Fluid and Particle Mechanics
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Lecture - 65
Boundary Layer

So, we have almost done with the Fluid Mechanics part ok, where we learned how to calculate essentially pressure drop and the whole idea was to get the right pump that is suitable. Now, but as we saw in the beginning of the course ok, this fluid flow is not just confined to analysis of pipe flow.

There are several situations I mean in almost every place that you look at fluid flow becomes important ok. So, there are many more things that needs to be done and as chemical engineer one of the context in which again fluid flow will become important is in the context of boundary layers, so that is what we are going to start looking at from today onwards.

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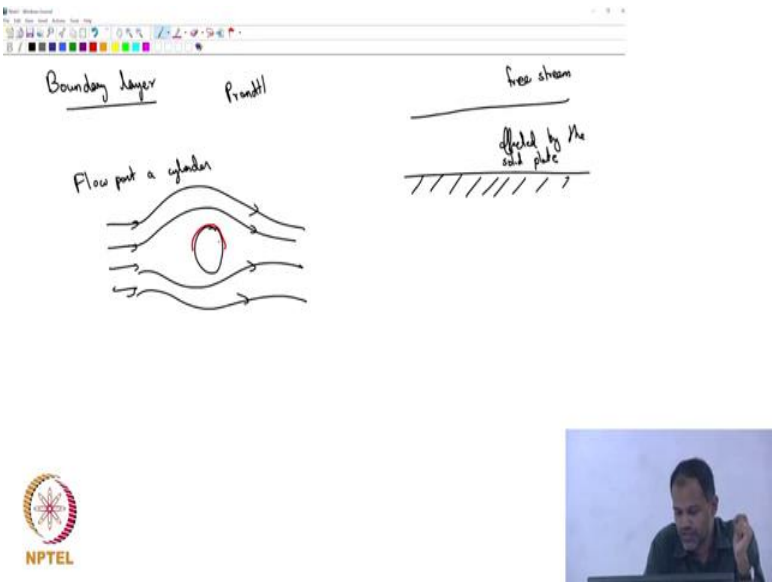

So, boundary layer is nothing but you know the region of the fluid flow that is affected by a nearby solid boundary ok. So, for example, if you have a flat plate, let us say a solid plate and if there is a fluid that is on top of it that is flowing, there will be a region which is affected by the solid plate and then there will be a free stream that is outside. So, when I said free stream that is a region which has which does not feel the effect of the solid plate, but there would be a region which is close to the plate where its experiences or in other words the this presence of a solid boundary will exert some drag, it will retard that fluid motion ok, so that region which is retarded by this by the solid plate that is what you call as the boundary layer.

Now, this boundary layer is very important for example, in heat and mass transfer. So, either you might have learned or you might be learning that you know you have situations where you know either heat is going to get conducted from the solid plate to a nearby fluid just like in a heat exchanger and that process ok. So, one of the ways of happening heat transfer is conduction, you could have as well as convection which means it is determined by the fluid flow right.

So, you would want to know so here we have a typical situation where the fluid is very near to a solid, let us say solid is at a different temperature from the fluid, you want to know how much is the heat transfer happening and you would then really want to know what is the velocity profile, because it is not just due to conduction, but it is going to be convection as well ok. Similarly, you can have mass transfer also.

For example, that is a solid ok and let us say it is dissolving into the fluid or let us say some crystallization is happening. So, there is going to be a mass transfer between the solid region and the fluid region. Again the rate of mass transfer is going to depend upon how the fluid flow is behaving there.

If the fluid flow is extremely slow, then diffusion is the only thing that will happen, but in general you would want to you know increase the speed of or rate of these processes, you would probably be you know pumping fluids its sufficiently large velocity. And therefore, there will be a boundary layer formula formation, and the mechanics in the boundary layer is actually going to determine how fast these you know transfer processes are going to take place ok, so that is one of the reasons why boundary layer is so much relevant to chemical engineers.

So, there is so this concept of boundary layer ok. It happened I mean the concept came up somewhere early 1900s and this was done by a person Prandtl. I think we heard this name sometime before or you would have definitely heard Prandtl number by now. So, see people were initially thinking about or let us talk about like this. So, Reynolds number right, Reynolds number is defined as the ratio of two forces. What are the two forces?

Inertial by viscous forces. And people thought if we keep increasing the Reynolds number, then inertial forces are going to be much larger than viscous forces right. Or we would think that the effect of viscosity becomes much smaller; much smaller, we can actually neglect all the viscous effects, and we can only talk about just inertial effects.

Or in other words, if you talk about Bernoulli's equation right, this Bernoulli's equation contain viscous effects or you neglect viscous effects when you derive Bernoulli's equation, you neglect it right. So, you would think that if I am at a very high Reynolds number, I should be able to solve my fluid flow by just using Bernoulli's equation and I can completely forget about the viscosity of the fluid ok. So, that was the initial thought, and then some of the experiments of what they did is they looked at flow past body. So, let us say we will talk about flow past a cylinder ok.

So, if you have a cylinder and then fluid is approaching and then it basically goes that way ok. So, we now know that the solid is going to you know exert or let us talk about this say the fluid is going to exert a force on the solid object ok. And that forces can have actually two components ok. A component that is parallel to the flow direction, and a component that is perpendicular to the flow direction.

So, the component that is parallel to the flow direction is what we call as drag ok; the component that is perpendicular to the flow direction is typically called a lift force and you know why the lift force is coming because that is typically the case when you talk about aeroplanes and it is that lift force that helps you to fly the aeroplane ok, so that is where the origin is.

So, you typically you can talk about a drag force and a lift force. Now, people thought, ok, we will calculate these forces and when they try to come up with you know try to compare experiments with Bernoulli's equation. They found that yes they are able to predict lift forces very accurately, but not drag forces. Now, we associate drag so much with viscous forces ok, but we also say that if you go keep increasing your Reynolds number, viscous if it should become lesser and lesser important and therefore, we should be able to predict that drag force just from Bernoulli's equation.

But when we did experiments, the experiments never matched with you know the predictions from Bernoulli's equation ok. If you neglect the specifics the calculations were never correct ok. So, then actually Prandtl came up with this idea of boundary layer. What he said is that however large Reynolds number is there is going to be always a thin layer of fluid very near to the boundary where you cannot neglect viscous forces.

So, what he said is that even in this case, there is going to be a very thin layer near the solid region, where you cannot you know neglect the viscous forces. And therefore, you need to account for that particular you know mechanics of whatever is happening in that layer to predict what is the viscous forces or really the drag force ok and that is way Bernoulli's equation was not able to give you a good result for the drag force. So, that is actually the origin of boundary layer ok.

So, the layer could be very thin. It was just like our roughness ok. You would think that roughness does not play any row, but roughness actually plays a major role. Similarly, here the boundary layer might be so thin, but it could actually be very important in determining what are the forces that are going to act on the particle ok, so that is where the origin of boundary layer has come from.

And now I mean at various places, you see this concept ok. You would think that the majority of the forces or the main mechanism that is going to be acted is something, but very near to something else there could be a thin thing which you might generally tend to neglect could actually be a very important thing.

So, it depends upon situations and so on. So, if people call all such things as you know boundary layer, boundary layer concept and so on. So, this is why this is how you know the origin of boundary layer happened and then of course, then you know in heat transfer and mass transfer everywhere you start seeing the importance of it, so that is the history.

So, let us look at. So, there are two things that we would be interested in this course ok. One is to see what is going to be the thickness of boundary layer ok, because that is a region where the fluid flow is seeing the solid plate ok and then we you want to of course it not in this course, but you may want to talk about heat transfer across that layer and so on. So, you want to calculate what is the thickness of the boundary layer; second is that we will also use this concept to explain flow separation.

So, you remember in one of the classes I was talking about expanding sections and so on and I said the flow could get separated ok. So, the analysis of boundary layer will actually tell you where does the flow separation happen you, why does it happen and so that is the another thing that we are going to look for. And it is important to know whether flow separates or not because that will also help you to determine how you know whether the transfer process of heat and mass transfer are going to be affected or not. So, these are the two aspects that we will look at.

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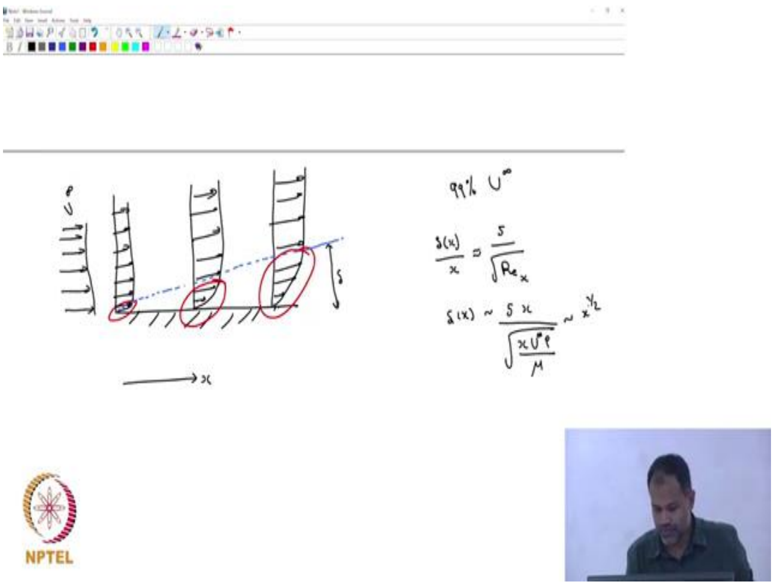

So, we will look at a very simple situation ok. We will only worry about a flat plate nothing else that is a flat plate and we will say there is a fluid that is coming at a constant velocity. So, the velocity profile is a constant ok and it is approaching the solid plate. And as soon as it approaches the solid plate what would happen the fluid element that is very close to so the last one when it approaches there at this region, it is immediately going to be retarded by the solid plate ok.

While everywhere else it is still going to be moving with so let us call this a velocity U infinity ok. So, the fluid is moving with a U infinity velocity doubt ok. And then as soon as it touch the edge of that plate that fluid element is going to get slowed down and then the next fluid element and it keeps going right.

So, if I look at if I draw the velocity, so up to here the velocity profile was flat or vertical, immediately as soon as it touches, there will be a small region let us say I should draw this, there will be a small region where velocity would the middle be you know affected so much and immediately after that it will regain its U infinity, so that is what is going to be the velocity profile ok, so that small region is where it is affected by the solid plate and everywhere else the velocity is U infinity ok. But now you as it you know goes downstream acid proceeds further, even more and more, layers are going to feel the effect of the solid plate ok, because this already is accelerating the drag.

So, after a while if you look at further downstream, you will see that it is something like that ok. So, that much is the region which gets affected by the solid plate and that will keep going. In other words, this, the thickness of this layer would keep expanding ok. So, now, it is even larger ok. So, the thickness of this thing keeps expanding. So, this is where we are going to define our boundary layer thickness. We want to define boundary layer thickness as the region which is affected by the solid plate ok. Now, in principle all the fluid up to infinity will also see it ok.

So, if you look at the solid fluid here ok, let us say this is the solid plate, and the fluid layer it gets slowed down, and it really reaches you know when it is really far, but we do not want to talk about that. So, we need to you know give some criteria of telling what is the boundary layer thickness and we can say that let us say whenever the velocity reaches 99 percentage of the free stream velocity, we will call this as a boundary layer thickness or you could call it 99.99 percent whatever you like ok. So, we will say that whenever the velocity is 99 percentage of U infinity, we will call that as a boundary layer thickness.

So, if you mark that boundary layer thickness, so here you are starting from the edge and then you are starting from the edge and then you know it keeps increasing ok. So, that is what the boundary layer is going to be, so that is just an imaginary surface that you have drawn to identify the boundary layer part of the fluid flow. So, your interest is to find out what is that right. So, typically we denote it with a with delta as the thickness, and it turns out this we will derive delta of x. So, this is my x-direction delta of x ok, delta at any place in x divided by x is approximately equal to 5 divided by square root of Re x.

$$\frac{\delta(x)}{x} = \frac{5}{\sqrt{Re_x}}$$

So, when I said Re x the Reynolds number is defined based on the distance from the leading edge ok. So, typically we will define Reynolds number based on a given length scale, it could be the radius of the pipe or you know the size of the particle, here there is no other length scale other than really the distance from the plate itself.

It does it decrease with x. So, let us see delta of x basically is 5 x divided by square root of x U infinity rho divided by mu, so that is actually going as x to the power of half. So, it in; it is increasing with. Why it increases? So, as we said so as soon as the fluid touches the leading edge ok, only that fluid particle that is touching the leading edge will experience the effect of the solid plate. Now, what is the problem with the guy who is just behind you?

So, the only the fluid particle that is touching the leading edge will get retarded, everywhere else it is going to be still coming with that velocity of U infinity right. But as it enters a little bit more, two of the layers will start experiencing it because this is. So, what is the solid plate doing, the solid plate is retarding the fluid motion right. So, it basically has to get transferred right across various layers. So, here there is a fluid that is coming with a strong velocity.

So, only the fluid particle that is going to touch is going to be experiencing that. But then the next layer we will see, but a little downstream, and the next layer we will see it a further downstream. So, therefore, the boundary layer will keep growing. Does that answer your question? Yes.

Yes, you are right. So, the well also at the moment in this picture, I have assumed that the fluid outside the boundary layer is actually going all the way to infinity ok, so that is the only reason I have assumed that ok. But you are right that yes the other region the velocity will increase and I will point out that a little later also today. Any other question?

So this is so for a laminar flow if so the flow field in the boundary layer could either be laminar or it could be turbulent ok. If the flow in the laminar in the boundary layer is laminar, then the thickness of the boundary layer is given by this expression delta of x by x is approximately equal to 1 by square root of Re ok, x, where Reynolds number is defined based on the distance from the leading edge ok. And if you simplify that, you will find that the boundary layer thickness actually increases with x in a square root form ok.

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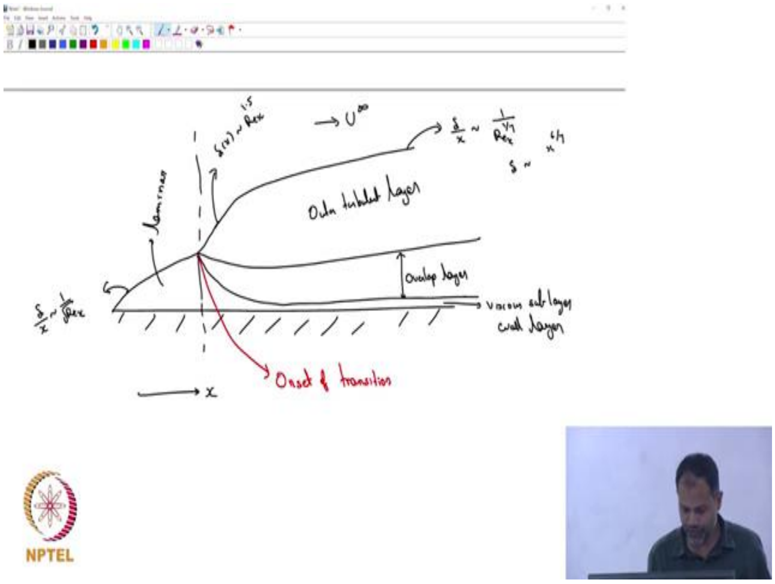

Now, what happens is that, so let us say that shows your growth of boundary layer ok. If the Reynolds number can get sufficiently large ok, then even though the flow would be laminar at the near the leading edge of the plate after a certain point, it will actually go to a turbulent ok. It can basically turn to a turbulent flow. So, you would see that if you were to actually track the boundary layer thickness you will see that it nicely you know grow like a root x and then suddenly there will be some point in which is it will start you know increasing suddenly ok, it will start increasing suddenly and then again it will slow down.

So, what happens is that up to here, the flow would be laminar. And beyond that point the flow would turn turbulent ok. Now, why that happens is not a part of this course, I, we cannot say. But what is going to happen is that the layer in the boundary layer ok, you already know that we have already talked about that, that will constituted by three different parts. There will be viscous sub layer very near to the wall, then there can be an overlap layer and then there will be turbulent boundary layer and then there can be a free stream ok.

So, from here what happens is that, so remember, so this region the region which is laminar basically will become smaller and smaller and will become a very thin layer which is what we called as the viscous sub layer or wall layer. And then there if there will be another region which is our overlap layer.

And then we have our outer turbulent layer and then outside is where we have our velocity as U infinity or the free stream velocity. So, this region the delta by x is approximately going as 1 by square root of Re x. This region is where delta of x goes as Re x to the power of 1.5 much faster and much downstream delta by x goes as 1 by Re x to the power of 1 by 7, which is much weaker ok. So, this is really the onset of transition. So, this is the structure of the transition from laminar to turbulent layer, turbulent flow in a boundary layer. Do you want to clarify something? Ask.

Re x for 1.5, yes. So, in the laminar, it is going to be root ok, it is x to the power of half or I have written this delta by x is 1 by root of Re x. In the after immediately after the onset ok, it is going to be much faster, it is x to the power of 1.5 and then much further down, it will be delta by x is 1 by Re x to the power 1 by 7. So, delta will go as x by x to the power of 1 by 7. So, it is x minus x to the power of 6 by 7. Now, 1 minus 1 minus 1 by 7, so that is how the growth is going to be.

What we are going to do now, let me then now answer his question. So, we have seen this before ok, correct. So, we earlier we have looked at you know somewhere there it is so much further downstream ok. We never talked about the region, which are very close to where the transition is happening.

So, it is really those three layers are formed and stabilized and so on. See from the left if you see is laminar. So, it is not that at one point it turns turbulent ok. It is a top actually that turns turbulent and then that affects the cascades down. So, it basically takes a while for things to stabilize. So, it is just going to say that this boundary layer formation ok, that is really what we talked also when we talked about the entrance length.

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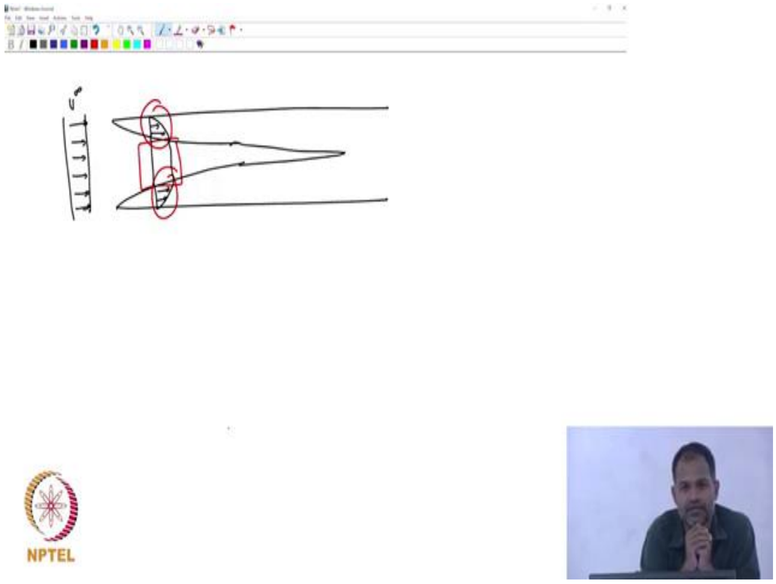

If you remember in the first class, I might have spoken that if you have a channel and if I say that if fluid of constant velocity comes as soon as it touches the edge, it basically slows down, and then the region which is affected by that solid plate will grow like that ok. So, inside this layer, so the fluid is the velocity profile is going to be like this. So, the regions which are very close to those plates, the velocity slowed down, but in between the fluid is still coming with the velocity which is uniform ok, so that is really basically the boundary layer growth ok.

And whenever that fluid flow reach the when the boundary layers actually merged beyond a point, so this will happen; this will happen and it will merge and then after that we had everywhere in some sense boundary layer only that is what we called as the fully developed flow and that is what we analyzed and so on. And to answer your question, this is the region where you know you would see that, so if this was coming with a velocity of U infinity ok.

Here this region and this region has slowed down, so that means the this part of velocity field should be having a velocity larger than U infinity. In fact, you can use that mass conservation to calculate the velocity in that region ok. So, this is what you know really boundary layer was in the context of what we had earlier. What we are going to do now is try to derive an expression for boundary layer thick.

What is larger?

So, see something is coming with a uniform velocity ok, the entire mass that is coming should go out, but part of it has retarded, that means, the other part should get accelerated right, then only you can maintain the total flow rate. Otherwise, there is a mass loss right or in. So, as long as the fluid that is coming in has to go out, part is slowed down means part is sped up.

Because, the other parts have been slowed down.

Yes, which would be actually in terms of pressure rather than anything else. So, it is not that we so viscous forces are only going to anyway slower down or you can think about it as you know things so you so fluid was coming, but it is getting squeezed. So, it is actually speeding up that is what really it is ok.