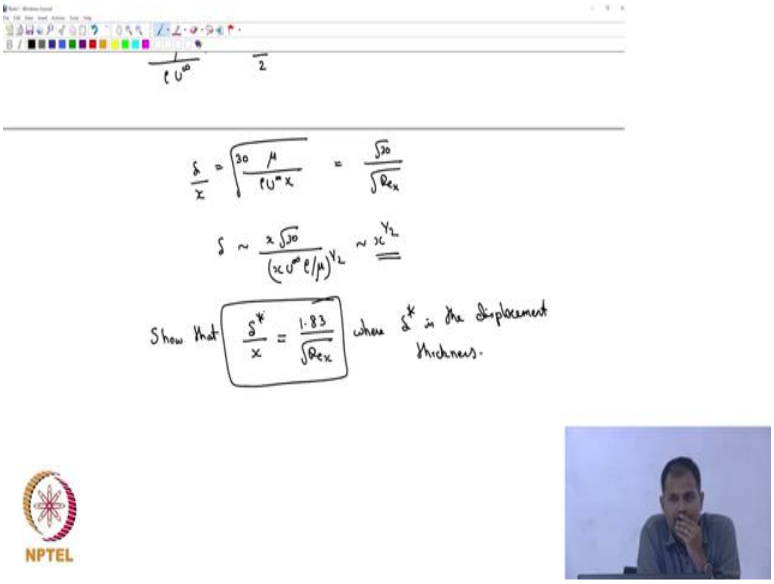

So, homework for you, you see use this use these expressions and try to show that delta star by x is equal to 1.83 by square root of R e x where delta star is the displacement thickness. This is the equations that we have already written down and try to manipulate you see whether, so you understand what is delta star right is just delta minus h. We know how delta changes with R e x, so see whether you are able to do that, so useful expression ok, any doubts.

$$\frac{\delta^*}{x} = \frac{1.83}{\sqrt{Re_x}}$$

So, theta is called this is the momentum thickness, it is just a measure of the drag force but express there is a length ok, so that is all it is. It is a in fact, if you look it, it just a introduced in this calculation for convenience instead of dealing with that particular integral. Integral 0 to delta U by this thing you have just called a theta and if you look at it is basically d by rho b U infinity square ok. So; that means, it is drag force, but non dimensionalized. So, that quantity is called theta, but it has the dimensions of length now, so it is not completely non dimensionlized which has the dimensions of length.

Delta minus h, so the displacement thickness.

It will change with.

H will not, h is the height in which a fluid element is coming and it is getting displace by a distance of delta star ok. So, h, so what matters this how much it is dis, so delta minus h is what matter. So, delta star will be a quantity which will be a function of x.

Because, it is going to match with a fine the real expressions ok, so one thing is that whenever you have laminar flow confined between two things the it has to if the viscous mechanism is important like in pipe flow we typically get something close at a parabolic profile. So, here also if you actually had a complete derivation you get a profile which is very close to parabolic. So, typically whenever you have flow in very thin sections is reasonable the an approximation of a parabolic profile often gives you good results, so that is the only reason.

So, we could have done the same calculation assuming linear and you would have gotten a good estimate ok, parabolic is slightly better and a because the actual profile is very closed to a parabolic profile. And if you want to do turbulent then you would not want to do parabolic because you know that it is going to be a much flatter one.

So, it is basically is a better to assume something which is like you know going to the power of n where n could be some number which gives you a much more flatter profile ok. So, we understand a little bit about how to approach boundary layer and the thickness, we will look at differential approach now.

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