

Fluid and Particle Mechanics
Prof. Sumesh P. Thampi
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture - 64
Pressure Drop in Pipes Which Connected at Junction

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Junctions & deviations

and change f_1, f_2, f_3

$$h_1 = \frac{\Delta P_1}{\rho g} = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_1 \frac{L_1}{D_1} \left(\frac{Q_1}{A_1} \right)^2 \frac{1}{2g}$$

$$h_2 = \frac{\Delta P_2}{\rho g} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} = f_2 \frac{L_2}{D_2} \left(\frac{Q_2}{A_2} \right)^2 \frac{1}{2g}$$

$$h_3 = \frac{\Delta P_3}{\rho g} = f_3 \frac{L_3}{D_3} \frac{V_3^2}{2g} = f_3 \frac{L_3}{D_3} \left(\frac{Q_3}{A_3} \right)^2 \frac{1}{2g}$$

$\Delta P_1 = \Delta P_2 = \Delta P_3 = \Delta P$

Assume ΔP , $\frac{\Delta P}{\rho g} + \Delta z_1 = f_1 \frac{L_1}{D_1} \left(\frac{Q_1}{A_1} \right)^2 \frac{1}{2g}$
 Assume f_1 , calculate Q_1
 Check Q_2 , Check.

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Let me give a new example; can you know which has got junctions and elevations. So, you have let us say some reservoir from which a pipe is coming. Let us call this reservoir 1, this is another one, and then there is another one ok. So, this reservoir 1, this as reservoir 2, that is reservoir 3, and all of them are filled with some fluid ok. Let us say this is at an elevation is z_1 , this is at an elevation is z_2 , and this is at an elevation is z_3 .

Now, you, so the what is going to happen is that the fluid from the highest reservoir or the highest two reservoir is going to flow into the third reservoir. You do not know what it is, and let us say your interest is to calculate what is the flow rate through each of these pipes ok. So, let us assume that Q_1 , Q_2 , Q_3 are the flow rates, but you do not know what is Q_1 , Q_2 , Q_3 , you do not know what is the pressure drop except that you just know that it is all you know at different elevations. So, here we will have P atmospheric here we will have P atmospheric here we will have P atmospheric.

So, the flow rate Q_1 is determined by the atmospheric pressure or the and then this hydrostatic head. So, it is really is z_1 that determines what is the flow rate through Q_1 ,

and that would generate a pressure difference between the top of the reservoir and this point at the junction. Similarly, there will be a pressure drop across your second pipe, there will be a pressure drop across your third pipe.

So, you can write down general equations. You can say that your h_1 which is the pressure drop across the first pipe is given by $\Delta p_1 / \rho g$ that is a pressure drop plus you know Δz_1 , so that is essentially so that is the definition of our head loss ok. So, so far we have not been taking you know z into consideration, because we never worried about elevation changes, and that is equal to $f_1 L_1 / D_1 \times v_1^2 / 2g$. Where v_1 we do not know, but we know it in terms of Q_1 . So, let us write it as $f_1 L_1 / D_1 \times Q_1^2 / A_1^3$ and then there is $A_1^3 / 2g$ ok.

Similarly, h_2 is equal to $\Delta p_2 / \rho g$ plus Δz_2 that is equal to $f_2 L_2 / D_2 \times v_2^2 / 2g$ or that is equal to $f_2 L_2 / D_2 \times Q_2^2 / A_2^3$ into $1 / 2g$. Similarly, h_3 equal to $\Delta p_3 / \rho g$ plus Δz_3 equal to $f_3 L_3 / D_3 \times v_3^2 / 2g$ that is $f_3 L_3 / D_3 \times Q_3^2 / A_3^3$ into $1 / 2g$ ok.

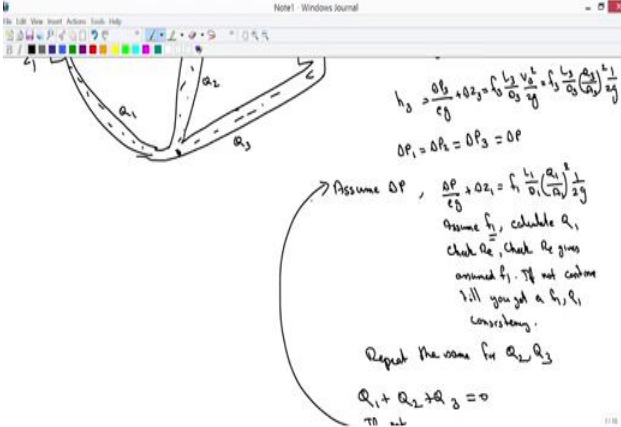
$$h_i = \frac{\Delta P_i}{\rho g} + \Delta z_i = \frac{f_i L_i}{D_i} \frac{v_i^2}{2g} = \frac{f_i L_i}{D_i} \left(\frac{Q_i}{A_i} \right)^2 \frac{1}{2g} ; i = 1, 2, 3$$

Now, what we know is that h_1 , h_2 , h_3 should be different, because the z_1 is z_2 , z_3 are different, but Δp_1 , Δp_2 , Δp_3 should be same. Because on one side, it is all exposed to atmosphere and Δp_1 is what the pressure difference between the top of this reservoir and this junction. Similarly, Δp_2 will be the pressure difference between the top of the reservoir, and this junction in which three pipes join and then so it is Δp_3 .

So, we would know Δp_1 is equal to Δp_2 is equal to Δp_3 , so that is one thing that we know and therefore, what you can do is you can start again you need an iterative procedure let us say assume Δp . So, let us call it equal to Δp assume Δp . So, if you know Δp , then what you can do is that let us take the first equation which is that.

So, let us write it down if you assume Δp , then $\Delta p / \rho g$ plus Δz_1 is equal to $f_1 L_1 / D_1 \times Q_1^2 / A_1^3$ into $1 / 2g$. So, you assume f_1 , calculate Q_1 , check Reynolds number, check Re gives assumed f_1 that is this f_1 if not continue till you get a $f_1 Q_1$ consistency ok.

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The diagram shows a junction where three pipes meet. Pipe 1 is on the left, Pipe 2 is on the top right, and Pipe 3 is on the bottom right. Arrows indicate flow directions: into the junction from Pipe 1, and out of the junction into Pipes 2 and 3. The flow rates are labeled Q_1 , Q_2 , and Q_3 .

Handwritten notes in the journal include:

$$h_3 = \frac{\Delta p}{\rho g} = f_3 \frac{L_3}{D_3} \frac{V_3^2}{2g} = f_3 \frac{L_3}{D_3} \left(\frac{Q_3}{A_3} \right)^2 \frac{1}{2g}$$

$$\Delta p_1 = \Delta p_2 = \Delta p_3 = \Delta p$$

Assume Δp , $\frac{\Delta p}{\rho g} = \Delta z = f_1 \frac{L_1}{D_1} \left(\frac{Q_1}{A_1} \right)^2 \frac{1}{2g}$
 Assume f_1 , calculate Q_1 ,
 Check Q_2 , check Q_3 gives
 assumed f_1 . If not continue
 till you get a f_1, Q_1
 consistency.
 Repeat the same for Q_2, Q_3
 $Q_1 + Q_2 + Q_3 = 0$
 to solve

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So, then you can do that and repeat, the same for Q_2 and Q_3 . And once you get Q_1 , Q_2 , Q_3 , then you know that Q_1 plus Q_2 plus Q_3 should be equal to 0. If not, then you have to go and change your assume delta p, and the process to be continued. So, what am I trying to say I am trying to say that you can for example, assume delta p, you know the equation for each pipe, so you can solve for what is velocity in each pipe. But when you try to do that you would not know what is the friction factor, so you have to assume a value friction factor find out the velocity in each pipe ok, and you have to do that in an iterative manner, so that you get an f_1 v 1 or f_1 and Q_1 combination that satisfies the assumed delta p.

And then you can do the same thing for second pipe, you can do the same thing for third pipe that means you have gotten the flow rate for first second and third pipe. But you also know that you are actually looking at a junction and there cannot be a net flow, so Q_1 plus Q_2 plus Q_3 should be equal to 0. So, you can check that condition and if that condition is violated, that means, this delta p that you have assumed the one which you started with is actually the wrong one, and you need to change that value. And then continue this procedure still till you find that Q_1 plus Q_2 plus Q_3 equal to 0.

So, this for example one way of solving the set of equations. As I mentioned earlier you can come up with any procedure, so that this condition is satisfied. So, the point in note is that as far as the junction is concerned, there is no net flow through that or whatever is

coming into that junction the flow it should be such that that fluid flow should go out or in incoming flow should be equal to outgoing flow. As long as you impose that condition and your solution satisfies that that is going to be your solution.

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$Q_1 + Q_2 + Q_3 = 0$
if out

- ① There cannot be a net pressure drop across a closed loop
- ② There cannot be a net flow through any junction
- ③ All velocities/flow rates should satisfy friction factor correlations, Moody's chart

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So, in general whenever you have a system it is you know you let us say you have a pipe, it might be complicated one, this is something like this comes. So, there is a fluid that is going to go like that, there is a bifurcation, and then it goes, it goes that way, it goes this way and so a complicated system. What you can do is that given this that, if I take any loop let us say a loop that you start from there, and finish that loop. And if I am coming back at that point then you can insist that there cannot be a pressure drop across a closed loop, because I am starting from the same point and ending at the same point.

So, you can has insist that there cannot be a net pressure drop across a closed loop. Second thing that you can say is that there cannot be net flow through any junction. So, these are the two constraints that you should impose. And if you are trying to impose that constraint and then you are going to get all values for velocity, you are going to get Reynolds number for each pipe and then you can insist that all you know friction, all velocities, all velocities that you calculate or flow rates that you calculates should satisfy friction factor correlations Moody's chart.

So, as long as you write down you know set of correlations that satisfy these three constraints, then you basically will have a system that needs to be solved ok. And most of

the times you would end up the solution being in an iterative form because you will have more than one unknowns, and the equations may not be straightforward to do it. And remember when I have done all these calculations, I have assumed all minor losses to be 0, but the minor losses should be accounted for because we know that some it can be it can play a major role.

So, then it will really depend upon the system that you are looking at. So, so for example, when you write down the total head loss, you should write down the total head loss due to the length of the pipe as well as that due to the minor losses like bends and valves and so on in the system.

So, it becomes a very system specific configuration. But what we have done is really looked at what are the constraints that you should impose ok. So, the constraints would be one is coming from the mass conservation which is about how a flow gets divided or added up. The second is about coming from the force balance, which is about your pressure loss or how does the pressure drop whether they get added or equally divided and so on.

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Handwritten equations in the journal:

$$h = h_1 = h_2 = h_3$$

$$Q = Q_1 + Q_2 + Q_3$$

$$Q = \left(\frac{h_1}{f_1 \frac{L_1}{D_1} \frac{1}{A_1^5}} \right)^{1/2} + \left(\frac{h_2}{f_2 \frac{L_2}{D_2} \frac{1}{A_2^5}} \right)^{1/2} + \left(\frac{h_3}{f_3 \frac{L_3}{D_3} \frac{1}{A_3^5}} \right)^{1/2}$$

$$Q = h^{1/2} \left[\left(\frac{1}{f_1 \frac{L_1}{D_1} \frac{1}{A_1^5}} \right)^{1/2} + \left(\frac{1}{f_2 \frac{L_2}{D_2} \frac{1}{A_2^5}} \right)^{1/2} + \left(\frac{1}{f_3 \frac{L_3}{D_3} \frac{1}{A_3^5}} \right)^{1/2} \right]$$

$$h_1 = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_1 \frac{L_1}{D_1} \left(\frac{Q_1}{A_1} \right)^2 \frac{1}{2g}$$

$$h_2 = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} = f_2 \frac{L_2}{D_2} \left(\frac{Q_2}{A_2} \right)^2 \frac{1}{2g}$$

$$h_3 = f_3 \frac{L_3}{D_3} \frac{V_3^2}{2g} = f_3 \frac{L_3}{D_3} \left(\frac{Q_3}{A_3} \right)^2 \frac{1}{2g}$$

$$Q_1 = \left(\frac{h_1}{f_1 \frac{L_1}{D_1} \frac{1}{A_1^5}} \right)^{1/2}$$

$$Q_2 = \left(\frac{h_2}{f_2 \frac{L_2}{D_2} \frac{1}{A_2^5}} \right)^{1/2}$$

$$Q_3 = \left(\frac{h_3}{f_3 \frac{L_3}{D_3} \frac{1}{A_3^5}} \right)^{1/2}$$

So, these are the two rules that you should remember. I just realized that while I wrote the first problem, I have not taken a one by 2 g into consideration. So, here when I replace v 1 with Q 1 by A 1 there is a factor of 1 by 2 g is a factor of 1 by 2 g is a factor of 1 by 2 g which would appear in expressions for Q 1, Q 2 and in Q 3. And when you have substituted

that would again come, so that is 1 by 2 g, 1 by 2 g, 1 by 2 g, 1 by 2 g, 1 by 2 g and 1 by 2 g, it is just coming as a factor ok.