

Lecture - 60
Tutorial - 08

The system shown in Fig. 1 consists of 1200 m of 5 cm cast iron pipe, two 45° and four 90° flanged long radius elbows, a fully open flanged globe valve, and a sharp exit into a reservoir. If the elevation at point 1 is 400 m, what pipe pressure is required at point 1 to deliver 0.005 m³/s of water at 20° into the reservoir? Useful data: Roughness of the pipe(ϵ) = 0.26 mm; Loss coefficient(K) for

- 45° flanged long radius elbow = 0.2
- 90° flanged long radius elbow = 0.3
- Open flanged
- Sharp exit = 1

Figure 1

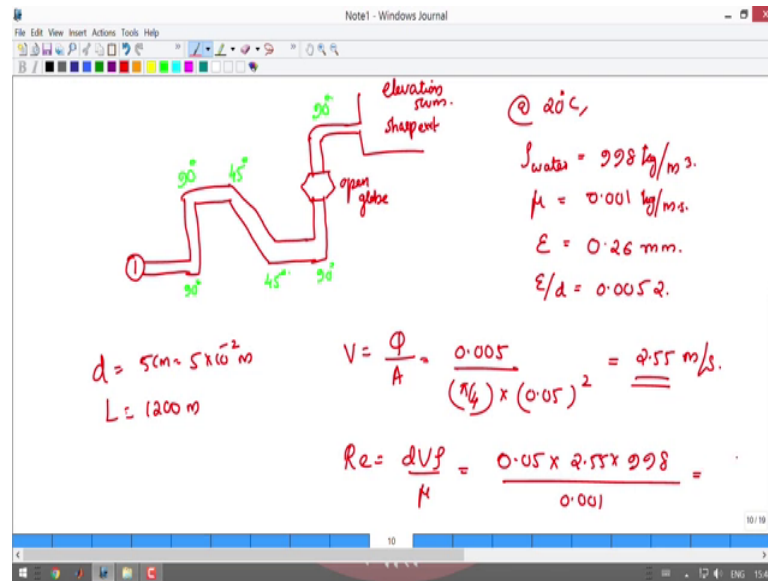
On 2

The pressure drop through a particle bed can be used to determine the external surface area and the average particle size. Data for a bed of crushed ore particles show $\Delta p/L = 84 \text{ lbf/in.}^2/\text{ft}$ (which is equal to 1900130.709 Pa/m) for air flow at superficial velocity of 0.015 ft/s (which is equal to $4.572 \times 10^{-3} \text{ m/s}$). The measured void fraction is 0.47 and the estimated sphericity, ϕ_{est} is 0.7. Calculate the average particle size and the surface area per unit mass if the solid has density of 4.1 g/cm^3 . How sensitive is the answer to an error of 0.01 in.



Hi, everyone I will discuss about the question number 8. I will read the question, the system consists of a 1200 meter of 5 centimetre cast iron pipe 245 degree and 490 degree flanged long radius elbows, a fully open flanged globe valve, and a sharp exit into a reservoir. If the elevation at 0.1 is 400 meter. what gage pressure is required at point 1 to deliver 0.005 meter cube per second of water at 20 degree Celsius into the reservoir ?

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So, this is our system, elevation 500 meter. So, as given in the question here you can see, that 245 degree flanged long radius elbows, those are here; one is 45 degree, one is a here. And, for 90 degree long radius flanged those are one 90 degree, 90 degree, then 90 degree and 90 degree.

So, we have to calculate gage pressure at the point 1. So, for water and 25 20 degree Celsius, we can take the density of water as at 20 degree Celsius, at 20 degree Celsius density of water. We can take it as 998 kilogram per meter cube and also the viscosity equal to 0.001 kilogram per meter per second and for the cast iron, we can take the roughness factor as epsilon equal to 0.26 millimetre.

So, that epsilon by d the roughness ratio which will be 0.0052; where d mean, in the question it is given d like diameter is equal to 5 centimetre equal to which is equal to 5 into 10 raise minus 2 meter, ok and L equal to 1200 meter. So, if we know the flow rate, we can calculate the velocity and also from that we can calculate the Reynolds number. So, V velocity equal to flow rate divided by area.

In the question phi it is given which is equal to 0.005 divided by area equal to pi by 4 d square into 0.005 whole square. So, velocity will be equal to 2.55 meter per second. And, from that we can calculate the Reynolds number; Reynolds number equal to d V rho by mu; so, it will be equal to 0.05 into 2.55 into density 998 divided by 0.001 which is equal to and from the moody's chart, we can get the friction factor.

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Handwritten notes in a Windows Journal window:

$f_{mood} = 0.0315$

Minor losses:

- 45° long radius elbow, $K = 0.2$
- 90° " " $K = 0.3$
- open flanged globe valve, $K = 8.5$
- submerged exit, $K = 1.0$
- sharp entrance, $K = 0.5$

Equations:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 =$$

$$\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + (h_f + \sum h_m)$$

$$\frac{V^2}{2g} \left(\frac{fL}{d} + \sum K \right)$$

So, that you have to calculate and you will get and there are minor losses and which can be listed as 445 degree long radius elbow; 45 degree minor losses, 445 degree long radius elbow, k equal to 0.2; similarly, for 90 degree, k equal to 0.3.

And, for open flanged globe valve, k equal to 8.5. Similarly, for submerged exit, k equal to 1; 1 and also for the entrance, sharp entrance k equal to 0.5 and if you are writing the energy equation between the 0.1 and the 2 which is the reservoir surface we can write their relation as. Since we are considering 2 as the reservoir, the gage pressure will be 0.

So, here we can neglect this and also velocity equal to 0 and these are the head loss due to friction and this term is due to some a bends elbows and all. So, this term we can replace it as V square by $2g$ into fL by d plus $\sum K$ [noise].

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The image shows a handwritten derivation of the Bernoulli equation with friction and minor losses. The equation is written as:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = 0 + Z_2 + \frac{V_1^2}{2g} \left(\frac{fL}{d} + \sum K \right)$$

Annotations in the image include:

- An arrow pointing to $\frac{fL}{d}$ with the text "due to friction".
- An arrow pointing to $\sum K$ with the text "due to minor losses".

The derivation then proceeds to solve for P_1 :

$$\frac{P_1}{\rho g} = Z_2 - Z_1 + \frac{V_1^2}{2g} \left(\frac{fL}{d} + \sum K - 1 \right)$$

$$= 500 - 400 + \frac{2.55^2}{2 \times 9.81} \times \left(0.0315 \times \frac{1200}{0.05} \right) + 0.5 + 2 \times (0.2) + 4 \times (0.3) + 8.5 + 1 - 1$$

$$= \underline{353 \text{ m}} \rightarrow \textcircled{P_1}$$

So, P_1 by ρg plus V_1 square by $2g$ plus Z_1 equal to 0 plus Z_2 plus V_1 square by $2g$. I mean, this term is due to friction, due to minor losses. So, from this we have to calculate the gage pressure at 0.1 , that is P_1 ; this one we have to calculate. For that P_1 by ρg equal to Z_2 minus Z_1 plus V_1 square by $2g$ fL by k plus σk minus 1 .

So, when we are substituting all the values, we will get 500 minus 400 plus velocity we got it as 2.55^2 into 9.81 into and from the Moody's chart, we have calculated the friction factor which is equal to point 0 , which equal to 0.0315 into length equal to 1200 divided by and this is d , diameter equal to 0.05 plus and we are taking all the summation of this loss coefficients. Those are here we have 245 degree, 490 degree and one open flanged globe valve and also one submerged exit and also we have one entrance, sharp entrance.

So, we are taking all the summation, which is equal to for the sharp entrance 0.5 plus for the 245 degree we are taking 2 into 0.2 and for 490 degree 0.3 plus, this is for the exit and this is no this is for the open flanged globe valve and this is for the submerged exit. After doing the calculation you will get 353 meter and from that we have to calculate P_1 .

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Handwritten calculations in a Windows Journal window:

$$\begin{aligned}
 P_1 &= 353 \times g \\
 &= 353 \times 9.81 \times 998 \\
 &= \underline{\underline{3.46 \text{ MPa}}}
 \end{aligned}$$

So, P 1 equal to 353 into rho g; which is equal to 353 into 9.81 into 998 which is equal to 3.46 mega Pascal.

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Handwritten calculations in a Windows Journal window for question 9:

⑨ Kozeny-Carman eqn
 $Re < 1$

$$\frac{\Delta P}{L} = \frac{150 \mu V (1 - \epsilon)^2}{(\phi D_p)^2 \epsilon^3}$$

$$(\phi D_p)^2 = \frac{150 \mu V (1 - \epsilon)^2}{\epsilon^3 (\Delta P / L)}$$

Given values:

$$\mu_{air} = 1.81 \times 10^{-5} \text{ g/(m.s)}$$

$$\epsilon = 0.47$$

Calculation:

$$\begin{aligned}
 (\phi D_p)^2 &= \frac{150 \times 1.81 \times 10^{-5} \times 4.572 \times 10^{-3}}{(0.47)^3 \times 1900130.709} \\
 &= \underline{\underline{1.89 \times 10^{-10}}}
 \end{aligned}$$

Coming to question number 9, I will read out the question; the pressure drop through a particle bed can be used to determine the external surface area and the average particle size. Data for a bed of crushed or particles show, delta P divided by L equal to 84 pound per in square per feet. For air flow at superficial velocities 0.015 feet per second the measured void fraction is 0.47 and the estimated sphericity is 0.7. Calculate the average

particle size and the surface area per unit mass, if the solid has a density of 4.1 gram per centimetre cube. How sensitive is the answer to an error of 0.01 in σ ?

So, here what we have to like mainly, we have to calculate the size of the particle and then surface area of the particle and also I mean, how sensitive it is if the error is 0.01 in σ . So, first thing what we have to do is, we know Kozney Carman equation. So, I will write that equation, which is applicable for Reynolds number equal to I will write the Kozney Carman equation, where Reynolds number less than 1.

So, $\Delta P \text{ by } L$ equal to $150 \mu V \text{ into } 1 \text{ minus } \epsilon \text{ the whole square divided by } \phi D_p \text{ the whole square } \epsilon \text{ cube}$. Where μ is the viscosity, V is the superficial velocity and ϵ is the porosity of the bed, ϕ is the sphericity, D_p is the diameter of the pellets and ΔP is the pressure drop, L is the length of the tube.

So, here we have to substitute all the values, if I am taking $\phi D_p \text{ the whole square}$ in the left hand side we can rewrite this above expression as $\mu V \text{ into } 1 \text{ minus } \epsilon \text{ in the whole square divided by } \epsilon \text{ cube into } \Delta P \text{ by } L$. So, $150 \text{ into } \mu \text{ of air is, air equal to } 1.81 \text{ into } 10 \text{ raise to minus } 5 \text{ kilogram per meter second}$.

So, if I am writing sign in SI units $1.81 \text{ into } 10 \text{ raise to minus } 5$ and velocity is given in the question; which is equal to I mean, in the question is given in FPS units we have to converted into SI units. So and velocity will equal to $4.572 \text{ into } 10 \text{ raise to minus } 3 \text{ into } \epsilon \text{ equal to } 0.47$, so $1 \text{ minus } 0.47 \text{ in the whole square divided by } 0.47 \text{ in the whole cube into } \Delta P \text{ by } L$ which is equal to 84.

So, finally, you will get so, after doing the calculation you will get 1.89 no, 84 is in sorry [noise], 84 is in FPS unit. So, here we are taking SI units; so, I have to write in SI unit $\Delta P \text{ by } L$ is 1900130.709. So, after doing the calculation you will get this as $1.89 \text{ into } 10 \text{ raise to minus } 10$. So, from this we have to calculate D_p which is size of the particle.

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Handwritten calculations in a Notepad window:

$$\phi = 0.7$$

$$(\phi D_p)^2 = 1.89 \times 10^{-10}$$

$$D_p = 5.99 \mu\text{m}$$

$$\rho_p = 4.1 \text{ g/cm}^3$$

$$Re = \frac{D_p v_s}{\mu} = \frac{1.265 \times 10^{-5} \times 0.015 \times 0.075}{0.018 \times 6.72 \times 10^{-4}} = 1.8 \times 10^{-3} < 1$$

$$\phi_s = \frac{6V_p}{D_p S_p} \rightarrow S_p$$

$$S_p / V_p = 6 / (\phi_s D_p) = 6 / (0.7 \times 5.99 \times 10^{-6})$$

$$S_p = 1.43 \times 10^6 \times 10^{-6} \text{ m}^2 / 4.1 \text{ g/cm}^3 = 5.9 \text{ m}^2/\text{g}$$

So, in the question sphericity is given as 0.7 and we also got the value of ϕD_p the whole square, which is equal to 1.89×10^{-10} . So, after substituting the sphericity, we should we will get the size of the particle.

So, sorry ϕD_p will be equal to mean, first you can take root of this and after substituting this sphericity you will get D_p equal to 5.99 micrometer. So, this is first part of the question. And, initially we have assumed the Reynolds number is less than 1.

So, we have to check whether we are getting Reynolds number which is less than 1. So, for that we are just substituting the all the values ah. So, after substituting all the values even if I am writing in FPS system, 1 or you can write in SI unit, whatever it is you will get the same. In question it is given both in SI unit and FPS unit so, you can use whatever you want.

So, after substituting all this you will get Reynolds number equal to 1.8×10^{-3} which is less than 1. So, we can use the above equation, which we have used initially, so, our assumption is correct. So, I will like so, that we can say that the size of the particle which we got is correct.

And, the next thing is we have calculate the surface area of the particle. For that we have the relations sphericity equal to $6 V_p$ divided by from this we will get, we have to calculate surface area of the particle. So, we have to calculate the surface of the particle so, we can

write the this expression as S_p by V_p equal to 6 divided by ϕ S into D_p ; which is equal to 6 divided by 0.7 into 5.99 into 10 raise to minus 6.

In the question the density of the particle is given as 4.1 gram per centimetre cube. So, here you will get the value as 1.43 into 10 raise to 6 meter square per meter cube. So, after substituting the volume, you will get S_p equal to 1.43 into 10 raise to 6 into 10 raise to minus 6 into 4.1 gram per centimetre cube. So, which is equal to 5.9 meter square by per gram.

So, the surface area of the particle is 5.9 meter square per gram, and the others part is that if we are taking error of 0.01 in σ how sensitive it is? That we have to find out; so, in the question initially, it is given that σ equal to 0.47 so, if you are taking an error of 0.01 this σ will be point 0.46.

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The image shows a handwritten derivation in a Notepad window. The text is as follows:

$$\begin{aligned} \sigma_2 = 0.46 &\rightarrow \sigma_1 = 0.47 \\ D_p^2 &\propto \frac{(1-\sigma)^2}{\sigma^3} \\ \left(\frac{D_{p2}}{D_{p1}}\right)^2 &= \left(\frac{1-\sigma_2}{1-\sigma_1}\right)^2 \times \left(\frac{\sigma_1}{\sigma_2}\right)^3 \\ &= \left(\frac{1-0.46}{1-0.47}\right)^2 \times \left(\frac{0.47}{0.46}\right)^3 \\ &= 1.086 \\ \left(D_{p2}/D_{p1}\right) &= 1.042 \Rightarrow D_{p2} = (1.042) D_{p1} \Rightarrow \text{error is } 5\% \end{aligned}$$

If a σ equal to 0.46, what is the like how sensitive it is? That we have to calculate, for that we have the relation from the Kozney Carman equation that D_p square equal to 1 minus σ whole square divided by σ cube.

So, D_{p2} divided by D_{p1} the whole square equal to 1 minus σ_2 divided by 1 minus σ_1 the whole square into σ_1 divided by σ_2 the whole cube. So, which is equal to 1 minus initially, it was point σ_1 equal to 0.47, σ_2 equal to 0.46, 0.46

divided by $1 - 0.47$ the whole square into 0.47 divided by 0.46 the whole cube and you will get is equal to 1.086.

So, D_p by this is so, you have to take the square root of this. D_p^2 by D_p^1 equal to 1.042 implies that D_p^2 equal to 1.046 of D_p^1 . Which implies that there is 5 percentage increment in the size, or we can say, error is 5 percentage. I mean, error is about 5 percentage for an error of 0.01 in sigma value. This is the final answer, ok.

Thank you.