Fluid and Particle Mechanics
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Lecture - 63
Pressure Drop in Pipes Which Connected in Parallel

So, we so, what how does one proceed in terms of calculating the core connecting Pressure Drop versus volumetric flow rate, when the fluid is flowing through pipes which are connected in series. Now, let us look at the other configuration that comes or that one often encounters is when pipes are connected in parallel.

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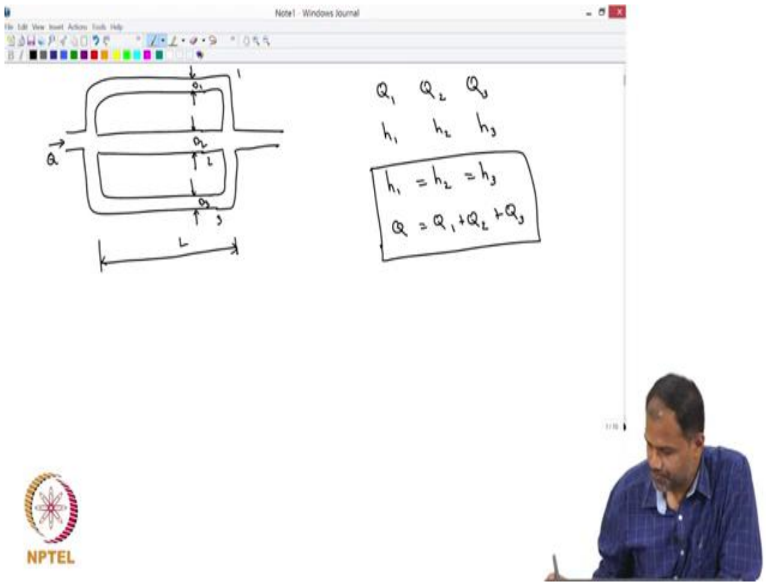

So, let us take an example. So, let us say you have a fluid that is entering the pipe and let us its divided into 3 parts ok. So, the pipe is divided into 3 parts ok. So, let us call the first 1 pipe 1, pipe 2 and pipe 3, all of them let us say have a length L and let the diameter be different.

So, let us say this is D 1, this is D 2 and then this is D 3 and fluid or flow rate Q is coming. So, it is through the flow there is a fluid that is entering that junction the flow rate is Q and then it is passing in through 3 tubes before joining again and coming out. So, this is the configuration that you have and your interest is to connect the relation between Q and pressure drop. So, let us say Q 1 Q 2 Q 3 are the flow rates through each of the pipes and let us say h 1 h 2 h 3 are the head losses through each of the pipes. So, again you can take the same analogy as we saw in case of pipes connected in series, that the volumetric flow rate it is like your current, the pressure losses, the head losses it is like your voltage.

So, you can see that the current should be divided into 3 while the voltage across the section should remain same. In other words in this particular configuration h 1 should be equal to h 2 is equal to h 3, in other words the pressure drop across the inlet and the outlet will be same whether you take pass 1, 2 or 3. However, the flow rate the total Q should be divided into Q 1 plus Q 2 plus Q 3, just the fact that the other volumetric flow rate will be divided into 3 parts and going in. So, these are the two rules that will dictate the flow configuration in this system.

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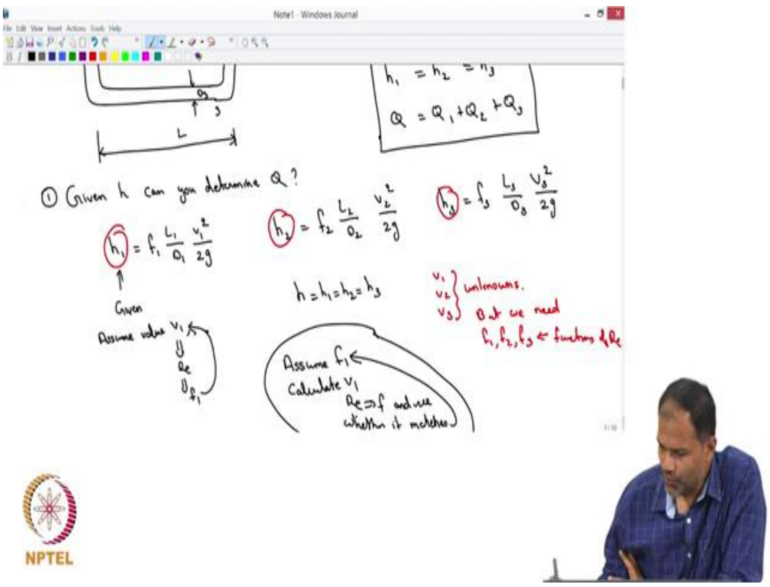

Now, we could ask the question given h can you determine Q ? So, let us say you have got a particular pump so, you know that it has got a certain head loss associated with it. And, then you were interested in finding out what is the maximum flow rate that you can pump through the system.

Now, you could write down the relations you can say that h 1 is f 1 into let us say L 1 by D 1 into V 1 square by 2 g; h 2 is equal to f 2 L 2 by D 2 V 2 square by 2 g, h 3 equal to f 3 L 3 by D 3 V 3 square by 2 g. Now; so, this is how you would calculate h 1 h 2 and h 3, but we do not know it what is h we know h 1 h 2 h 3, but we do not know V 1 V 2 V 3.

But that is simple because, actually if you look at it since h 1 is given, we can we need to just calculate V 1, but to calculate V 1 you need to know what is f 1. And f 1 of course, depends upon the Reynolds number and Reynolds number again depends on V 1. So, there is no simple way to calculate it, except that you have to assume value for V 1, calculate a Reynolds number or let us say calculate Reynolds number calculate f 1. And, then see whether that f 1 is consistent with V 1 or in other words you've to calculate a V 1 and f 1 that is consistent with each other that can give you a measure a pressure head loss that is given by h 1; because h 1 is prescribed in the problem.

Note I have I am specifying h so, h is equal to h 1 equal to h 2 equal to h 3 so; that means, we know h 1, we know h 2, we know h 3; we do not know V 1 V 2 or V 3 are unknowns. And our idea is to calculate V 1 V 2 V 3, but we need f 1 f 2 f 3 which are all functions of Reynolds number which are in turn functions of V 1 V 2 V 3 so, this are really a non-linear equation.

So, you assume a value of V 1, calculate the Reynolds number, calculate f 1 and see whether that f 1 is actually for the for the Reynolds number that you have calculated. And, this procedure can be repeated or the another way of doing it is to start by looking at a value of f 1 you could also do this assume f 1.

So, since you know h 1 calculate V 1, then calculate Reynolds number and then calculate f. And see whether it matches with the f 1 you have assumed if not correct it and so on. And, this might be probably a better way of doing things either way you can do either you start by assuming f 1 and calculate V 1 or start by assuming V 1 and calculate f 1 and continue that procedures till you till you get a prescription.

And, you have to do the same thing for h 2 or rather V 2 and you can do the same thing for V 3 and then you can calculate V 1 V 2 V 3. So, this is the procedure to evaluate ah you know the flow rate through a parallel connection if h is given; you could also ask the reverse question.

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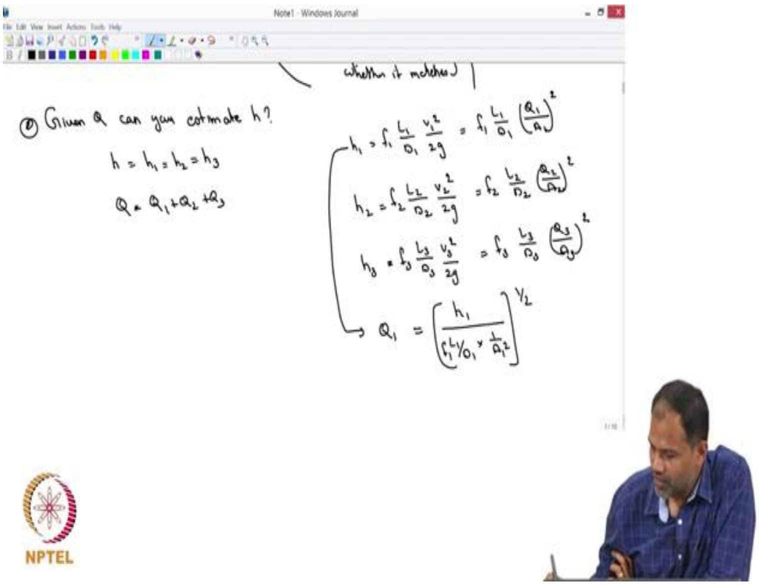

Given Q can you estimate h? So, what are the things that we know? h is given by h 1 plus h 2 sorry h is equal to h 1 equal to h 2 equal to h 3, Q is equal to Q 1 plus Q 2 plus Q 3. We know h 1 is f 1 into L 1 by D 1 into V 1 square by 2 g, h 2 is f 2 into L 2 by D 2 into V 2 square by 2 g, h 3 equal to f 3 into L 3 by D 3 V 3 square by 2 g. The second rewrite as f 1 into L 1 by D 1 into V 1 that is Q 1 divided by A 1 square. So, because if I divide flow rate with area I am going to get velocity.

This is f 2 L 2 by D 2 into Q 2 by A 2 whole square, this is f 3 L 3 by D 3 Q 3 by A 3 whole square. Then I can write so, this expression tells me that I can write Q 1 is equal to h 1 divided by L 1 by D 1 and then you have 1 by A 1 square to the power 1 by 2. So, I am just calculating Q 1 from the expression. So, I have got h 1 and f 1 sorry into f 1 into L 1 by D 1 into 1 by A 1 square that is going to be my Q 1.

$$Q_i = \left[\frac{h_i}{f_i \dfrac{L_i}{D_i} * \dfrac{1}{A_i^2} \dfrac{1}{2g}}\right]^{1/2} \quad ; i = 1,2,3$$

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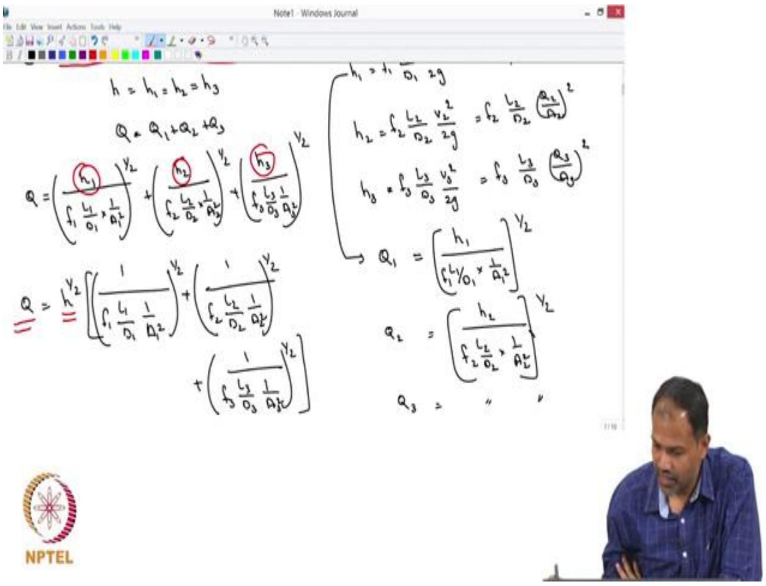

Similarly, I am going to get an expression for Q 2 is going to be simply h 2 by f 2 into L 2 by D 2 into 1 by A 2 square to the power 1 by 2 and I will also get an expression for Q 3 in a similar fashion. Then if I substitute here I will write Q is equal to the expression for Q 1 which is really h divided by f 1 into L 1 by D 1 into 1 by A 1 square to the power of 1 by 2 plus h 2 by f 2 into L 2 by D 2 into 1 by A 2 square to the power of 1 by 2 plus h 3 by f 3 into L 3 by D 3 into 1 by A 3 square to the power of 1 by 2.

But, we know that h 1 h 2 h 3 are equal therefore, I will write Q is equal to h to the power of half into 1 divided by f 1 L 1 by D 1, 1 by A 1 square to the power of half plus 1 by f 2 L 2 by D 2 1 by A 2 square to the power of half plus 1 by f 3 L 3 by D 3 1 by A 3 square to the power of half.

$$Q = h^{\frac{1}{2}}\left[\left(\frac{1}{\frac{f_1 L_1}{D_2}\frac{1}{A_1^2}\frac{1}{2g}}\right)^{1/2} + \left(\frac{1}{\frac{f_2 L_2}{D_2}\frac{1}{A_2^2}\frac{1}{2g}}\right)^{1/2} + \left(\frac{1}{\frac{f_3 L_3}{D_3}\frac{1}{A_3^2}\frac{1}{2g}}\right)^{1/2}\right]$$

So, what I have done really is that I have now gotten a connection between Q and h ok. So, our problem originally was that given Q can you estimate h? So, by manipulating the expressions we have got an expression between Q and h, again we have a problem because we do not know what is f 1, f 2 and f 3 are.

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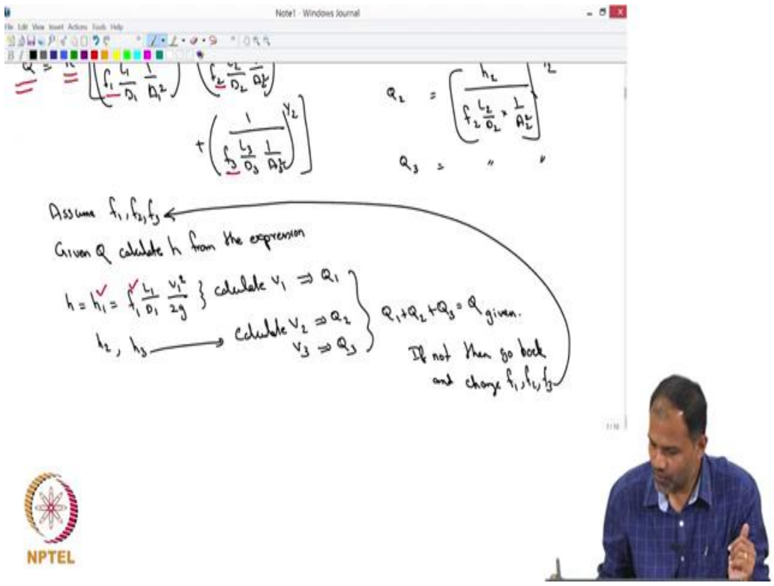

So, the only way one can do is that assume f 1, f 2, f 3 given. So, what is the question given Q? Given Q calculate h from the above expression so, but we do not know whether f 1, f 2, f 3 are right, but we have calculated h. But, we know that h is same as h 1 same as f into L 1 by D 1 into V 1 square by 2 g ok. So that means, you can now calculate V 1 because, in this expression you know h 1 you have assumed f 1 so, you can calculate V 1.

Similarly, from h 2 h 3 calculate V 2 V 3 and if you know V 1 V 2 V 3 you also know Q 1 Q 2 Q 3. And, if you know Q 1, Q 2, Q 3 you know that Q 1 plus Q 2 plus Q 3 should be equal to Q given. If not then go back and change f 1, f 2, f 3 there and repeat the procedure. So, this is a so, this is an iterative procedure and this iterative procedure needs to be done because there is no other way to solve these expressions. So, you can start by assuming f 1 f 2 f 3, then you can calculate a Q from that sorry given the Q you can calculate an h from that.

And, then you can calculate V 1 V 2 V 3 and calculate again see whether the V 1 V 2 V 3 that you get is balancing your constraint Q 1 plus Q 2 plus Q 3 equal to 0. So, let me tell that again we start by assuming f 1, f 2, f 3 then we use the expression connecting Q on h to calculate h. And, since we know h you can use the individual expressions like h 1 equal to f 1 L 1 by D 1 V 1 square by 2 g, those expressions to calculate V 1 which means you can calculate Q 1, Q 2, Q 3. And, then you can check the constraint whether Q 1 plus Q 2 plus Q 3 is given to be the total flow rate and this procedure can be repeated.

So, this is just one way of doing it, essentially what you have is a system of equations and you need to solve them and it is your choice really whichever way you want to do it, this is just one really algorithm for that ok. So now, given that so, we have seen how one should think about when pipes are connected in series, how should one think about when pipes are connected in parallel, you could also have junctions.

So, let us look at a case where you have you know junctions, you can have elevations. So, far we basically said we are going to look at pipes that are all you know horizontally aligned and therefore, there was no elevation change.