Fluid and Particle Mechanics
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Lecture - 61
Momentum and Kinetic Energy Correction Factor

So, I want to introduce you to the two terms that we did not talk about, but you might actually come across these terms. And the when you are reading let us say books again you are going to see this; one is momentum correction factor and another is a kinetic energy correction factor ok. These two terms should really appear in any of the force balances or energy balances that you write, but I have not been really taking that into reconsideration. So, let me just introduce them to you.

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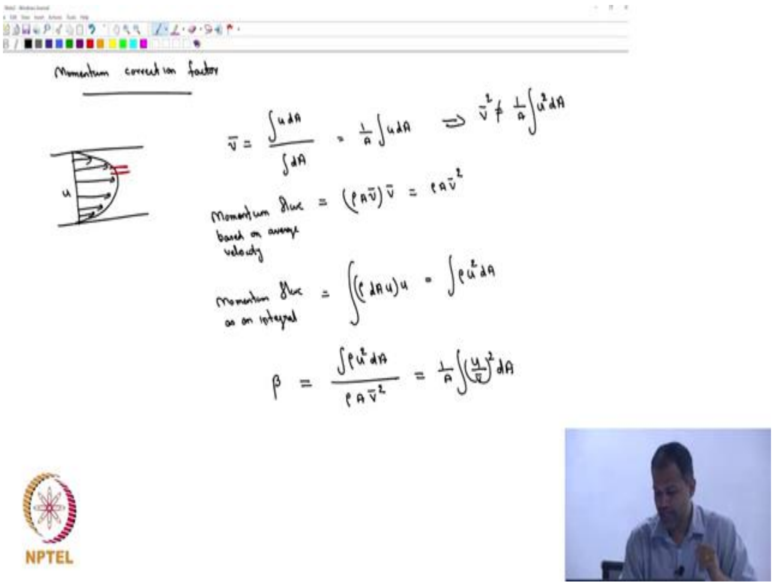

So, the first one is momentum correction factor. So, this will come whenever we talk about an integral force balance or integral energy balance. The reason is that let us say we go back to our situation let us say the pipe flow and you have fluid that is flowing fully developed.

And now you know that the profile is parabolic ok, then we define an average velocity as integral; let us say if the velocity profile is given by u; u dA divided by integral dA; that is what you would have done to calculate your average velocity right. Now, let us say we are worried about; so let me write that as 1 by A; something that you have integrated integral u dA.

$$\bar{v} = \frac{\int u\,dA}{\int dA} = \frac{1}{A}\int u\,dA$$

Let us say we want to calculate average momentum flux or not; no point in telling average actually momentum flux. We can calculate momentum flux either as an integral or we could just use our averaged velocity itself ok. If you do an average velocity what would you do? So, you if you want to calculate momentum flux, you would basically take the volumetric flow rate times the; sorry not volumetric flow rate, mass flow rate times the velocity right.

That is going to be the momentum flux. So, you would say that if I have you know a fluid of density rho is going with the velocity with an average velocity V bar; so A into V bar is going to be my volumetric flow rate, times rho will be my density, times V bar is going to be my momentum in right. So, you would simply write the momentum flux in as rho A V bar square that is a momentum flux in.

$$Momentum\ flux\ based\ on\ averaged\ velocity = \rho A\bar{v}^2$$

However, if you take let us say a small section. So, I will take a small section here and try to write this as an integral. So, this is momentum flux based on average velocity, but if I do momentum flux as an integral; I will say its rho times dA, times some velocity. So, u into dA is going to be a differential volumetric flow right; in a differential element if I consider of cross sectional area A; u into dA is going to be my volumetric flow rate through that section, times rho is going to be density, times u is going to be my velocity; my momentum flux and integrated over all area is going to give me the total flux and that is nothing, but integral rho u square dA right.

$$Momentum\ flux\ as\ an\ integral = \int \rho u^2\,dA$$

So, what have I done? I have just calculated the momentum flux either based on an average velocity or based on the actual velocity. And if I have done it using the actual velocity; I would have written down it as an integral ok; the question is; are these two quantities same or not? So, if we have a flat velocity profile meaning that, if u was a constant; there is no question u v equal to u average and everything will be same ok, but now we are saying that this the if the velocity profile is not flat ok; if it is not a constant these two quantities in general will be different ok.

So, that is because the integral; so because we have defined V bar as an integral of u dA we do know that; if I do V bar square that should really be 1 by A integral u dA square that will not be equal to 1 by A square, integral u square dA right. So, they are not same actually I should write this. How do I do this? V bar square is this and that is not equal to; I do not know, this is u square this is not integral of u square dA.

So, you can only write it like that; V bar is V bar square is not equal to 1 by A integral u square dA right; average of square of average is not equal to integral of square. So, therefore whenever you write down momentum flux you have to be careful. If you are writing down your momentum flux based on your average velocity, you are actually not doing the correct value you will be making a; you will be making a difference.

So, that difference is denoted by factor beta; beta is the ratio of integral rho u square dA divided by rho A V bar square or that simply 1 by A integral; u by V bar whole square dA ok; this is called the momentum correction factor.

$$\beta = \frac{\int \rho u^2 \, dA}{\rho A \bar{v}^2} = \frac{1}{A} \int \left(\frac{u}{\bar{v}}\right)^2 dA$$

Now, the question that I wanted to ask is should you have considered beta in our previous derivation? And if we should have where should have come where should it have come can you check hm. Yeah, so where we basically wrote down? Our force balance ok, I think on the right hand side we said it is mass flow rate times velocity ok. So, that is really the total momentum in and I said V is the average velocity, but writing down that expression we have made an assumption that you know V square is actually giving you the total momentum in.

But that is not really right there should have been a factor that should have come which that factor is beta. So, in our derivations we have assumed beta to be 1 ok; so, therefore, that calculation is not exactly right. For what kind of flow would you expect beta to be 1 or closer to 1; laminar or turbulent because you know that the profile is much flatter ok.

So, the calculations are expected to be better for a turbulent flow rather than a laminar flow.

We have made a similar error now in the energy equation also; can you guess what that would be? Because we have estimated kinetic energy based on average velocity should we do that now because we know kinetic energy cannot be just average velocity square.

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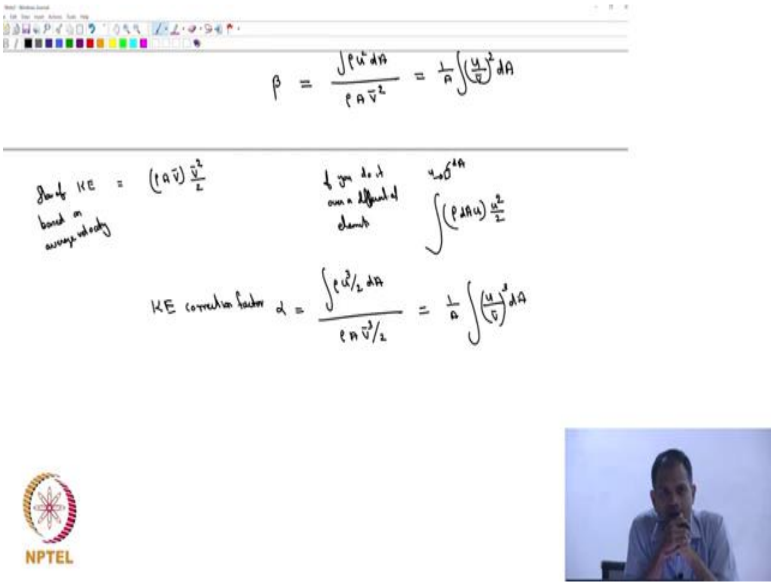

In fact, let us try writing down what would be the Kinetic energy, you know flow of kinetic energy ok; if you do it based on average velocity you would write it as flow rate which is rho into A into V bar into V bar square by 2. So, mass flow rate into V bar square by 2 is the kinetic energy; in if you do it based on an average velocity.

But if you do it over a differential element, so you have a small area through which something is going with a velocity u; area dA, you would say that is rho times dA, times u, times u square by 2 and then integrated ok. So, the quantity that you have on the right hand side is going to be the total kinetic energy in and that is the way you should have calculated.

But you would simply write it as rho A V bar is the; if you do it based on the average velocity you would have done it on the left hand side way ok. So, therefore, you know that these two quantities are not same and therefore, you define a kinetic energy correction factor; alpha as integral rho u cube divided by 2 dA divided by rho A V bar cube divided by 2. And; that means, you have 1 by A integral u by V bar cube dA ok; so that is the kinetic energy correction factor.

$$\alpha = \frac{\int \frac{\rho u^3}{2} dA}{\frac{\rho A \bar{v}^3}{2}} = \frac{1}{A} \int \left(\frac{u}{\bar{v}}\right)^3 dA$$

Should we have used the kinetic energy correction factor also in our previous derivation; yes no? Yes; where; so let us write down our actual equations that we should have used.

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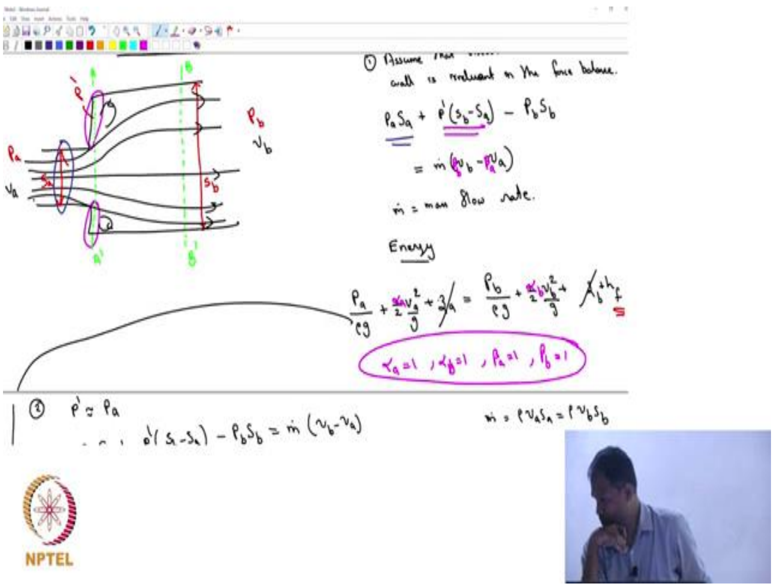

Yeah, we should have had a factor of beta b they should have been a factor of beta a, that should have given us our original or actual momentum flux. And here I should have had an alpha a and I should have had an alpha b ok. And we have really said alpha a equal to 1, alpha b equal to 1, beta a equal to 1, beta b equal to 1.

So, the flow the calculations are better when these things are closer to 1; the ones which we have derived. Any questions? In fact, I have not tried you could probably try if you introduce these factors would you be able to simplify the equations this way and get something extra or does it become impossible to simplify further. The idea is clear now? The idea of a kinetic energy and momentum correction factor?

Contraction, you study by yourself ok; there is nothing really you can do theoretically, but there are some arguments that you can see in the literature.

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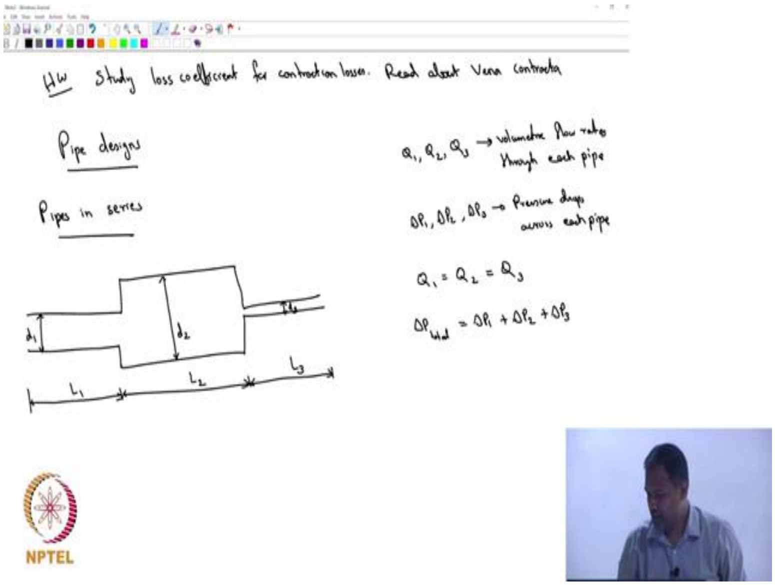

So, if you have done your assignment correctly; you might already have come across these words and so on ok.