Fluid and Particle Mechanics

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Lecture - 58
Minor Losses, Sudden Expansion and Contraction

So, the last class we looked at the effect of roughness on pipe flow and we also looked at how to deal with non circular cross sections right. So, just to write down we started by looking at laminar flow in pipes, then we realized that that is not always the case, if Reynolds number is sufficiently large then the flow does not remain laminar.

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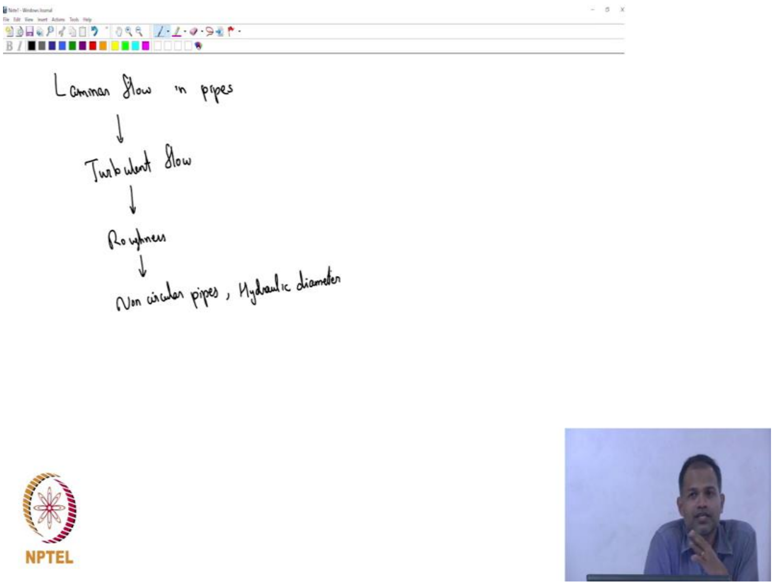

The flow goes to turbulent and therefore, it was necessary to look at the relation between friction factor and Reynolds number for turbulent flow in pipes, then we realized that if you are looking at turbulent flow then the roughness of the pipe can be important. So, we looked at roughness, we defined a roughness factor. We calculated what is the relationship between friction factor and roughness.

And then we sort of generalized our procedure by looking at noncircular pipes and to apply all the formalism that we derived for circular pipes. We actually defined a hydraulic diameter and then we said that we would just replace use the hydraulic diameter to deal with the non circular pipes. So, that has really been the course so far; so we defined a hydraulic diameter.

So, is that all that we need if you want to talk about a pipe design in a practical case. So, do we know everything now how to select a pump for a given application? Is there something that you think is missing? Something is missing and that is what is the next thing that we want to look at and that is called the minor losses.

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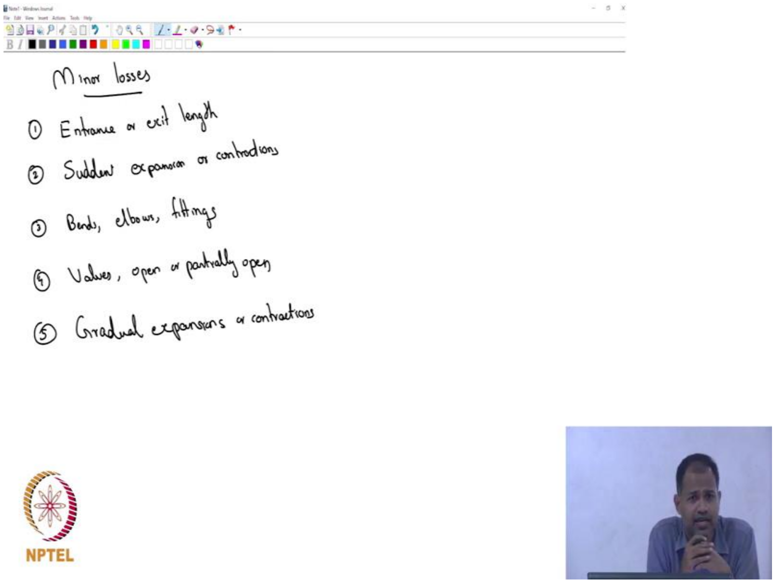

So, the minor losses is pressure drop that can happen that we have not accounted for h and the possible reasons could be what happens at the entrance or exit length. It could be due to sudden expansion or contractions. You could have bends, elbows, fittings and so, you could have valves you know open or partially open you could have a gradual expansions or contractions.

So, when you think about a general pipe system where you are pumping fluid from one place to another many of these things could come and all of them can contribute to additional pressure drop. So, we also need to know what those losses are and the losses that happen due to any of these things are called minor losses, though its called minor losses it may not be really minor ok, many times that would be the major losses. So, its important that we account for this. So, that we need to just look at some of these situations and then see what is the way in which we can really attack it. So, the first thing that we would look at entrance and exit length. Entrance length is the only thing that I will talk about.

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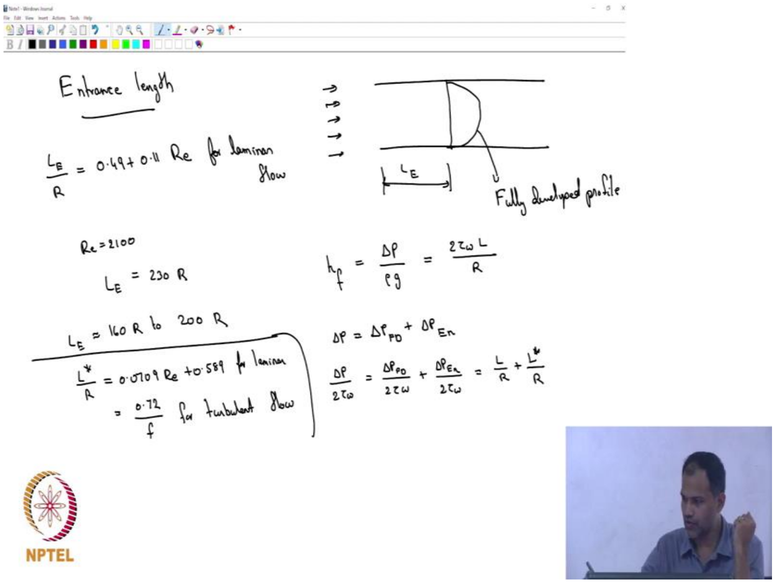

So what is entrance length? So, when we started our class you may remember that we talked about pipe and a fluid that is coming in with a uniform velocity and as soon as the fluid touches the wall, the fluid will get slowed down and then it will take a it will take some length before it will reach the fully developed flow right and the length that is required to reach the fully developed flow is called the entrance length right. So, and all our analysis so far has been assuming that the fluid flow is fully developed.

So, the flow or the pressure loss pressure loss that would happen during the entrance loss is something that we have not calculated yet and therefore, that should be accounted for additionally whenever you have. So, the point is can that be significant ok. So, let us see what are the typical entrance lengths that you will have ok.

There is no real theory, but there are correlations which one could use. So, if so let us say this is entrance length let us say L E beyond which you will start seeing fully developed profile. So, that is L E by R where R is the radius of the pipe is given by 0.49 plus 0.11 Reynolds number for laminar flow. So, if the fluid flow is laminar, then you can estimate what is the approximate entrance length.

$$\frac{L_E}{R} = 0.49 + 0.11 * Re$$

So, let us say the highest Reynolds number at which you will get laminar flow in a pipe would be approximately 2000 2100. If Reynolds number is 2100 let us say what would be

L E some 2000 into 0.11 plus 0.49 something like 200 plus comes out to be 230 R ok. So that means, is there anything? Ok. So, 230 R ok; that means, the entrance length is going to be 200 times longer you know compared to the radius. So, its not a small number; so, if you are dealing with 1 meter sized pipe the first 230 meter would be entrance length. Only after that you are going to get a fully developed profile ok.

So, therefore, the calculations are the calculations that we have done so far is not true for the first 230 R length and therefore, it has to be separately accounted for. Even in turbulent flow its L E is found to be approximately 160 R to 200 R ok. So, that is also sufficiently large. So, whether its laminar or turbulent there is enough length of the pipe in which the flow is just in its developing state. So, the way typically that you would see is that we need to talk about a head loss which is related to your pressure drop right that is the way we defined our head loss.

We can say our delta P is now coming from the fully developed flow plus a delta P that is additionally there due to the entrance effect. So, how did we write delta P in terms of tau w the wall shear stress, what is the definition of what is the connection between a head loss and wall shear stress? Is it all ok? So, therefore, its a good idea to define delta P divided by 2 tau w is delta P FD divided by 2 tau w plus delta P En divided by 2 tau w that is equal to L by R plus let us say some L star by R.

$$h_f = \frac{\Delta P}{\rho g} = \frac{2\tau_w L}{R}$$

$$\Delta P = \Delta P_{FD} + \Delta P_{En}$$

$$\frac{\Delta P}{2\tau_w} = \frac{\Delta P_{FD}}{2\tau_w} + \frac{\Delta P_{En}}{2\tau_w} = \frac{L}{R} + \frac{L^*}{R}$$

So, what I am just trying to rewrite is there the pressure difference that the pressure drop ok. Can be really thought about as an extra length and that extra length is called L star and what is what you will find in literature is expressions for L star which you can use and calculate what is the extra pressure drop that can come from entrance length. Is that ok?

So, not L star is not same as L E ok. LE is actually the original entrance length, L star is that that length which you would have contributed to the pressure drop that an extra additional length L star by R is equal to 0.0709 Reynolds number plus 0.589 for laminar flow and yeah is equal to 0.72 divided by f for turbulent flow. So, you can use those to calculate what is the additional pressure loss. So, that is what typically happens with entrance and we can talk about sudden expansion and contraction ok.

$$\frac{L^*}{R} = 0.0709 * Re + 0.589 \; (For\ laminar)$$

$$\frac{L^*}{R} = \frac{0.72}{f} \; (For\ turbulent)$$

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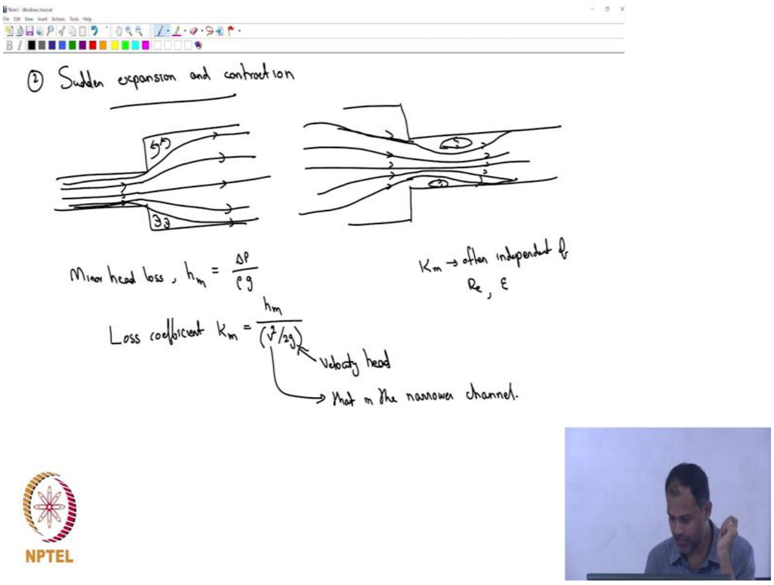

So, it is very common for example, you work on pumping fluid through some pipe and let us say it needs to be divided or it needs to be connected to a larger pipe ok. So, it is very usual that the pipe dimensions will immediately change. So, you will see that up to some length you had something and then suddenly you will connect it to a bigger pipe to which through which the fluid needs to go ok. So, that is where you have a sudden expansion.

So, what would happen is at the fluid that is coming like that? It does not stick to the wall. In fact, the one which is coming at the center will go at the center the one which is next to it will expand a little bit and go. The one which is next to it will expand more and go like that ok. So, that is what the fluid profile is going to look like ok.

So, the fluid layer that is very close to the wall would expand and then go. So, here what happens is that there will be fluid, but those fluids would be just recirculating like that.

There will be fluids which are just getting recirculated the fluid that is coming would expand and occupy the pipe only after a certain length. So, that means, that you the fluid that is occupying this region right, the region at the corners are just getting rotated, but they are not moving anywhere. So, you are actually losing additional energy because of that right. You are not getting anything from that you are having extra energy to just generate those vortices there and those losses are actually huge ok.

So, therefore, sudden expansions will result in additional losses that we have not captured yet from our friction factor calculations. So, we need a way of calculating friction factor associated with this sudden expansion. So, similarly this similar thing could happen in case of a sudden contraction. So, in if you have a pipe with a large cross section and is suddenly changing to a small cross section the fluid that is coming would start actually so the one at the center would go, the one would actually contract a little bit before its expanding something like that.

So the fluid that is coming as soon as it see a contraction it will continue like that it will contract a little bit before it expands ok. So, again you will have regions where you will have this kind of circulations, I am not going to write ok. So, this is these are called flow separated regions and we will see the and the regions why that happens later, but those regions are again created ok. So, these losses are again something that we have not considered. So, therefore, there can be expansion losses, there can be contraction losses whenever the cross section changes, which needs to be accounted for in addition to the non losses.

So, really you know whenever the so see originally for example, if you look at laminar flow, where is the friction really coming from? The friction was coming from the walls right that is what we accounted for. Now, here the additional friction is coming because either the magnitude of velocity changes or the direction of velocity changes. Actually both change here right the magnitude and the direction both of them change. So, if you look at 2 consecutive fluid elements their velocities might be different both in direction and in magnitude; that means, that they are going to experience extra friction ok. So, it is really that extra friction that is resulting in additional losses, which was not the case when we had a uniform pipe the there.

The only the only way in which the velocity is changing is when you are going perpendicular to the flat plate or perpendicular to the pipe cross section. So, that is the only way friction was exhibiting. If you are going along the pipe there was no difference in the velocity of fluid elements and there was no friction, but now that is not the case. Here any 2 consecutive elements you are looking at there is going to be difference in the velocity and if 2 things are moving with a relative velocity you would expect that there is going to be friction acting between them ok. So, therefore, whenever there is a change in the magnitude or direction of velocity you would expect that there is going to be additional losses in addition to the circulations that we have already talked about.

So, there are like two different contributions really, one is because of the change in the magnitude and direction of the velocity and second is because the flow is separated like you know near the corners or near this corner, which I have drawn here. So, these are various contributions that are going to give additional pressure drops and are going to be something major also. So, again the one cannot really do any calculations and figure out these things, so it is very difficult to do it. So, typically one goes with again correlations and often you would write that you will talk about head loss let us say minor head loss h m is yeah this is the pressure drop ok.

So, that was, so our original definition of h f the friction factor were we defined it as delta P by rho g. I am not including delta is at because delta is at contribution is going to be small for any of these changes. So, h m is delta P by rho g and we often define a loss coefficient K m as h m divided by v square by 2 g.

So, v square by 2 g is the velocity head. So, we often represent our minor head loss as a ratio of minor head loss to the velocity head and therefore, that at K m is going to be a non dimensional quantity and the numbers that you will have often access to is K m. So, the moment you have K you can multiply it with the v square by 2 g the velocity head and get your friction loss yeah.

$$h_m = \frac{\Delta P}{\rho g}$$

$$K_m = \frac{h_m}{\left(\frac{V^2}{2g}\right)}$$

So, this is loss coefficient that is velocity head. K m again often independent of Reynolds number roughness and so on. So, you will see that the dependencies are v; so you will actually get it as some number like 0.1 or 10 or things like that ok. It is very it is a function of the geometry. So, for example, you will find that K m could be dependent upon the you know the radius of the smaller section and the radius of the bigger section and so on, but you will not generally see it as written as functions of the flow field or Reynolds number. And the other thing is that what is the v that I have written there velocity head, what is that v?

Student: Average velocity.

Average velocity, but average velocity where? So, there is a convention involved and that convention is that the average velocity in the smaller pipe ok. So, that is what typically one should use. So, in the expansion it will be the inlet average velocity in the contraction it will be the outlet average velocity. So, that v associated is that in the narrower channel. So, we will look at expansion a little more detail maybe next class, but before that let us look at other things bends, elbows, fittings bend why would you know.

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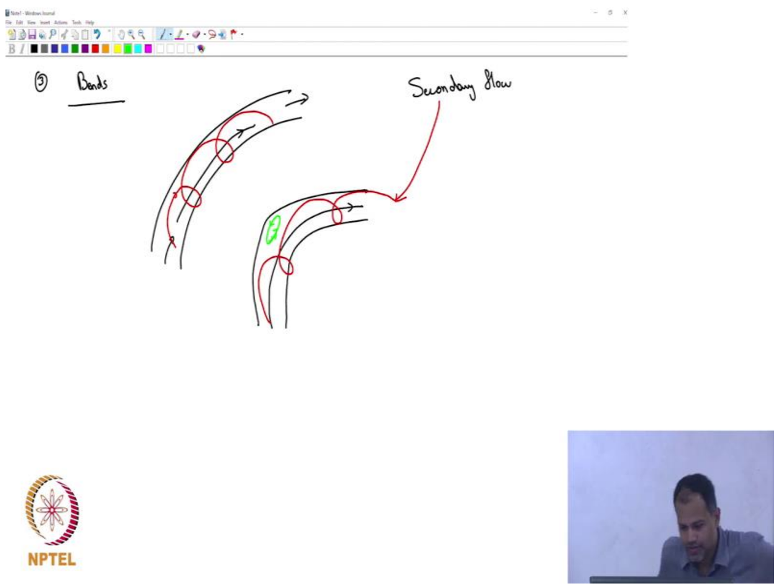

Let us say a bend in the pipe or an elbow that you will collect. For example, you need to take the pipe like this and then take a corner and go. Why would that cause additional losses? Something here the fluid is going like that. It could be a sharp 90 degree bend or anything. So, what would the fluid experience when it goes through something like this? The additional force that it would experience would be centrifugal force ok.

So, in addition to the fluid that is going to go like that because the fluid experiences centrifugal force it will start exhibiting additional flows ok. We have actually seen one example when we did Taylor Couette flow where we had flow between 2 parallel plates and one sorry 2 concentric cylinders and one of the cylinder was rotating and we saw that when the Reynolds number was really small it was only like this, but as it was increasing additional flow fields developed ok.

So, similarly what happens is that even though the flow field is the primary flow is along the bend it will develop a secondary flow and that secondary flow will be actually you know across the section. So, something like that you know some sort of a helical pattern. So, the fluid will start exhibiting flow like that. So, that is something that we have not accounted for yet ok; so that gives rise to additional losses friction losses yeah. Also depending up on how strong this bend is you could have again flow circulation. So, maybe I should draw another one. So, the primary flow is like that then it could develop a secondary flow ok. So, that is secondary flow and then also you could have flow separations like this ok.

Near the bend there could be certain regions which are not moving they would be separated from the main flow. So, these all will additionally contribute to frictional losses and therefore, you need to take care of. So, you when you design something you need to k. Now, therefore, if you are actually giving a bend you should think about what is the loss. So, again you will define a loss coefficient ok. You have to find out what is the loss coefficient for a 90 degree bend or for a smooth bend and then add that to the total frictional losses.

So, you might want to for example, think about I will just avoid having all the you know bends at all. I will try to give as smooth as possible so, that you can reduce the frictional losses, but many times that may not be a good idea because you need additional space you are thinking about giving longer curvatures; that means, longer pipes. So, that long length would also cause you know additional pressure drop.

So, you will have to estimate if the space is available do you need actually a long pipe ok. With the minimum you know bend or do you need actually a short pipe with a 90 degree bend what is best required; so you need to do a comparison. So, again the cost will the operating cost will change the initial you know investment will change. So, you need to know which one is suitable. So, that is why it becomes important to estimate these things and small differences here and there over a month would make a lot of difference. So, that is why people real worry about what are the numbers associated with, any questions.

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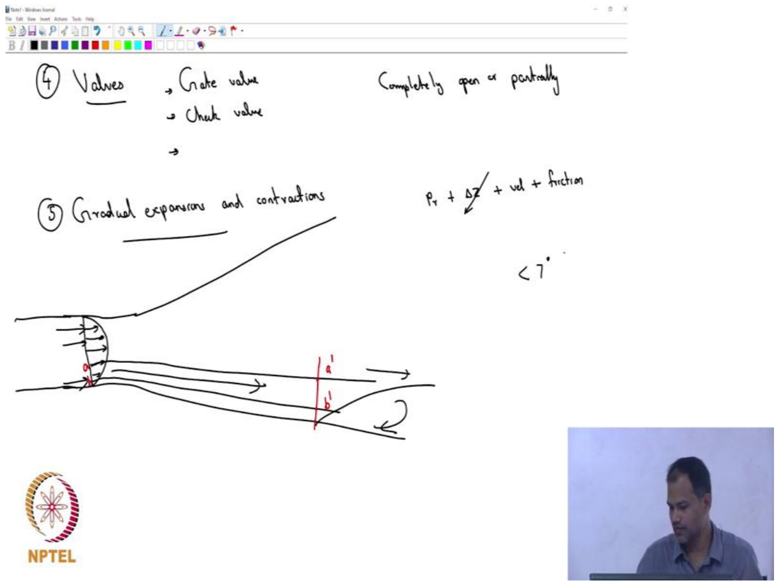

That is about bends valves; valves you must already be you have submitted assignment no i have not looked at it yet. So, you were; so dealt with different kinds of valves right what all valves did you write about? Gate valve ok then?

Check valve, then maybe we should have had a surprise quiz on valves today yeah therefore, for final exam there will be questions from assignments. So, yeah so, there several like you know disc valve, butterfly valve, globe valve I think all of you must have written different things. So, and when you have written it you might have probably found that certain kinds of valves are useful for gases, certain kind of valves are useful for liquids, some of them have a better control, some of them are like you know completely open and completely closed or you want to fine control over your changing. Why do we need flow valves by the way?

To control the flow rate we always. So, for example, you know you have you know for a plant and there are fluids that are coming from outside ok. Let us say for a heat exchanger you are taking water from the nearby river or let us say you have a petroleum which is getting you know which is coming from some port somewhere which is getting injected.

So, there are lots of external influences and all of them are going to affect the fluid properties that are coming in the temperature might be different, the composition might be different. So, the equipment that is under operation must be you must be able to change your flow rate. So, that the equipment works the best way ok. So, you would always want to therefore, control your flow rate that is why you would use valves and depending upon what fluid it is you may use different valves.

The valves may be completely open or it may be partially open and its important whether it is completely open or partially open or partially because often a partially opened valve will generate more pressure drop than a completely opened valve. So, therefore, one needs to know how much it is open and then the pressure drop calculation should really depend upon that. Yeah and the valves the pressure the again the frictional losses are coming because of its complicated geometry ok.

So, you must have seen all sort of complicated geometries associated with for example, a globe valve and so on or a butterfly valve ok. There will be the geometry is complicated the velocity is changing quite dramatically in the valve the both magnitude and direction there could again be regions of separation. So, all of them will contribute additional losses and all of them will be calculated based on the loss coefficient that you get.

The loss coefficient just the way we defined for our sudden expansion or contraction; we defined a loss coefficient. Similarly when you buy a valve from the manufacturer you will get what is the loss coefficient or in other words you should ask them about it they would have done an experiment and would have calculated.

So, you get those numbers and use it in your design that is the way you should go ahead those numbers may not be so accurate and therefore, you should give always some margin, say typically the error could be as large as 50 percentage in those numbers because it depends all lot on you know the flow conditions. So, it might be very different the way the manufacturer would have calculated the loss coefficient, the conditions may be very different from your operating conditions. So, you should worry about those details, but typically there are some guidelines would be available which you can take for granted and do the first calculation; so valves and gradual expansions and contractions.

So, we talked about sudden expansion and contraction a pipe a small pipe suddenly getting enlarged. So, you would think that you know and you realize that yeah of course, the major losses are coming because the you know the direction the magnitude is suddenly changing also there are regions of the separation and so on. So, why should we have a sudden expansion or a sudden contraction?

We would rather go for something that is smoothly changing. So, that we can avoid the you know sudden changes in the velocity and therefore, reduce the frictional losses. The idea is ok except that the feasibility is very little ok. So, let us see if we talk about a sudden expansion, sorry its not sudden expansion a gradual expansion; fluid is coming this side.

So, the because the its an expansion ok, the velocity downstream would be smaller or bigger you have an increased area. So, the velocity is going to be smaller or in other words the velocity head goes down and we have seen the energy equation now the energy equation was what? There is a pressure head plus there is a contribution that will come from the elevation, then there is a velocity head and these 3 basically balance or in or you could have a friction right. These are the 4 contributions that would have come up in your energy equation.

Now, we are talking about something, let us say we do not care about the elevation change ok. So, you could have a pressure head and a velocity head. So, if the velocity comes down that can get converted into a pressure that is what Bernoulli's equation really says right velocity decreases the pressure increases. So, what happens is that if you have an expansion many much of the kinetic energy can actually get converted into a pressure form ok. It becomes pressure head, but the difference here is that if you look at the fluid that is coming in the velocity profile is going to be that right. The velocity at the center is large the velocity near the wall is small. So, let us draw 2 streamlines maybe one is that and one is very close ok.

So, let us call this a so difficult this is a and this is b and somewhere else this is this a prime, this is b prime ok. So, a prime is a streamline, b plus b prime is a streamline. At location a the velocity is going to be definitely larger than the velocity at location b agreed because of the presence of the wall. At location a not a prime at location a which is also actually part of that uniform pipe a is slightly away from the wall and therefore, a will have a larger velocity then if I if I look at a fluid particle at location a, it will have a larger velocity than velocity of a fluid particle that is located at b or in other words the velocity head of particle a at location a is going to be larger than at b.

Now both a and b are the fluid particle velocities are going to slow down as it goes downstream ok. By the time it reaches a prime, b prime part of the velocity head would have been converted into pressure head agreed. Now, it so happens that the pressure itself is not going to change much across the cross section. If you look at the pressure at location a prime and pressure at location b prime the pressure remains almost constant.

And the only way that would have happened is that, because the fluid particle at location b had a smaller velocity head it would have lost much more than the one which is at a prime along the streamline, or you could say that for example, the fluid particle at location b would have come to rest by the time it reached b prime, but the one at location a would still be moving ok, because the velocity head is actually getting converted into pressure head.

So, because a was coming with a larger velocity the particle at location a was coming with a larger velocity it gets converted into pressure head, the one with at b also got converted into pressure head, but the velocity at location a would be larger than the velocity at location b even though both of them have been the velocity heads have been converted into pressure head ok.

Actually taking that to the limit in which you would see that if b prime actually comes to you know a stagnant situation if b prime actually comes to a halt a prime would still be moving. So, what you would see is that as you proceed along this direction you will see that b prime gets slower and slower while a prime will start moving faster and I mean compared to b prime a prime will be moving faster ok.

So, the relative velocity between a prime and b prime keeps increasing. After some point what happens is that the b prime will come to 0 velocity while a prime would still be moving and beyond some point you would see that b prime would actually move in the opposite direction compared to a prime. Now, whatever I said just now we will see how the equations predict that may be when we do the boundary layer for the time being let us just get this fact. That therefore, there can be a region where the fluid will actually start flowing in the opposite direction even though the main flow is in a different direction. So, along the center the fluid would be actually flowing from left to right, but near the wall the flow could be in the opposite direction ok.

So, that is coming because of this differences in which how much velocity head is converted getting converted into pressure head. So, the numbers let us say we will see later. So, what happens is that because of that the fluid. So, even though outside this layer the fluid is in one direction, inside the fluid could actually start going in the opposite direction. So, that is what you really call as flow separation. So, whenever I do flow separation there; there; there; there, you know here in all these cases this is what is happening even though you have a major flow a primary flow there could be a flow separation which would which would be in an opposite direction and can result in much more pressure drop ok.

So, even though you would want to change your sudden expansion or a sudden contraction there could still be regions where this flow separation can occur and it turns out that if you want to really stop flow separation the angle of divergence that you need is less than 7 degree. So, if it is less than 7 degree you will be able to reduce the that separation, but otherwise it is difficult to stop the separation; that means, if you want to get rid of a sudden expansion you should think about an expansion which is having a 7 degree; that means, that expansion would be over a very large length because the angle is really small where you can stop separation ok.

So, you if you want to replace a sudden expansion with a gradual expansion your pipe should be really from you know the expansion should be happening over a really long length. Now, that becomes a problem because you want to get a pipe which is slowly changing in its cross section over a large length that is going to be really a custom made thing that you are asking for and you need to also plan how to you know space it out and so on. So, other course will actually become important and material cost will also go up ok. So, therefore, it is also not always a good idea to replace a sudden expansion with a gradual expansion unless its you know really going to help us.

So, therefore, you would want to again compare whether you want sudden expansion or a gradual expansion when you are designing something ok. So, it is all due to the separate the fluid that get separated. So, these regions where the reverse flow happens is called flow separation ok. Its separated from the main flow and that flow separation is a reason for additional losses. So, when you add when you install something the thing that you would look for is to not have any flow separation. Do not worry if you did not get the idea of why it happens, we will learn about it in detail, we go to the boundary layer theory.

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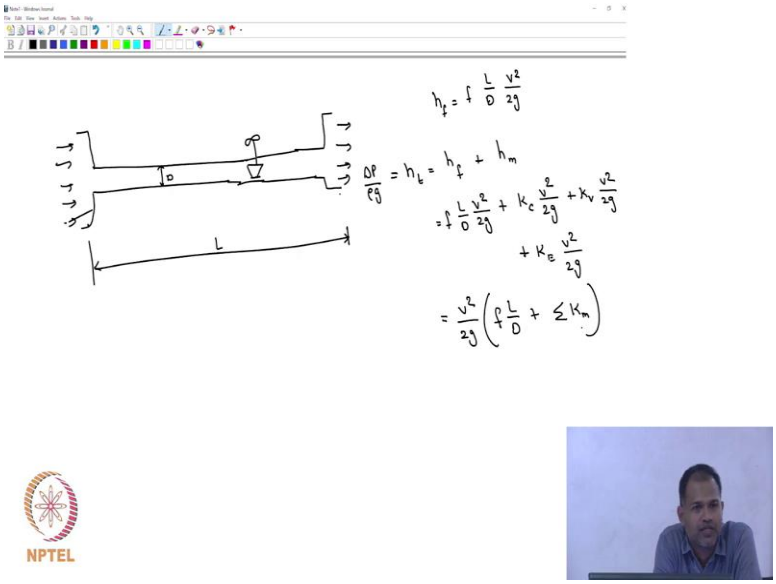

So, let us look at a situation fluid is coming in a larger pipe, then it got contracted and let us say you have some valve and then the fluid is going and then its suddenly expands. Let us say this is the particular section that you are interested in and you want to know what is the pressure drop across this section, let us say this is very small so; some valve is there, so something is coming here fluid and going out.

So, if you did not have that valve and if you did not have that expansion and contraction you would have just written down your usual loss which we called as the friction loss h f as f times L by D times v square by 2 g right. Where we relate now f with the Reynolds number, so you would calculate it. So, remember this diameter is that diameter, this length is the length of the small pipe, v is the average velocity in the small pipe.

Now, you have got an expansion, you have got a contraction and you have a valve. So, you need to add 3 minor losses to this expression. So, your total loss h t let us say we can write it as h f plus h m the friction loss that is coming from the fully developed part of the flow plus additional minor losses and h f you will write it as f into L by D into v square by 2 g and minor losses could be many of them. So, you have a minor loss that is going to come from the contraction which you will write sorry yeah contraction which you can write as K c into v square by 2 g. Remember how we defined our loss coefficient, we defined our loss coefficient as the minor loss divided by the velocity head.

So, the loss coefficient times the velocity head is going to give you the minor loss. So, K c which is the contraction. So, I have the c subscript is for contraction K c into v square plus 2 g plus there is a valve. So, let us say K v into v square by 2 g plus there is an expansion K e into v square by 2 g. So, that is the total pressure drop or the total head loss which is equal to delta P by rho g; so, you know the pressure drop. Can you if you like you can write it as v square by 2 g into f L by D plus sum of all minor losses ok.

$$\frac{\Delta P}{\rho g} = h_t = h_f + h_m = \frac{fLV^2}{2Dg} + K_c\frac{V^2}{2g} + K_v\frac{V^2}{2g} + K_e\frac{V^2}{2g} = \frac{V^2}{2g}\left(\frac{fL}{D} + \sum K_m\right)$$

So, all we have said today is essentially how to attack this problem ok. If you are you have got a pipe design, then the way to proceed would be look at the loss coefficients associated with each element that you would put it in the pipe and add it your frictional losses that is a take home message hm .