Fluid and Particle Mechanics
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Lecture - 59
Friction Losses in Sudden Expansion

So, we learned about minor losses in the last class and we said minor losses would come into the picture whenever you have additional fittings in a pipe. So, we said it could be sudden expansions, contractions or gradual expansions, contractions, valves; you know elbows, turns, bends any of them could give rise to additional frictional losses and that could be huge and we need a way of a characterizing it. And we said we so, we are talking about loss coefficient ok.

Now, typically look calculation of loss coefficient is a pretty difficult and you need to depend on literature or published data to figure out what is the loss coefficient that is useful for you, except for you know sudden expansion; they exist a nice calculation. So, I think it is a good idea that we just look at it to get a feel of what it means, it also might give you a little more insight into the definition of loss coefficient.

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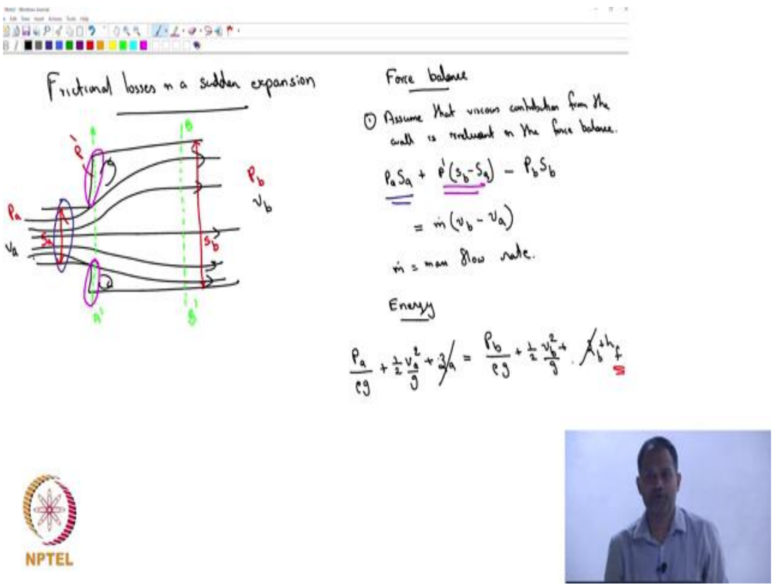

So, what we are going to do today is to first look at frictional losses in a sudden expansion. So, what we have in mind is fluid entering through a small pipe and then exiting through a large pipe. So, that is a fluid stream line at the center; others as we said would expand as it go along ok. So, that is what the fluid stream lines are going to look like. Let us say this fluid that is entering, let us say the fluid that is entering as a velocity v a and it is exiting with a velocity let us say v b where v and v b are the average velocity.

And, let us say this cross section the cross sectional area is Sa and the leaving cross sectional area is Sb. And, let us say the pressure with which is entering is P a and the pressure with which it is not leaving is P b. And, we want to calculate what might be the losses associated with it, what I am going to do is I am going to. So, dealing with differential equations in this case, it is not going to be useful because, you cannot do any simplification. So, here the flow is very complicated so, we can do an integral approach.

So, that is what we will do, let us say we will take a small section very close to the wall and developed part. So, let us call it let us say section AA prime and B B prime. So, how do I am basically looking at a control volume which is you know between A A prime and B B prime and I am going to write down a force balance and an energy balance for that region ok. So, first is the force balance so, you have a fluid entering at some pressure, fluid is leaving at some other pressure.

So, there is definitely going to be a some forces that might come from pressure, you could have shear stress the wall shear stress that is acting. So, that is something that you should consider and then the fluid is changing its momentum because its velocity is changed right. So, these are the three relevant things. What we are going to do is we are going to neglect the contribution that is coming from wall shear stress ok that is an approximation. We assume that wall or the viscous contribution really contribution from the wall is irrelevant in the force balance.

And, I can find out what is the net force acting on this fluid that is only going to come from the pressure forces. The pressure that is acting on the left hand side let us say is so, we know the pressure in the pressure of the incoming stream is P a. We do not know the pressure here which is very near the wall let me just call it P prime ok, then the force that is acting along x direction is P a times S a plus P prime times S b minus S a ok. So, there is a force acting along x direction minus the force at the section B B prime P b into S b that is ok. And, just multiplying pressure with area to get my force is equal to change in the momentum which is going to be let us say some mass flow rate m dot into v b minus v a.

$$P_a S_a + P'(S_b - S_a) - P_b S_b = \dot{m}\,(v_b - v_a)$$

So, I have said the rate of change of momentum which is I have written on the right hand side is equal to the total forces acting in that control volume where, I have neglected the force that will come from the wall region ok. And, I am doing that assuming that that contribution is going to be small, other losses are going to be large and also I know that that tau w the contribution that will come from the wall is really over a small section ok. So, that is not going to be very large compared to other losses so, that is the assumption that is going in that is.

So, m dot is the mass flow rate. Energy I will simply use a you know energy flow equation which again you have seen I will apply it between section A A prime and B B prime. So, I will say P a by rho g plus half v a square plus z a is equal to P b by rho g there is a g know g we said yeah, P b by rho g plus half v b square plus g z b plus the other losses which is what we want to calculate. Let us just applying between section A prime and B B prime. They missed something? Yes yes yes.

$$\frac{P_a}{\rho g} + \frac{v_a^2}{2g} + z_a = \frac{P_b}{\rho g} + \frac{v_b^2}{2g} + z_b + h_f$$

So, you have the fluid that is coming, it is expanding and going so, that there is a force that is going to come from that section that is neglected yeah should be P by rho. So, P by rho plus half v square is it ok, this is correct now.

Wait it cannot be correct now because this is v square so, that is v square plus second square. So, it should be g is z that sounds correct.

Oh ok. So, wait let me just check what should be then right. So, it should be P a by rho g plus v square by 2 g plus is z is equal to P b by rho g plus g this, is this now? Yeah.

So, we are looking at the total force right exerted in the flow direction so, it is just force sorry pressure times area. So, you have a this is coming from that part and this is coming from this part, pressure on the wall so ok. So, one way to argue is that I am going to look at a section which is very near to the wall, but why would pressure on the wall be 0?

No no no. So, that is you are blindly applying your Bernoulli's equation, but no why should still it be 0? It need not be 0 so, let me give you a counter example, let us say you take a fluid it is a static fluid there right. What would be the pressure on any side of the wall? Ok, so its there is nothing like pressure should be 0 on the wall, top meaning this region again oh. So, in the static case you mean here so, even if the pressure here is 0 ok, you have a hydrostatic pressure that will arise right which will act on all sides of the wall right and that pressure will continue to increase. So, the pressure need not be 0, it is completely independent of whether the fluid is moving not moving anything ok. So, the pressure could be some quantity ok.

So, the forces that are acting on a solid surface need not be just viscous, it could be pressure force also. I have not made any assumption yet, but the derivation that we are going to see you would see that it would hold better for a turbulent flow than laminar flow which I will argue later which one. So, I have a h f no now? So, so you can say that it is just a so, see P a v everything is an average quantity here ok. So, it is just writing down what is the net energy in is equal to net energy out plus extra force extra losses that is yeah.

Yes yes.

No h f is not so, there can be additional losses as we said in last time. So, the velocity magnitude is changing, the velocity direction is changing both of them will contribute to additional losses plus there will be re-circulations that will happen at the corner which will also contribute. So, what h f will give you is you know the sum of all those things yeah ok. So, the only thing is to just so, we will throw away the z a z b because we are not going to worry about the change in elevation. So, let us simplify so, there comes the second assumption ok.

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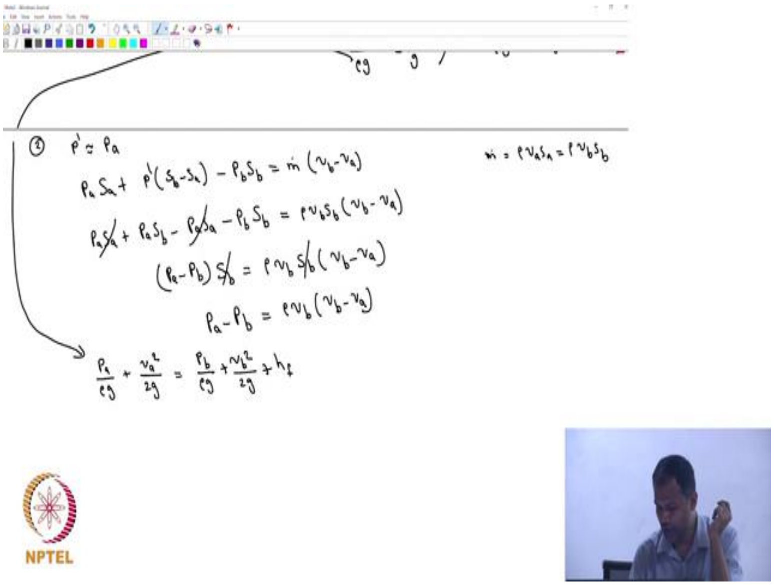

Now, we assume the about something about the wall shear stress, the second assumption is that we do not know what is P prime ok. And, I am going to claim that P prime is going to be approximately equal to P a that is the inlet pressure. Now, if you ask me why do we do that we really do not know, but if you do experiments you find that it is very close by. And, the results that you are going to get from this calculation also seem to be validating this assumption ok. So, that is the second assumption that is going in.

So therefore, I can simplify our pressure balance so, that is P a S a plus P prime into S b minus S a minus P b S b is equal to m dot into v b minus v a. So, if I substitute P prime is equal to P a P a S a plus P a S b minus P a S a minus P so, this is P a S b minus P b S b correct is equal to m dot; m dot is the mass flow rate, m dot is either rho v a S a or is also equal to rho v b S b mass flow rate ok, volumetric flow rate into density. So, I will substitute rho v b S b into v b minus v a. P a minus P b into S b is equal to rho v b S b into v b minus v a or P a minus P b is equal to rho v b into v b minus v a.

$$P' \sim P_a$$

$$P_a S_a + P'(S_b - S_a) - P_b S_b = \dot{m}(v_b - v_a)$$

$$\dot{m} = \rho v_a S_a = \rho v_b S_b$$

$$(P_a - P_b) = \rho v_b (v_b - v_a)$$

I just simplified the force balance so now, I am going to look at our energy balance ok; simplifying that P a by rho g plus v a square by 2 g is equal to P b by rho g plus v b square by 2 g plus h f; just by substituting P a minus P b and simplifying and see what where what do you get for h f.

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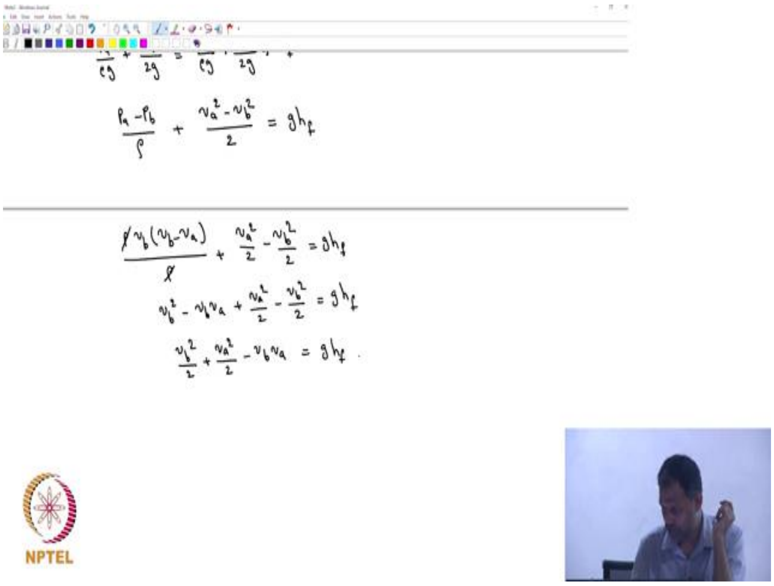

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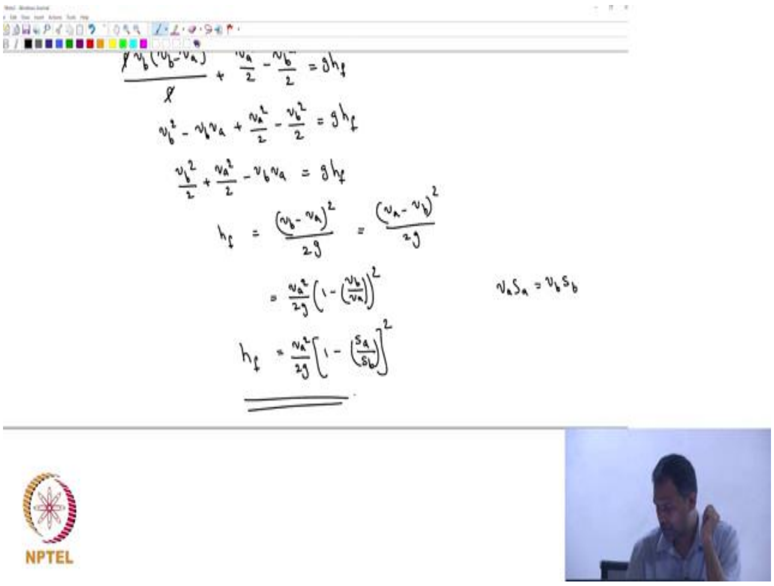

I am going to write this as v a minus v b because, v a is going to be the larger velocity; v a minus v b all squared divided by 2 g is equal to v a square by 2 g into 1 minus v b by v a whole a sorry 1 minus v b by v a square. But, we do know that v a S a is v b S b. So therefore, v a square by 2 g into 1 minus v b so, it is S a by S b square.

$$h_f = \frac{v_a^2}{2g}\left[1 - \left(\frac{S_a}{S_b}\right)\right]^2$$

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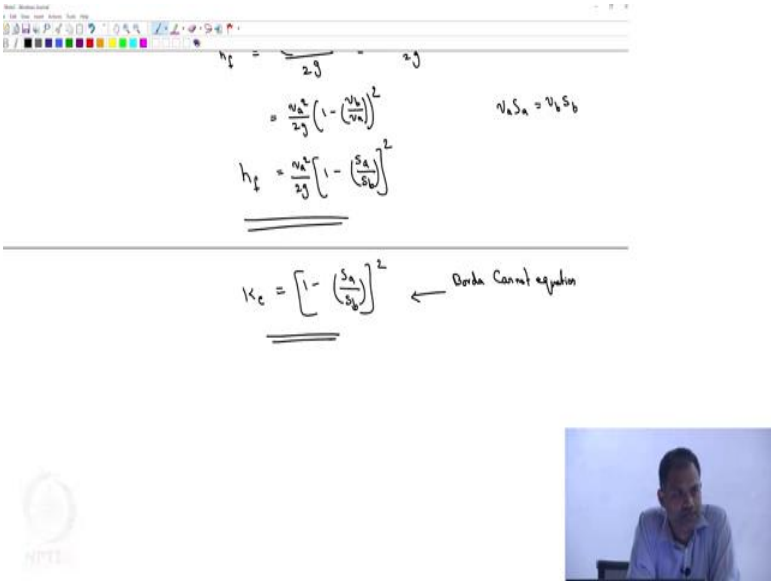

So, what would be our loss coefficient therefore, K e will be equal to so, that c expansion loss coefficient and seems to be working fine.

$$K_e = \left[1 - \left(\frac{S_a}{S_b}\right)\right]^2$$

So, the assumptions that we might have made should be ok, this equation is has a name yeah. So, so this is one of the very rare ones ok. So, again more any of the other cases as I said you may have a to just take the values from other places, but for expansion loss coefficient this equation should work.